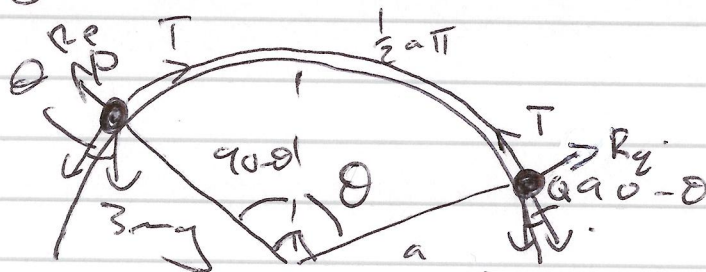


STEP III 2011

Mechanics Questions

- 9) Resolving tangentially works (rather than resolving towards the centre as in most M3 questions). See M3 book pages 137 to 139 for details.



Taking moments initially — when system is in equilibrium.

$$Ta + 4mg \sin \theta = Ta + 3mg \cos \theta.$$

$$\Rightarrow 4 \sin \theta = 3 \cos \theta.$$

$$\Rightarrow \tan \theta = \frac{3}{4} \quad \Rightarrow \quad \cos \theta = \frac{4}{5} \quad \sin \theta = \frac{3}{5}.$$

Effectively $t=0$ conditions.

Resolving tangential when system is moving. Tangential acceleration = $r\ddot{\theta}$

Note $r\ddot{\theta} = r \frac{d^2 \dot{\theta}}{d\theta^2}$ just like $\ddot{x} = \frac{d^2 \dot{x}}{dx}$.

$$\text{For P} \Rightarrow 3ma\ddot{\theta} = T - 3mg \cos \theta$$

$$\text{For Q} \Rightarrow 4ma\ddot{\theta} = 4mg \sin \theta - T.$$

Adding eqns.

$$7ma\ddot{\theta} = 4mg \sin \theta - 3mg \cos \theta$$

$$\Rightarrow 7a\ddot{\theta} = 4g \sin \theta - 3g \cos \theta$$

$$\Rightarrow 7a \frac{d^2 \dot{\theta}^2}{d\theta} = 4g \sin \theta - 3g \cos \theta.$$

$$\Rightarrow 7a \frac{1}{2} \dot{\theta}^2 = -4g \cos \theta - 3g \sin \theta + c.$$

when $t=0$ $\theta=0$ $\cos \theta = \frac{4}{5}$ $\sin \theta = \frac{3}{5}$.

$$\Rightarrow 0 = -\frac{16g}{5} - \frac{9g}{5} + c.$$

$$\Rightarrow c = \frac{25g}{5} = 5g$$

$$\Rightarrow 7a \frac{1}{2} \dot{\theta}^2 = -4g \cos \theta - 3g \sin \theta + 5g.$$

$$\Rightarrow \frac{7a}{2} \dot{\theta}^2 + 4g \cos \theta + 3g \sin \theta = 5g$$

$$\Rightarrow 7a\dot{\theta}^2 + 8g \cos \theta + 6g \sin \theta = 10g \quad (1)$$

i) Resolving towards the centre for Q gives

$$4mg \cos \theta - R_g = 4ma\dot{\theta}^2.$$

$$R_g = 0 \Rightarrow a\dot{\theta}^2 = g \cos \theta$$

Inserting into (1) gives. Changing θ to β

$$7g \cos \beta + 8g \cos \beta + 6g \sin \beta = 10g$$

$$\Rightarrow 15 \cos \beta + 6 \sin \beta = 10$$

ii) Resolving to centre equations

$$3m \ddot{\theta} = T - 3mg \cos \theta$$

$$4m \ddot{\theta} = 4mg \sin \theta - T$$

$$\Rightarrow \frac{T - 3mg \cos \theta}{3} = \frac{4mg \sin \theta - T}{4}$$

$$\Rightarrow 4T - 12mg \cos \theta = 12mg \sin \theta - 3T$$

$$\Rightarrow 7T = 12mg (\sin \theta + \cos \theta)$$

$$\Rightarrow T = \frac{12}{7} mg (\sin \theta + \cos \theta)$$