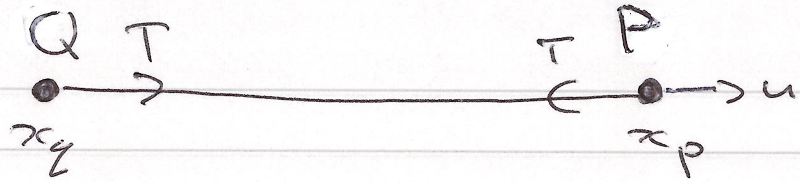
(i)



$l = a$

$\lambda = \frac{1}{2} m a \omega^2$

Applying Hooke's law.

$$T = \frac{\lambda (x_p - x_q - a)}{a} \Rightarrow T = \frac{1}{2} m \omega^2 (x_p - x_q - a)$$

Applying F=ma for both particles.

$$m\ddot{x}_p = -T = \frac{1}{2} m \omega^2 (a - (x_p - x_q))$$

$$m\ddot{x}_q = T = \frac{1}{2} m \omega^2 ((x_p - x_q) - a)$$

Adding gives $m\ddot{x}_p + m\ddot{x}_q = 0$.

$$\Rightarrow \ddot{x}_p + \ddot{x}_q = 0.$$

let $y = x_p + x_q \Rightarrow \ddot{y} = \ddot{x}_p + \ddot{x}_q = 0$.

$\Rightarrow y = At + B$

when $t = 0$ $y = a \Rightarrow B = a$.

when $t = 0$ $\dot{y} = u \Rightarrow A = u$.

$\Rightarrow y = ut + a$

substracting $z = x_p - x_q$

$$\Rightarrow \ddot{z} = \ddot{x}_p - \ddot{x}_q$$

$$m\ddot{x}_p - m\ddot{x}_q = m\omega^2 (a - (x_p - x_q))$$

$$\Rightarrow \ddot{x}_p - \ddot{x}_q = \omega^2 (a - (x_p - x_q)).$$

In terms of z.

$$\ddot{z} = \omega^2 (a - z)$$

$\Rightarrow \ddot{z} + \omega^2 z = \omega^2 a$.

$\Rightarrow z = A \sin(\omega t + B)$ complementary.

P.I. $z = \text{const} \Rightarrow z = a$.

$\therefore$ solution $z = A \sin(\omega t + B) + a$

$t = 0 \Rightarrow z = a$

$\Rightarrow A \sin B + a = a \Rightarrow \sin B = 0$
$\Rightarrow B = 0$.

$\Rightarrow z = A \sin(\omega t) + a$

$$\frac{dz}{dt} = A\omega \cos \omega t$$

$t = 0 \quad \dot{z} = u. \Rightarrow A\omega = u$
$A = \frac{u}{\omega}$.

$$z = \frac{u}{\omega} \sin(\omega t) + a.$$

$y = x_p + x_q \qquad z = x_p - x_q$.

$\Rightarrow x_p = \frac{1}{2}(y + z) = \frac{1}{2}\left(ut + a + \frac{u}{\omega}\sin(\omega t) + a\right)$

$= \frac{1}{2}ut + \frac{1}{2}\frac{u}{\omega}\sin(\omega t) + a$.

$$x_q = \frac{1}{2}(y - z) = \frac{1}{2}\left(ut + a - \left(\frac{u}{\omega}\sin(\omega t) + a\right)\right)$$

$$= \frac{1}{2}ut - \frac{1}{2}\frac{u}{\omega}\sin(\omega t).$$

clearly particles with return to a separation of 'a' when $t = \frac{\pi}{\omega}$

$t = 0 \qquad x_p = a \qquad x_q = 0.$

$t = \frac{\pi}{\omega} \qquad x_p = \frac{1}{2}u\frac{\pi}{\omega} + a \qquad x_q = \frac{1}{2}u\frac{\pi}{\omega}$

So both particles have travelled a distance of

$$\underline{\underline{\frac{1}{2}\pi\frac{u}{\omega}}}$$

When string is no longer taut $z \leq a$

particles will move with the ~~sp~~ speed they hade at the instant the string become slack $(z = a)$

$t = \frac{\pi}{\omega} \qquad \dot{x}_p = \frac{1}{2}u + \frac{1}{2}u\cos(\omega t).$

$$= \frac{1}{2}u + \frac{1}{2}u\cos(\pi).$$

$$= 0.$$

$t = \frac{\pi}{\omega}$ $\quad \dot{x}_q = \frac{1}{2}u - \frac{1}{2}u\cos(\omega t)$.

$= \frac{1}{2}u - \frac{1}{2}u\cos(\pi)$.

$= u$.

So P is station & Q moves with no forces acting on it with speed u.

$$\text{speed} = \frac{\text{dist}}{\text{time}} \Rightarrow u = \frac{a}{t}.$$

$$\Rightarrow t = \frac{a}{u}.$$

So total time from impulse to collision is.

$$\underline{\underline{\frac{\pi}{\omega} + \frac{a}{u}}}$$