Implausible Plot, Plausible Problem: An Application of Celestial Mechanics to the Film *Gravity*

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St. Joseph's College Student Research Symposium
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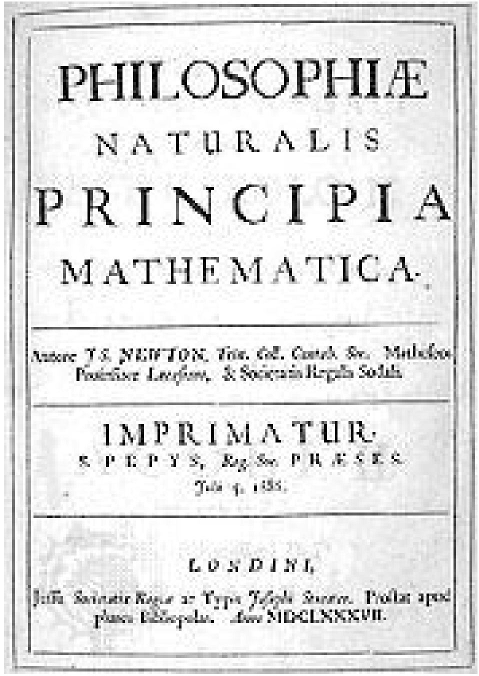
Answer: Gravity!



PHILOSOPHIÆ

NATURALIS

PRINCIPIA

MATHEMATICA.

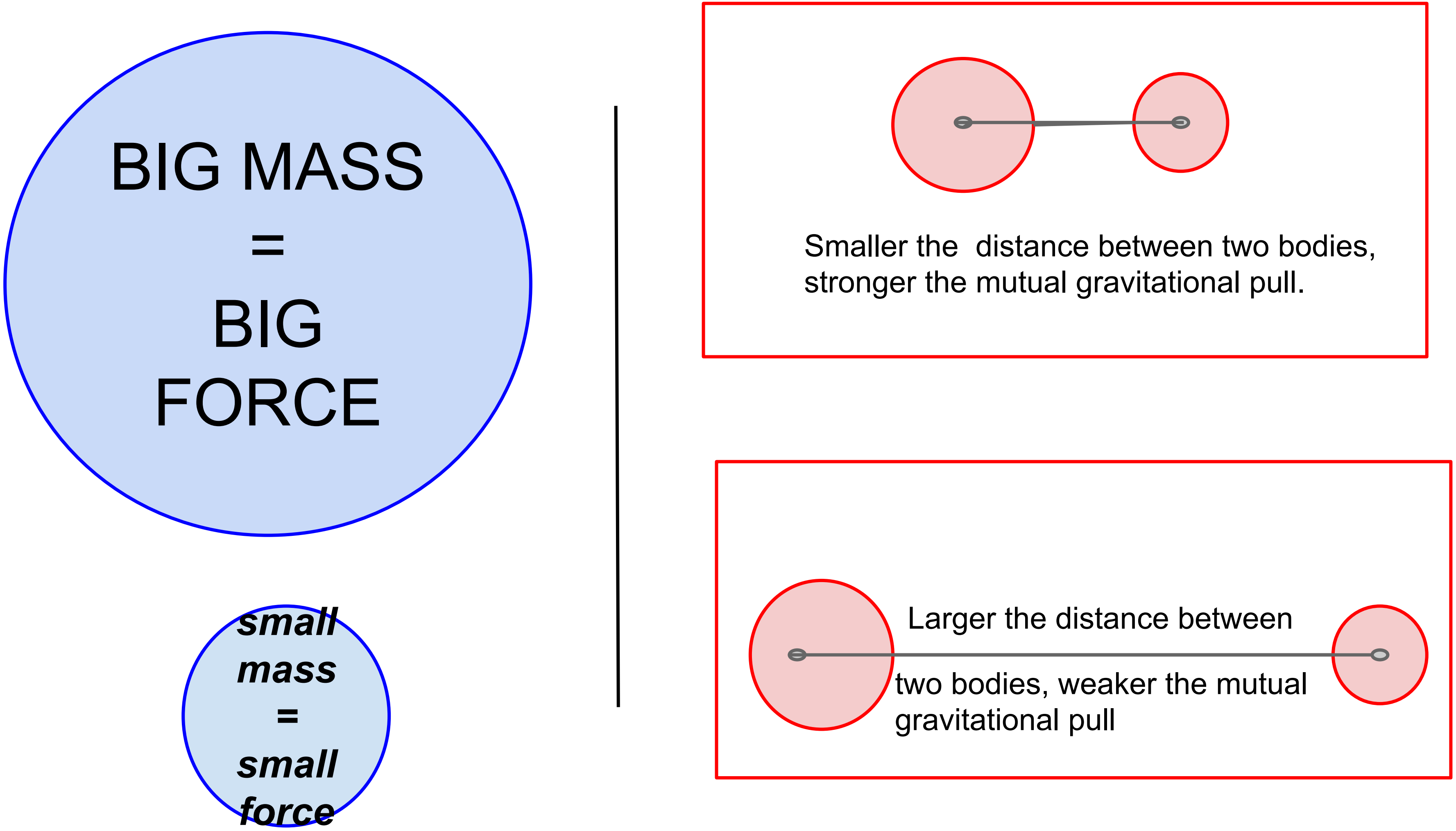
IMPRIMATUR.

LONDINI,

- The publication *Philosophiae Naturalis Principia Mathematica* by Sir Isaac Newton in 1687 contained Newton's three laws of motion and the universal law of gravitation.
- *Philosophiae Naturalis Principia Mathematica* essentially laid the foundation for modern science.

The Law of Universal Gravitation:

Every particle of matter in the universe attracts every other particle with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between their centers


BIG MASS
=
BIG
FORCE
small
mass
=
small
force
Smaller the distance between two bodies,
stronger the mutual gravitational pull.
Larger the distance between
two bodies, weaker the mutual
gravitational pull

These relationships between mass and force and the distance between the centers and force are a direct consequence of Newton's second law.

F = ma

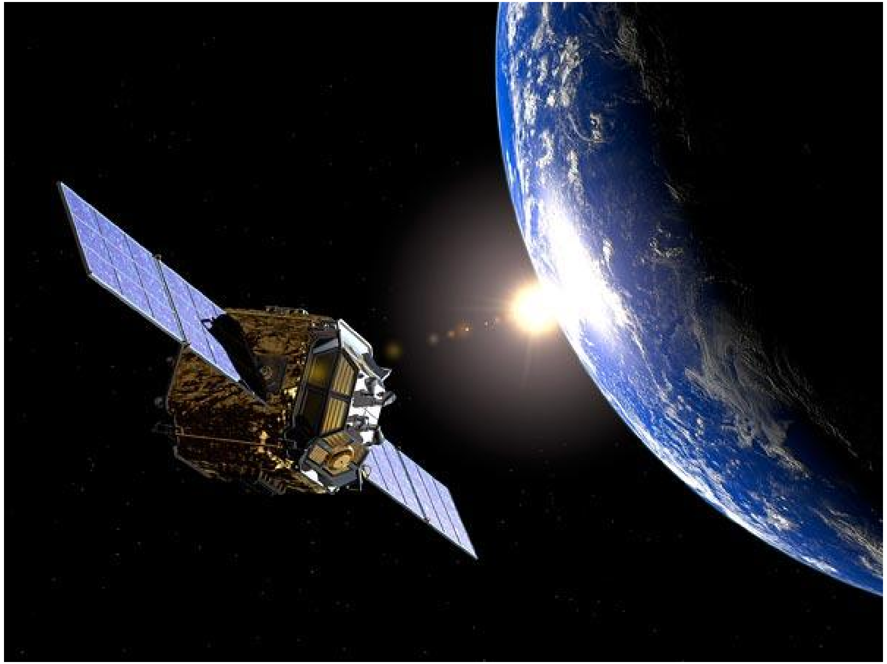
$$F = \frac{m1(m2)(G)}{\substack{2 \\ r}}$$

- *Celestial Mechanics:* the elaboration and application of the postulates expressed by Newton in *Principa*

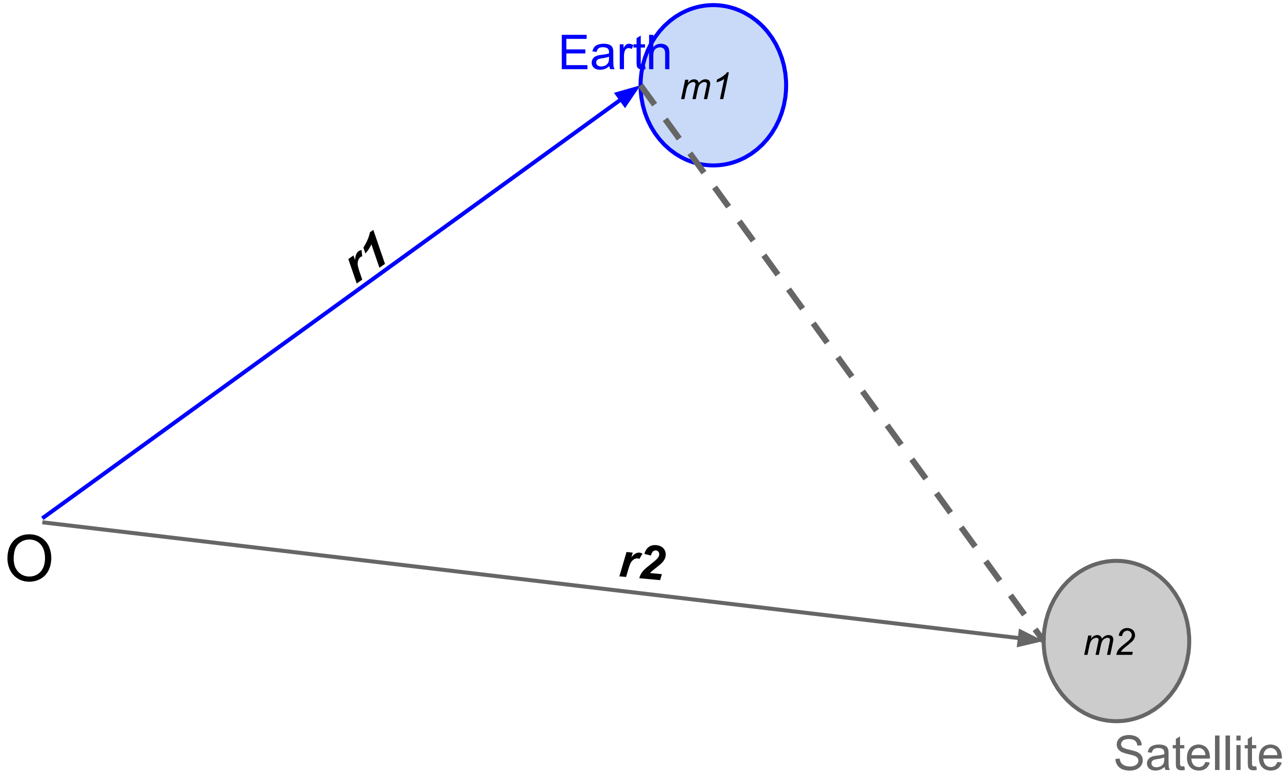
- *2-Body Problem:* describes the motion of a system of two mass particles moving ONLY according to their mutual gravitational attraction

There are several applications of the 2-body problem!

For example, the 2-body problem allows us to approximate the orbits of a satellite going around earth and earth of itself.



Visualizing the 2-Body Problem:



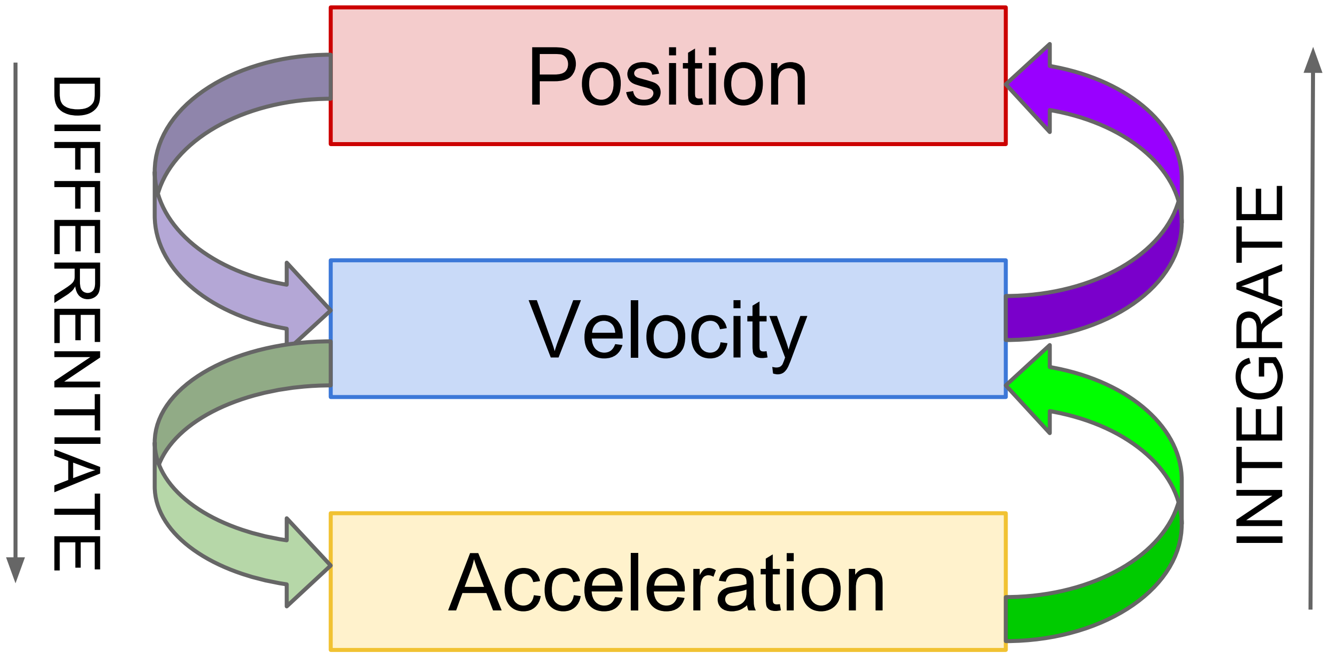
- According to the law of universal gravitation:

$$F = \frac{m1(m2)(G)}{\begin{matrix} 2 \\ r \end{matrix}}$$

- From the above and F = ma, we can determine the system of differential equations for the accelerations of mass 1 and mass 2:

$$\begin{cases} m_1 \cdot r''_1 = \dfrac{m_1 \cdot m_2 \cdot G}{|r_2 - r_1|^2} \cdot \dfrac{(r_2 - r_1)}{|r_2 - r_1|}, \\ m_2 \cdot r''_2 = \left(\dfrac{m_1 \cdot m_2 \cdot G}{|r_2 - r_1|^2} \cdot \dfrac{(r_1 - r_2)}{|r_2 - r_1|} \right) \end{cases}$$

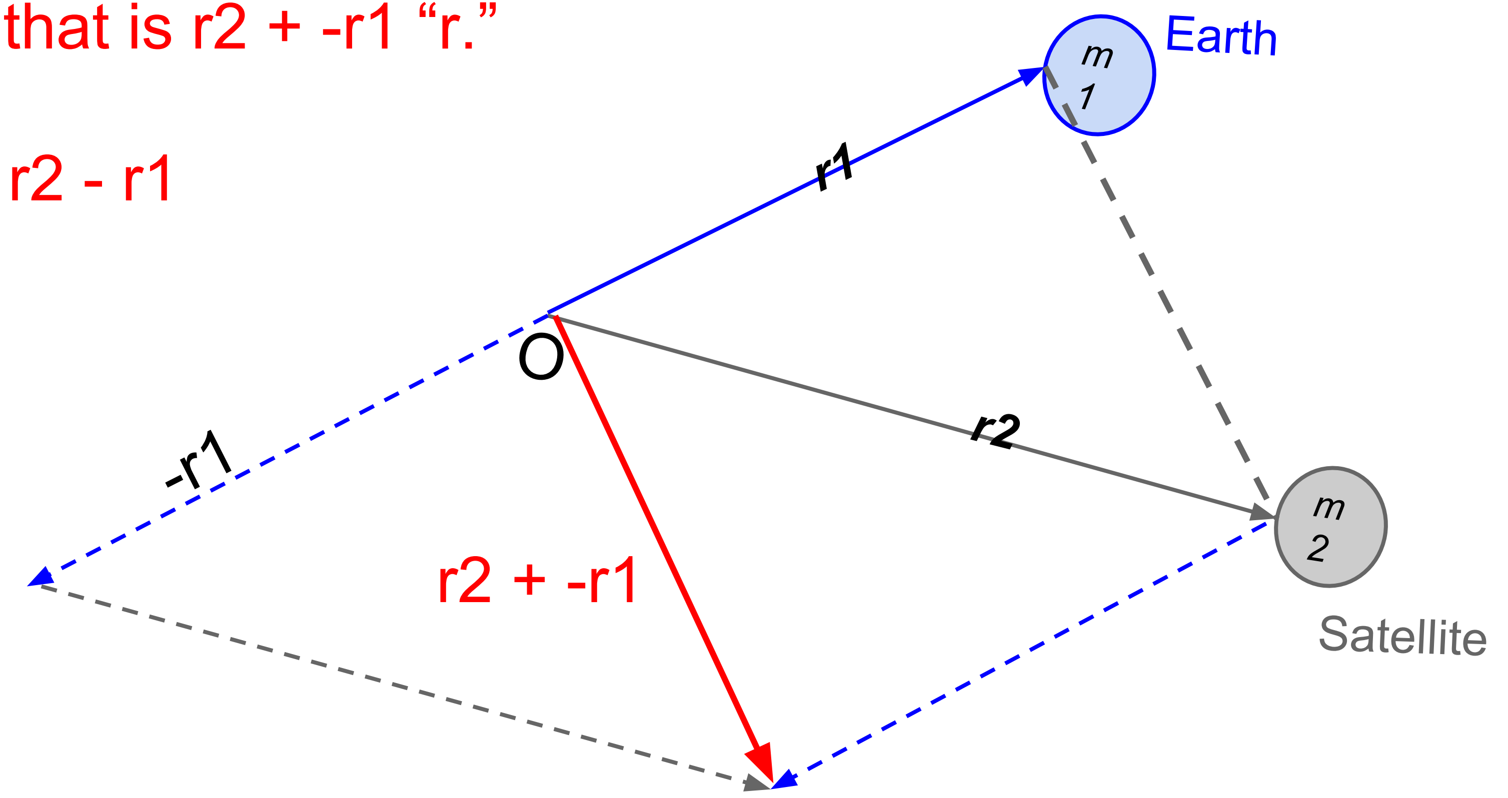
How position, velocity, and acceleration relate to one another and what the solution to our system of differential equation means:



$$
\begin{cases}
m_1 \cdot r''_1 = \dfrac{m_1 \cdot m_2 \cdot G}{|r_2 - r_1|^2} \cdot \dfrac{(r_2 - r_1)}{|r_2 - r_1|}, \\[2em]
m_2 \cdot r''_2 = \left(\dfrac{m_1 \cdot m_2 \cdot G}{|r_2 - r_1|^2} \cdot \dfrac{(r_1 - r_2)}{|r_2 - r_1|} \right)
\end{cases}
$$

We are going to call the vector that is r2 + -r1 “r.”

i.e. r = r2 - r1



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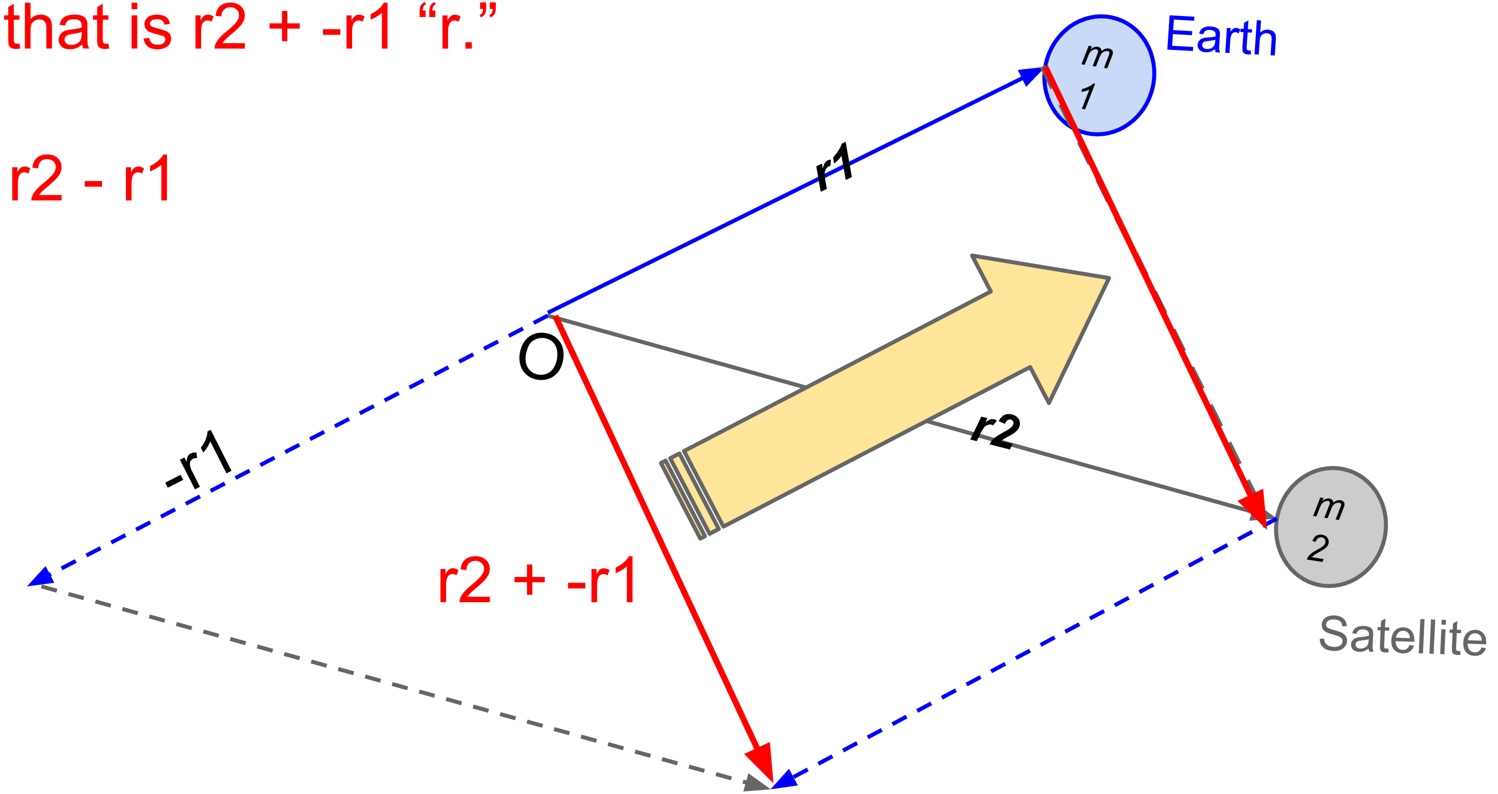


$$r = r_2 - r_1$$

$$r'' = r''_2 - r''_1$$

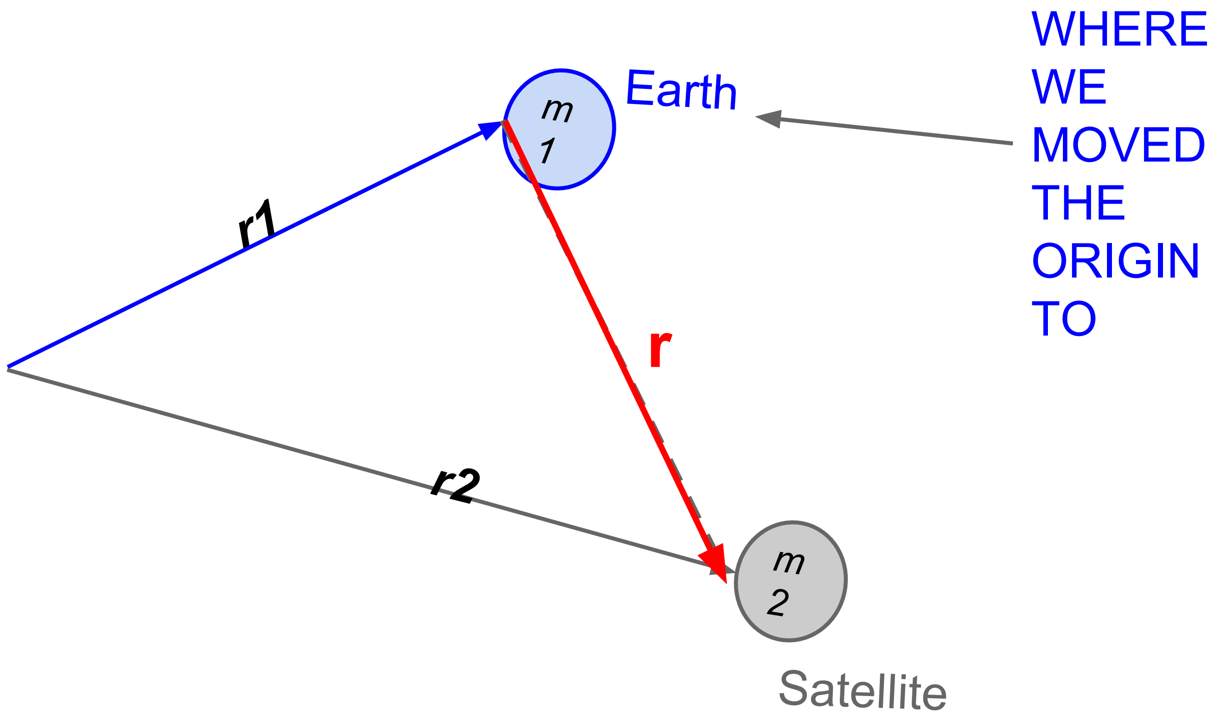
$$r'' = \left(\frac{G \cdot m_1}{|r_2 - r_1|^2} \cdot \frac{(r_1 - r_2)}{|r_2 - r_1|} \right) - \left(\frac{G \cdot m_2}{|r_2 - r_1|^2} \cdot \frac{(r_2 - r_1)}{|r_2 - r_1|} \right)$$

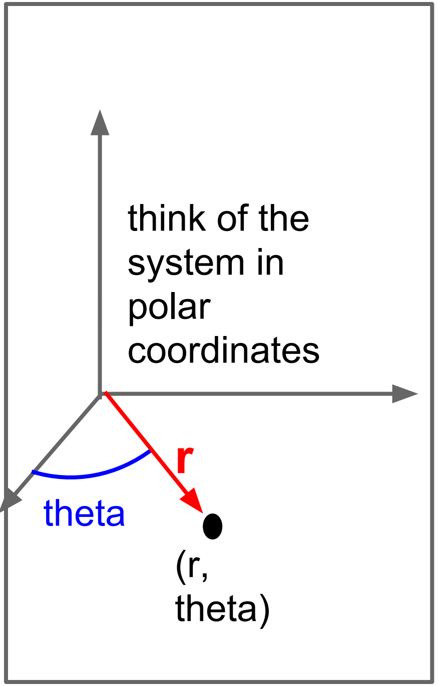
“Plugging in” the definitions of the r’’s from our differential system of equations.

simplify, and we find r’’

$$r'' = \frac{\mu \cdot r}{|r|^3}$$

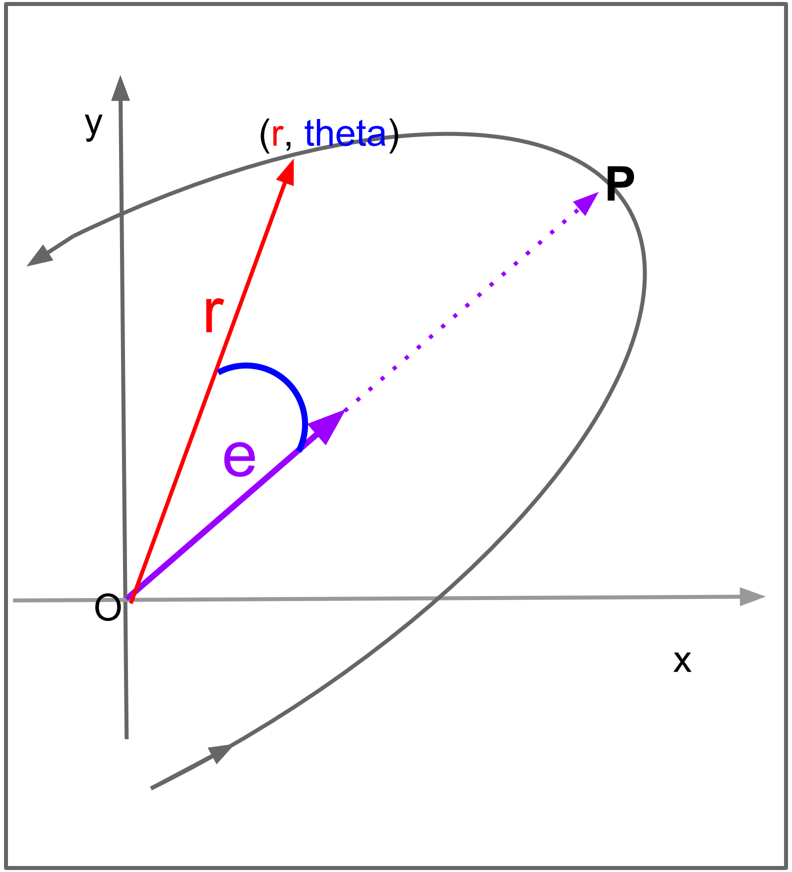
now we need to find an r that satisfies this equation!

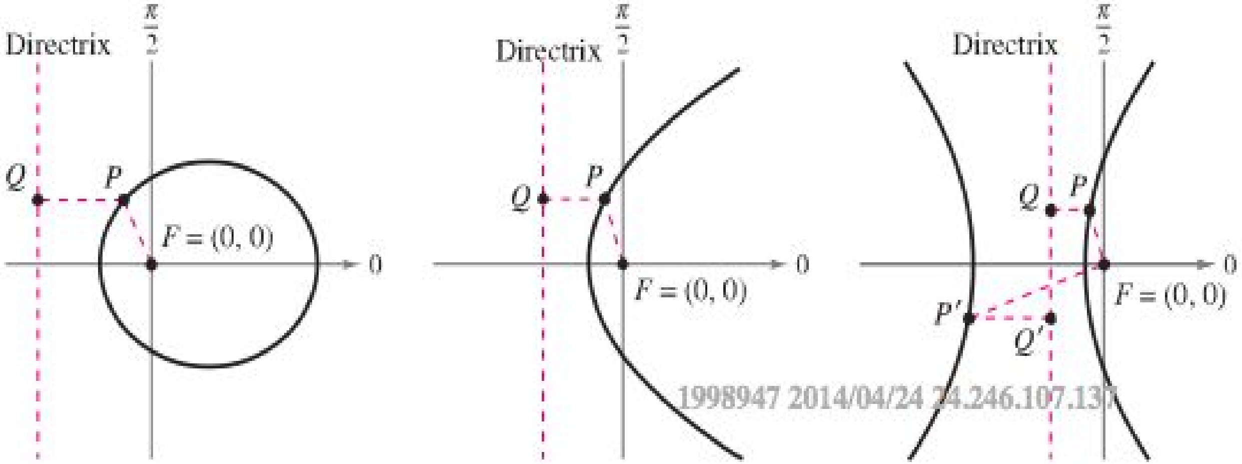

Earth
m
1
r1
r
r2
m
2
Satellite
WHERE
WE
MOVED
THE
ORIGIN
TO



our solution!

$$r = \frac{\left(\frac{c^2}{\mu}\right)}{(1 + e\cos(\text{theta}))}$$





Ellipse: $0 < e < 1$

$\frac{PF}{PQ} < 1$

Parabola: $e = 1$

$PF = PQ$

Hyperbola: $e > 1$

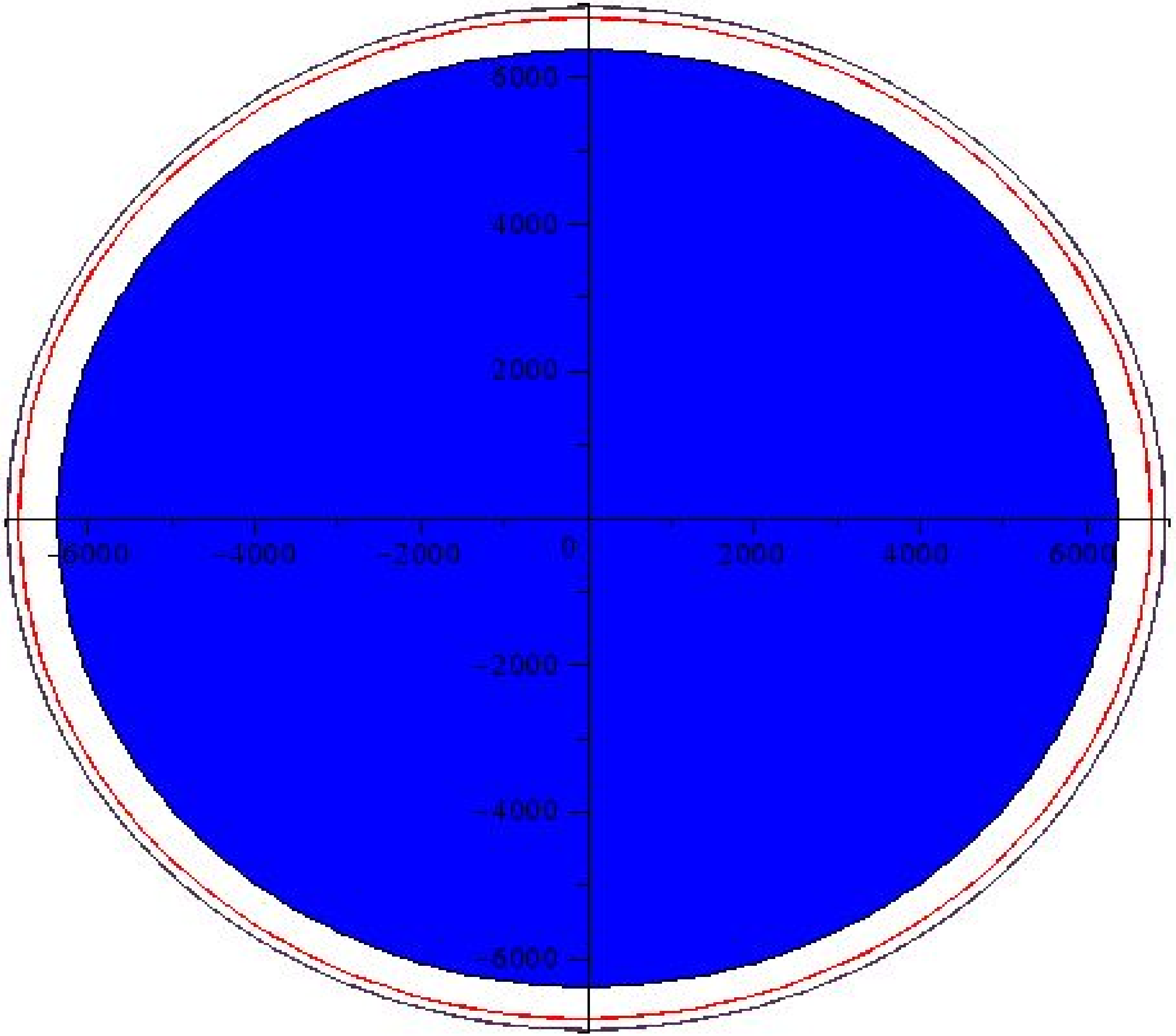
$\frac{PF}{PQ} = \frac{P'F}{P'Q'} > 1$

Figure 10.57

an orbit describes the set of positions occupied by the particle without any indication of the time at which a particular position is occupied

Gravity’s Plot:

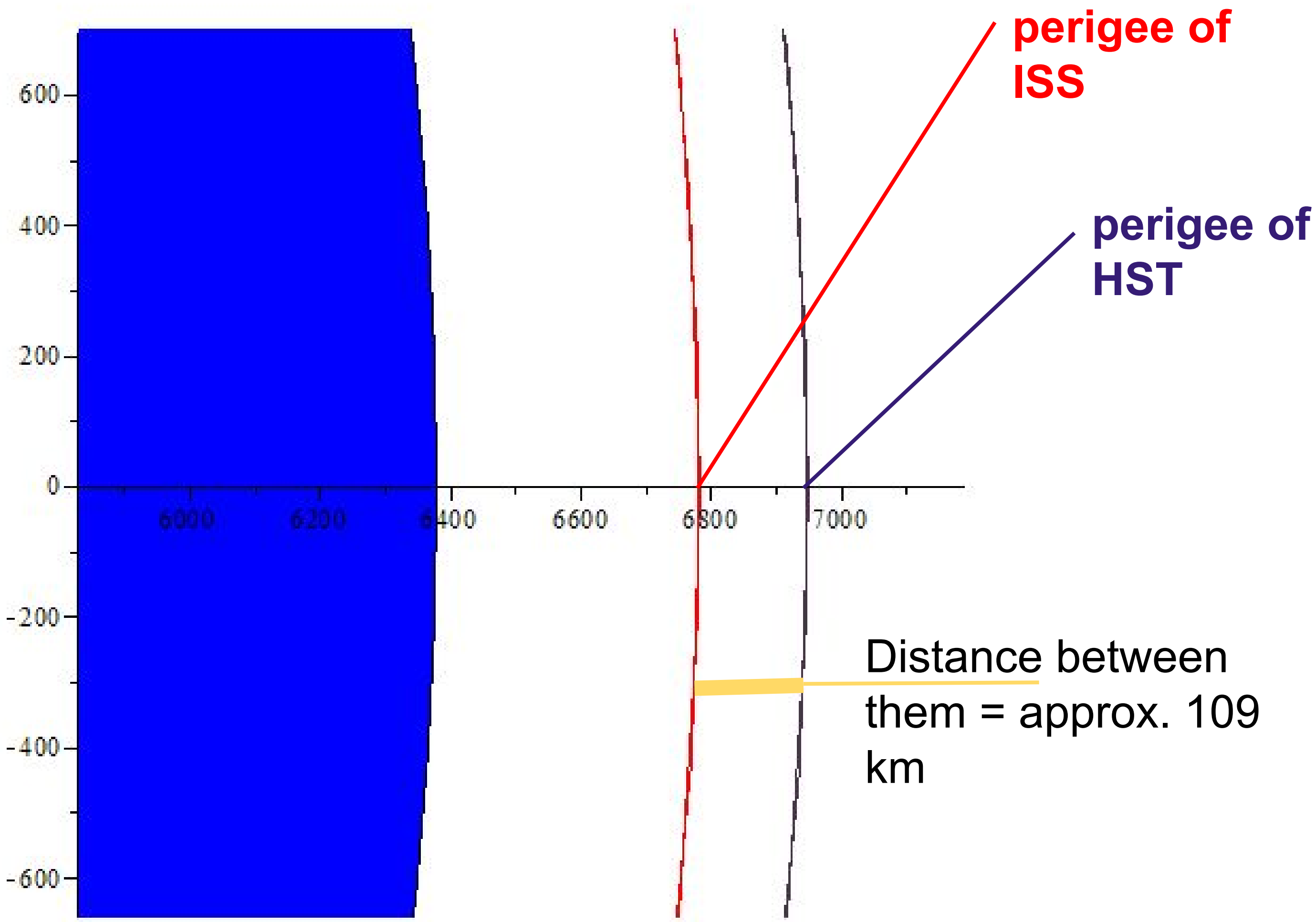
- Russians blow up defunct satellite
- Sandra Bullock is servicing the hubble telescope.
- High speed debris from blown up satellite strikes the Hubble and their spaceship.
- They are floating in space and try to get to the International Space Station
    - which, in the film is also under threat of the high speed debris



The outside, purple orbit = Hubble Space Telescope

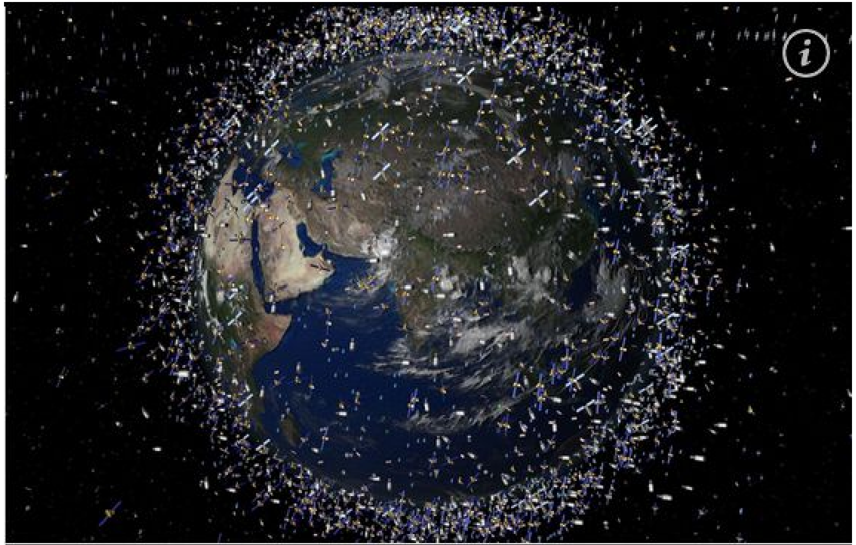
The inside, red orbit = International Space Station

Blue = Earth with radius of 6378.1 km



600
400
200
0
-200
-400
-600
6000
6200
6400
6600
6800
7000
perigee of ISS
perigee of HST
Distance between them = approx. 109 km

Implausible Plot, Plausible Problem:
Space debris is a three million body problem



A computer-generated image provided by the European Space Agency that shows an artist's impression of catalogued objects in low Earth orbit viewed over the equator. ESA/AP

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