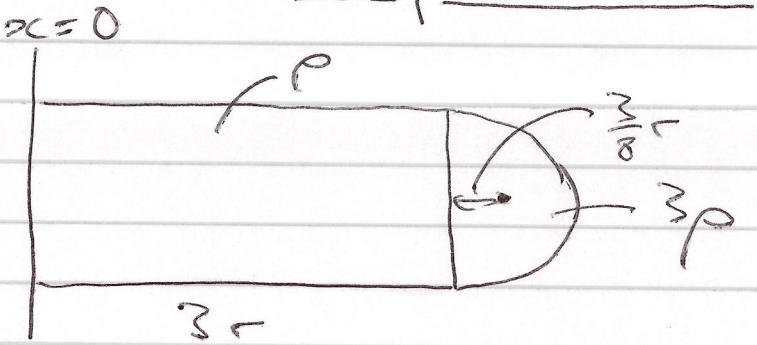
Step II 2007

10)



Cylinder $\quad C.G = \frac{3r}{2} \quad Mass = \rho \pi r^2 \times 3r = 3\rho\pi r^3$

Hemi-Sphere $\quad C.G = 3r + \frac{3}{8}r = \frac{27r}{8}$

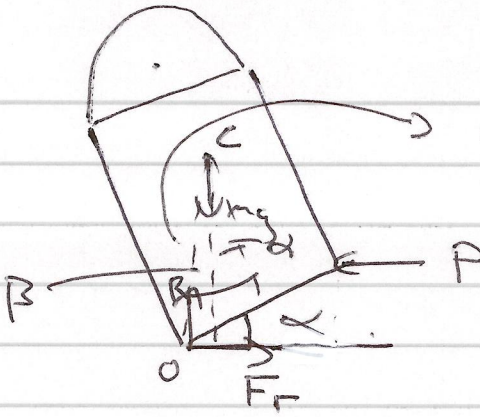
$$Mass = 3\rho \times \frac{2}{3}\pi r^3 = 2\rho\pi r^3$$

	Cylinder	Hemi:	Total
Mass	$3\rho\pi r^3$	$2\rho\pi r^3$	$5\rho\pi r^3$
C.G	$\frac{3}{2}r$	$\frac{27}{8}r$	$\bar{x}$

$$\frac{9}{2}\rho\pi r^4 + \frac{27}{4}\rho\pi r^4 = 5\rho\pi r^3 \bar{x}.$$

$$\Rightarrow \frac{9}{2}r + \frac{27}{4}r = 5\bar{x}$$

$$\frac{45}{4}r = 5\bar{x} \quad \Rightarrow \quad \bar{x} = \frac{9}{4}r.$$



$$CO = \sqrt{r^2 + \left(\frac{9r}{4}\right)^2} = \sqrt{\frac{16r^2}{16} + \frac{81r^2}{16}}$$

$$= \sqrt{\frac{97r^2}{16}} = \frac{\sqrt{97}r}{4}.$$

Case 1: Centre of gravity before point of pivot.

Resolving forces gives:

$R = mg$ & $F_r = P$.

& taking moments gives:

$$P \times 2r \sin\alpha = \frac{\sqrt{97}}{4} r \sin(\beta - \alpha)\, mg.$$

$$\Rightarrow P = \frac{\sqrt{97}}{8} \frac{\sin(\beta - \alpha)}{\sin\alpha} mg$$

$$\Rightarrow F_r = \frac{\sqrt{97}}{8} \frac{(\sin\beta\cos\alpha - \sin\alpha\cos\beta)}{\sin\alpha} mg.$$

$$= \frac{\sqrt{97}}{8}(\sin\beta\cot\alpha - \cos\beta)\, mg$$

$$\sin\beta = \frac{r}{\frac{\sqrt{97}}{4}r} = \frac{4}{\sqrt{97}} \qquad \cos\beta = \frac{9}{4}r \cdot \frac{4}{\sqrt{97}r}$$

$$= \frac{9}{\sqrt{97}}$$

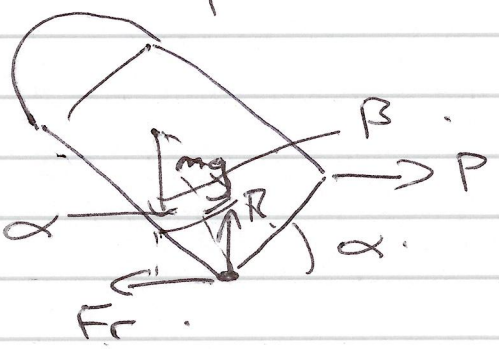
$$\Rightarrow F_r = \frac{\sqrt{97}}{8}\left(\frac{4}{\sqrt{97}}\cot\alpha - \frac{9}{\sqrt{97}}\right)mg.$$

$$= \left(\frac{1}{2}\cot\alpha - \frac{9}{8}\right)mg.$$

$$F_r \leq \mu R \Rightarrow \left(\frac{1}{2}\cot\alpha - \frac{9}{8}\right)mg \leq \mu mg$$

$$\Rightarrow \frac{1}{2}\cot\alpha - \frac{9}{8} \leq \mu. \quad -(1)$$

Case 2: Centre of gravity after point of pivot.



So forces resolve as before & moments gives $P \times 2r\sin\alpha = \frac{\sqrt{97}}{4} r\sin(\alpha-\beta)mg$

$\Rightarrow$ P is the opposite sign to previous.

Following this through the previous calculations gives.

$$\frac{9}{8} - \frac{1}{2}\cot\alpha \leq \mu. \quad -(2).$$

① & ② $\Rightarrow \mu \geq \left| \frac{a}{8} - \frac{1}{2}\cot\alpha \right|$

as $\left| \frac{a}{8} - \frac{1}{2}\cot\alpha \right| = \frac{a}{8} - \frac{1}{2}\cot\alpha$

$a - \frac{1}{2}\cot\alpha - \frac{a}{8}$.