



Name: Carlos Humberto Buitrago González Id: A01570142 Date: 26/09/19

1. If $f(5)=1$, $f'(5)=6$, $g(5)=-3$, $g'(5)=2$. Find the values of

a) $(f \cdot g)'(5) = f'(5)g(5) + f(5)g'(5) = (6)(-3) + (1)(2) = -18 + 2 = -16$
 b) $(f/g)'(5) = \frac{f'(5)g(5) - f(5)g'(5)}{(g(5))^2} = \frac{(6)(-3) - (1)(2)}{(-3)^2} = \frac{-18 - 2}{9} = -\frac{20}{9}$
 c) $(g/f)'(5) = \frac{g'(5)f(5) - g(5)f'(5)}{(f(5))^2} = \frac{(2)(1) - (-3)(6)}{1^2} = 20$

2. If $f(3)=4$, $g(3)=2$, $f'(3)=-6$ and $g'(3)=5$, find the following values

a) $(f+g)'(3) = f'(3) + g'(3) = -6 + 5 = -1$
 b) $(f \cdot g)'(3) = f'(3)g(3) + f(3)g'(3) = (-6)(2) + (4)(5) = -12 + 20 = 8$
 c) $(f/g)'(3) = \frac{f'(3)g(3) - f(3)g'(3)}{(g(3))^2} = \frac{(-6)(2) - (4)(5)}{2^2} = \frac{-12 - 20}{4} = -\frac{32}{4} = -8$

3. If $h(x) = f(x)g(x)$, use the table to find $h'(-1)$, $h'(0)$ and $h'(1)$

x	f(x)	f'(x)	g(x)	g'(x)
-1	2	1	1	2
0	-1	0	-1	3
1	2	-1	0	5

$h'(1) = g(1)f'(1) + f(1)g'(1) = (0)(-1) + (2)(5) = 10$

4. If $h(x) = f(x)/g(x)$, use the table to find $h'(-1)$, $h'(0)$ and $h'(1)$

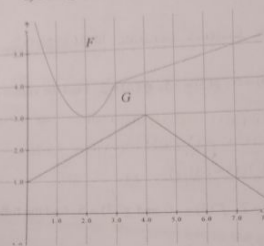
x	f(x)	f'(x)	g(x)	g'(x)
-1	2	1	1	2
0	-1	0	-1	3
1	2	-1	2	5

$h'(1) = \frac{g(1)f'(1) - f(1)g'(1)}{(g(1))^2} = \frac{(2)(-1) - (2)(5)}{2^2} = \frac{-2 - 10}{4} = -\frac{12}{4} = -3$

5. Considering that $P(x) = F(x)G(x)$ y $Q(x) = F(x)/G(x)$, where F and G are functions whose graphs are shown below.

a) Find $P'(2)$

b) Find $Q'(7)$



$f(2) = 5$
 $f'(2) = 4$
 $g(2) = 2$
 $g'(2) = 1$

a) $P'(2) = f'(2)g(2) + f(2)g'(2) = (4)(2) + (5)(1) = 13$

b) $Q'(7) = \frac{f'(7)g(7) - f(7)g'(7)}{(g(7))^2} = \frac{(1)(1) - (5)(-1)}{1^2} = 6$

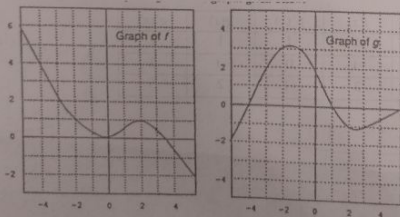
6. Consider that $h(x) = f(g(x))$, find $h'(-1)$, $h'(0)$ and $h'(1)$

x	f(x)	f'(x)	g(x)	g'(x)
-1	2	1	1	2
0	-1	0	-1	3
1	2	-1	0	5

$h'(-1) = f'(g(-1))g'(-1) = f'(1)(2) = 2$
 $h'(0) = f'(g(0))g'(0) = f'(-1)(3) = 0$
 $h'(1) = f'(g(1))g'(1) = f'(0)(5) = -1$

$h'(-1) = 2$
 $h'(0) = 0$
 $h'(1) = -1$

7. Consider that $h(x) = f(g(x))$, where f and g are functions whose graphs are shown below.



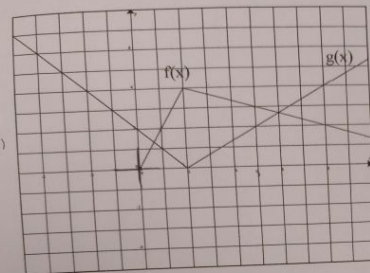
$g(-2) = 3$
 $f(3) = 1$
 $g(3) = -1$
 $f(-1) = 2.5$
 $h(-2) = 1$
 $h(3) = 2.5$

- a) Evaluate $h(-2)$ and $h(3)$
- b) Is $h'(-3)$ positive, negative or zero? Explain your answer. - 2 e 1 0
- c) Is $h'(-1)$ positive, negative or zero? Explain your answer. - 2 e 1 0

$g'(-1) = 3$
 $f'(-1) = \text{positive}$

8. If $f(x)$ and $g(x)$ are the functions whose graphs are shown, let $u(x) = f(x) \cdot g(x)$ and $v(x) = f(x)/g(x)$

a) Find $u'(1)$ b) Find $v'(5)$



$f(1) = 3$
 $f'(1) = -1/3$
 $g(1) = 2$
 $g'(1) = 1/3$

a) $u'(1) = f'(1)g(1) + f(1)g'(1) = (-1/3)(2) + (3)(1/3) = -2/3 + 1 = 1/3$
 b) $v'(5) = \frac{f'(5)g(5) - f(5)g'(5)}{(g(5))^2} = \frac{(1)(5) - (5)(1)}{5^2} = \frac{0}{25} = 0$



Rules of Differentiation- Product & Quotient Rule practice
By: Ing Ziad Najjar



OK

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Find the derivative of the following functions: **BOX YOUR FINAL ANSWER**

- 1) $f(x) = 4x^3(2x+5)^4$
 $f'(x) = (2x+5)^4(12x^2 + 4x^3(8(2x+5)^3))$
 $f'(x) = (2x+5)^4(12x^2 + 32x^3(2x+5)^3)$
- 2) $f(x) = 2x\sqrt{4x-2}$
 $f'(x) = (4x-2)^{1/2} + 2x \cdot \frac{1}{2}(4x-2)^{-1/2} \cdot 4$
 $f'(x) = \sqrt{4x-2} + \frac{4x}{\sqrt{4x-2}}$
- 3) $f(x) = (4x+1)^3(3-x^2)^4$
 $f'(x) = 3(4x+1)^2(12(4x+1)^2 + 4(3-x^2)^3(-2x))$
 $f'(x) = 3(4x+1)^2(12(4x+1)^2 - 8x(3-x^2)^3)$
- 4) $f(x) = x^3(x^2+1)^4$
 $f'(x) = 3x^2(x^2+1)^4 + x^3 \cdot 4(x^2+1)^3 \cdot 2x$
 $f'(x) = 3x^2(x^2+1)^4 + 8x^4(x^2+1)^3$
- 5) $f(x) = \frac{(x+1)^8(x^2-3)^4}{4(x^2-3)^3(4x+1)^2 + 8x(4x+1)^3(x^2-3)^2}$
- 6) $f(x) = \frac{2x-3}{3x(2x-3)^4 + 12x^2(2x-3)^3}$
- 7) $f(x) = \frac{(2x-1)^3}{2x^3}$
 $f'(x) = \frac{3(2x-1)^2(2) \cdot 2x^3 - (2x-1)^3 \cdot 6x^2}{(2x^3)^2}$
 $f'(x) = \frac{12x^3(2x-1)^2 - 6x^2(2x-1)^3}{4x^6}$
- 8) $f(x) = \frac{2x^2}{(x^2+1)^5}$
 $f'(x) = \frac{4x(x^2+1)^5 - 2x^2 \cdot 5(x^2+1)^4 \cdot 2x}{(x^2+1)^{10}}$
 $f'(x) = \frac{4x(x^2+1)^5 - 20x^3(x^2+1)^4}{(x^2+1)^{10}}$
- 9) $f(x) = \frac{8x}{(x+1)^2}$
 $f'(x) = \frac{8(x+1)^2 - 8x \cdot 2(x+1)}{(x+1)^4}$
 $f'(x) = \frac{8(x+1) - 16x}{(x+1)^3}$
- 10) $f(x) = \frac{1-2x}{(1-2x)(8(2x-3)^4) - 2(2x-3)^4(-2)}$
 $f'(x) = \frac{-2(1-2x) + 2(2x-3)^4}{(1-2x)^2 + 12(2x-3)^4}$
- 11) $f(x) = \frac{(2x+1)^5}{2x(2x+1)^2 - (2x+1)^2}$
 $f'(x) = \frac{5(2x+1)^4(2) \cdot 2x(2x+1)^2 - (2x+1)^5 \cdot 2}{(2x(2x+1)^2 - (2x+1)^2)^2}$
 $f'(x) = \frac{10x(2x+1)^6 - 2(2x+1)^5}{(2x(2x+1)^2 - (2x+1)^2)^2}$

13) Find the equation of tangent line to the given function at the indicated point:

$f(x) = x(2x-3)^4$ at $x=1$
 $f(1) = 1(2(1)-3)^4 = 1(-1)^4 = 1$
 $f'(x) = (2x-3)^4 + 4x(2x-3)^3(2)$
 $f'(1) = (-1)^4 + 8(1)(-1)^3 = 1 - 8 = -7$

14) Find the equation of tangent line to the given function at the indicated point:

$f(x) = \frac{(2x-1)^5}{x}$ at $x=1$
 $f(1) = \frac{(2(1)-1)^5}{1} = 1$
 $f'(x) = \frac{5(2x-1)^4(2) \cdot x - (2x-1)^5 \cdot 1}{x^2}$
 $f'(1) = \frac{10(1)(1)^4 - (2(1)-1)^5}{1^2} = \frac{10 - 1}{1} = 9$

- 1) $(2x+5)^4(12x^2 + 32x^3(2x+5)^3)$
 $(2x+5)^4(12x^2 + 32x^3(2x+5)^3)$
- 2) $\sqrt{4x-2} + \frac{4x}{\sqrt{4x-2}}$
- 3) $3(4x+1)^2(12(4x+1)^2 - 8x(3-x^2)^3)$
- 4) $3x^2(x^2+1)^4 + 8x^4(x^2+1)^3$
- 5) $\frac{10x(2x+1)^6 - 2(2x+1)^5}{(2x(2x+1)^2 - (2x+1)^2)^2}$
- 6) $\frac{-2(1-2x) + 2(2x-3)^4}{(1-2x)^2 + 12(2x-3)^4}$
- 7) $\frac{10x(2x+1)^6 - 2(2x+1)^5}{(2x(2x+1)^2 - (2x+1)^2)^2}$
- 8) $\frac{4x(x^2+1)^5 - 20x^3(x^2+1)^4}{(x^2+1)^{10}}$
- 9) $\frac{8(x+1) - 16x}{(x+1)^3}$
- 10) $\frac{8(1-2x)(2x-3)^3 + 20x^3(2x-3)^2}{(1-2x)^2}$
- 11) $\frac{-2x(2x+1)^5 + 10x^2(2x+1)^4}{(2x+1)^{10}}$
- 12) $\frac{10x(2x+1)^6 - 2(2x+1)^5}{(2x(2x+1)^2 - (2x+1)^2)^2}$
- 13) $\frac{10(1)(2(1)-1)^4 - (2(1)-1)^5}{1^2} = \frac{10 - 1}{1} = 9$
- 14) $\frac{10x(2x+1)^6 - 2(2x+1)^5}{(2x(2x+1)^2 - (2x+1)^2)^2}$

Rules of Differentiation- The Natural Logarithm Function

Selected by: Ing. Rosario I. González Canales
Reference: Taylor, Claudia D. y Gilligan, Lawrence. Applied Calculus. 4th edition. Pacific Grove: Brooks/Cole, 1996

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You must remember the following rule:

$$\frac{d(\ln x)}{dx} = \frac{1}{x} \quad \text{or} \quad \frac{d[\ln(u)]}{dx} = \frac{1}{u} \frac{du}{dx} = \frac{u'}{u}$$

I. Find the derivative for each of the following functions, frame your final answer.

1. $y = \frac{\ln 2x}{\ln 3x}$ $\frac{\frac{1}{x} \ln 2 - \frac{1}{3x} \ln(2x)}{(\ln(3x))^2} = \frac{\ln(3/2)}{x \ln^2(3x)}$	2. $f(x) = \sqrt{1 + \ln x}$ $f'(x) = \frac{1}{2\sqrt{1 + \ln x}}$ $f''(x) = \frac{1}{2} \frac{-1}{(1 + \ln x)^{3/2}} = -\frac{1}{4(1 + \ln x)^{3/2}}$
3. $h(x) = \ln(e^x)$ $h'(x) = 1$	4. $g(x) = x^2 + \ln x$ $g'(x) = 2x + \frac{1}{x}$
5. $y = \frac{\ln x}{x^2 + 1}$ $\frac{(\frac{1}{x})(x^2+1) - \ln x(2x)}{(x^2+1)^2} = \frac{1/x + 2x \ln x}{x^2+1}$	6.

II. Solve the following exercises, remember to show your procedure

1. The demand for a certain article is given by $p = 50 - 10 \ln x$, where x represents the price of the article and $x \geq 1$. Determine the rate of change of the demand with respect of the price.

$$p = 50 - 10 \left(\frac{1}{x}\right)$$

$$p' = \frac{10}{x}$$

2. The revenue by selling "x" hundreds of articles is given by $R(x) = 60\sqrt{\ln x}$, where $x \geq 1$ and R is measured in hundreds of dollars. Find the marginal revenue for 400 items.

$$R(x) = 60(\ln x)^{1/2}$$

$$R'(x) = \frac{30}{x\sqrt{\ln x}}$$

$$\frac{30}{400\sqrt{\ln 400}} = 0.0306$$

$$R'(x) = \frac{30}{x\sqrt{\ln x}}$$

Rules of Differentiation- Exponential Functions

By:

Ing Ziad Najjar

Name: Cecilia Humberto Briceno G22 ID: A0570146 Date: 2/10/19

Find the derivative of the following functions: **BOX YOUR ANSWER**

If $f(x) = e^{u'}$ then $f'(x) = U' \cdot e^u$

1) $y = \frac{3}{e^{2x^2}}$ $y' = 3e^{-2x^2}$ $y' = \frac{-12x}{e^{2x^2}}$	2) $y = \frac{e^{x^2}}{2x}$ $u = e^{x^2}$ $v = 2x$ $u' = 2x e^{x^2}$ $v' = 2$ $(2x)(2x e^{x^2}) - (e^{x^2})(2)$ $\frac{4x^2 e^{x^2} - 2e^{x^2}}{4x^2}$
3) $y = e^{3x}(2x-1)^4$ $4e^{3x}(2x-1)^4 + (2x-1)^3(2) + (2x-1)^4(3e^{3x})$ $8e^{3x}(2x-1)^3$	4) $f(x) = \frac{e^{2x}}{6} + 2x^5$ $u^1 = e^{2x}$ $u^2 = 2e^x$ $v^1 = 6$ $v^2 = 0$ $\frac{1}{6}e^{2x} + 2x^5$ $\frac{e(2e^x) - (e^{2x})(0)}{36}$ $\frac{12e^2 e^x}{36} = \frac{1}{3}e^{2x} + 10x^4$
5) $f(x) = e^{(x^2+3x)^4}$ $u = (x^2+3x)^4$ $u' = 4(x^2+3x)^3(2x+3)$ $v' = 4(2x+3)(x^2+3x)$ $f'(x) = (8x+12)(x^2+3x)^3 e^{(x^2+3x)^4}$	

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Name Carlos Humberto Briceno Gonzalez Mat. AV03014

I. Determine if true or false for each of the following statements (5 points each)

1. T The derivative of $y = e^{-x} - e^{-x^2}$ is $y' = -e^{-x} - 2e^{-2x}$

2. F The derivative of $y = \frac{1}{2} \ln(x-4)^{3/2}$ is $y' = \frac{3}{4} \ln(x-4)^{3/2}$
 $u = (x-4)^{3/2}$ $u' = \frac{3}{2}(x-4)^{1/2}$

3. F If $s(t)$ is the function of position of an object in motion, then $a(t) = s''(t)$ is equal to the function of the acceleration of the object.

4. T If the velocity of the car is a function of time, then the derivative of this function with respect to time, describes the acceleration of the car.

II. Circle the right answer. (10 point each)

1. C The derivative for $y = 2e^{3/x}$ is:

- A) $y' = 2e^{3/x}$ B) $y' = 2e^3$ C) $y' = -\frac{6e^{3/x}}{x^2}$ D) $y' = 6x^2 e^{3/x}$

2. C The derivative for $y = \ln \sqrt{2x-4}$ is:

- A) $y' = \frac{1}{2x-4}$ B) $y' = \frac{1}{2} \ln(2x-4)^{-1/2}$

- C) $y' = \frac{1}{2} \ln \frac{2}{\sqrt{2x-4}}$ D) $y' = \frac{1}{x-2}$

3. C If the equation that gives the velocity of an object is $v(t) = 2t^3 e^{6t}$, then the equation that gives the acceleration is:

- A) $a(t) = 6t^2 e^{6t} (2t+1)$ B) $a(t) = 6t^2 e^{6t}$
 C) $a(t) = 36t^2 e^{6t}$ D) $a(t) = 12t^3 e^{6t}$

$$(2t^3)(6e^{6t}) + e^{6t}(6t^2)$$

$$12t^3 e^{6t} + 6t^2 e^{6t}$$

$$6t^2 e^6 (2t+1)$$

III. Answer the following questions.

1) Find the SLOPE of the line tangent to $y = \frac{e^{3-2x}}{6}$ at $x = \frac{3}{2}$ (20 points)

$$u = e^{3-2x} \quad u' = -2e^{3-2x}$$

$$v = 6 \quad v' = 0$$

$$e^{3-2x} = \frac{-2e^{3-2x}}{6}$$

$$\frac{-2e^0}{6} - \frac{-2(1)}{6} = \frac{6(-2e^{3-2x})}{36}$$

$$\frac{-2e^0}{6} - \frac{-2(1)}{6} = -\frac{1}{3}$$

$$\frac{-2e^{3-2x}}{6} = \frac{-2e^{3-2x}}{6} = -\frac{1}{3}$$

2) Find the derivative of $f(x) = \frac{(2x-1)^5}{x}$ (15 points)

$$u = (2x-1)^5 \quad u' = 10(2x-1)^4$$

$$v = x \quad v' = 1$$

$$(x)(10(2x-1)^4) - (2x-1)^5(1)$$

$$f'(x) = \frac{10x(2x-1)^4 - (2x-1)^5}{x^2}$$

3) Find the derivative $g(x) = 3x^2 + \frac{1}{e^{2x}} + \ln(4x^2+3) + e^{\frac{8x}{x}}$ (15 points)

$$g'(x) = 6x - 2e^{-2x} + \frac{8y}{4x^2+3}$$

$$g'(x) = 6x - \frac{2}{e^{2x}} + \frac{8y}{4x^2+3}$$

$$u = \ln \quad u' = \frac{1}{x}$$

$$v = 4x^2+3 \quad v' = 8x$$

$$\ln(8x) + 4x^2+3 \left(\frac{1}{x}\right)$$

$$\ln 8x + \frac{4x^2+3}{x}$$

Bonus

1.

2. Left

$$\frac{8x(\frac{1}{x})}{4x^2+3}$$

$$\frac{8y}{4x^2+3}$$

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I. Circle the right answer. (5 point each)

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1) Find the slope for $f(x) = -5x^2$ at $x=3$

$-5(3)^2 = -45$

- A) 30 B) -75 C) -45 D) -30

2) What is the equation of the tangent line for the curve $y = x^3 + 2$ at the point $(-1, 1)$

- A) $y = -3x + 4$ B) $y = 3x - 4$ C) $y = 3x + 4$ D) $y = -3x - 4$

3) The following functions is not differentiable at $x = -4$

- a) $f(x) = |x+4|$ b) $f(x) = x^2 - 4$ c) $f(x) = \frac{x+2}{x-4}$ d) $f(x) = \sqrt{-x} + 4$

4) The following function is not differentiable at $x = 1$

- a) $f(x) = \frac{1}{x+1}$ b) $y = (x-1)^3$ c) $f(x) = |x+1|$ d) $f(x) = \sqrt[3]{x-1}$

II. Answer the following questions.

1. The position of an object, s , at any time, t , is given by: (15 points)

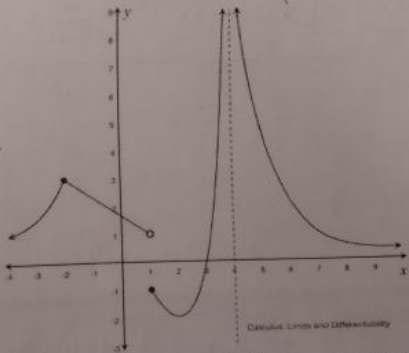
$s(t) = -18t^3 + 15t + 8$ where s is measured in feet and t is measured in seconds.

Find the equation of acceleration at any time $a(t)$.

$s(t) = -54t^2 + 15$

$a(t) = -108t$

2. The following graph shows the function $y = f(x)$ (20 points)



- a) Find the values of "x" where the function is not continuous $x = -2, x = 1, x = 4$
 b) Find the values of "x" where the function is not differentiable $x = 2, x = 4$

-2, 1, 4

III. Find the derivative by definition of the following function: (15 points)

$f(x) = 3x^2 + 5$

$\frac{f(x+h) - f(x)}{h}$

$\frac{3(x+h)^2 + 5 - (3x^2 + 5)}{h}$

$\frac{3(x^2 + 2xh + h^2) + 5 - (3x^2 + 5)}{h}$

$\frac{3x^2 + 6xh + 3h^2 + 5 - 3x^2 - 5}{h}$

~~$\frac{3x^2 + 6xh + 3h^2 + 5 - 3x^2 - 5}{h}$~~

$\frac{3h^2 + 6xh}{h}$

$\frac{h(3h + 6x)}{h}$

$3h + 6x$
 $3(0) + 6x$

$f'(x) = 6x$

IV. Find the derivative of the following:

a) $f(x) = 8\sqrt{x^3} - 2x^3 + \frac{5}{x^2}$ (10 points)

$f(x) = 8x^{3/2} - 2x^3 + 5x^{-2}$

$f'(x) = 6x^{-1/2} - 6x^2 - 10x^{-3}$

$f'(x) = \frac{6}{x^{1/2}} - 6x^2 - \frac{10}{x^3}$

b) $f(x) = 2(1-3x^2)^5 + \sqrt{4x-1}$ (10 points)

~~$f'(x) = 2 \cdot 5(1-3x^2)^4 \cdot (-6x) + \frac{1}{2\sqrt{4x-1}}$~~

$f'(x) = -60x(1-3x^2)^4 + \frac{2}{(4x-1)^{1/2}}$

c) $f(x) = 7(4x-5x^2)^6$ (10 points)

$42(4x-5x^2)^5 (4-10x)$

$f'(x) = 1680x(4x-5x^2)^5$

Bonus
1: Mercedes Stadium
2: Left