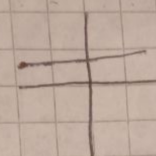
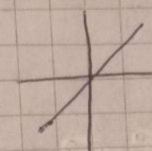
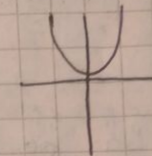
Carlos Humberto Bárcenas González A01370142 07/08/17
Andrés Marcelo Carranza Somera A01370321 GD: 403



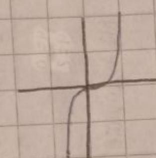
$y = C$



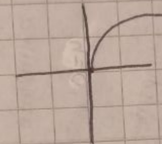
$y = x$



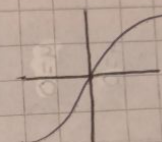
$y = x^2$



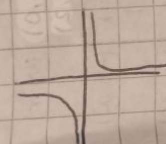
$y = x^3$



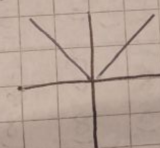
$y = \sqrt{x}$



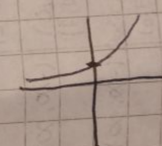
$y = \sqrt[3]{x}$



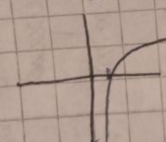
$y = \frac{1}{x}$



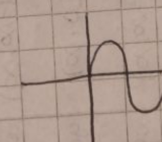
$y = |x|$



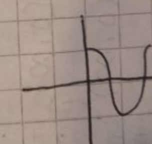
$y = e^x$



$y = \ln(x)$



$y = \sin x$



$y = \cos x$

		with both axis	Asymptotes	Key points	Domain	Range	Name of function	
1	$y=c$ $y=1$	$x=0$ $y=1$	No asymptote	$(-1,1)(0,1)$ $(1,1)$	$(-\infty,\infty)$	~~$(-\infty,\infty)$~~ 1	horizontal line	
2	$y=x$	$x=0$ $y=0$	NO Asymptote	$(-2,-2)(-1,-1)$ $(0,0)(1,1)$	$(-\infty,\infty)$	$(-\infty,\infty)$	linear function	
3	$y=x^2$	$x=0$ $y=0$	No Asymptote	$(-2,4)(-1,1)$ $(0,0)(1,1)$	$(-\infty,\infty)$	$[0,\infty)$	parabola	
4	$y=x^3$	$x=0$ $y=0$	NO asymptote	$(-1,-1)(0,0)$ $(1,1)$	$(-\infty,\infty)$	$(-\infty,\infty)$	cubic function	
5	$y=\sqrt{x}$	$x=$ – $y=$ –	No asymptote	$(0,0)(1,1)$ $(4,2)(9,3)$	$[0,\infty)$	$[0,\infty)$	get a form of ascending in the "y"	
6	$y=\sqrt[3]{x}$	$x=0$ $y=0$	NO asymptote	$(-8,-2)(-1,-1)$ $(0,0)(1,1)(8,2)$	$(-\infty,\infty)$	$(-\infty,\infty)$	cube root algebraic function	
7	$y=\frac{1}{x}$	$x=$ – $y=$ –	$y=0$ $x=0$	$(-5,-0.2)(-1,-1)$ $(1,1)(5,0.2)$	$(-\infty,0)\cup(0,\infty)$	$(-\infty,0)\cup(0,\infty)$	rational function	
8	$y=\lvert x\rvert$	$x=0$ $y=0$	No asymptotes	$(-2,2)(0,0)$ $(2,2)(4,4)$	$(-\infty,\infty)$	$[0,\infty)$	it is an increasing function	in the "y" axis
9	$y=e^x$	$x=0$ $y=1$	$y=1$	$(-2,0.135)(-1,0.368)$ $(0,1)(1,2.718)$	$(-\infty,\infty)$	$(0,\infty)$	only positive first decrease then increase	exponential function
10	$y=\ln(x)$	$x=1$ $y=0$	$x=0$	$(1,0)(2,0.69)$ $(3,1.099)$	$(0,\infty)$	$(-\infty,\infty)$	it is an increase basically "x" axis	logarithmic function
11	$y=\sin(x)$	$x=0$ $y=0$	No asimptote	$(0,0)(\frac{\pi}{2},1)$ $(\pi,0)(\frac{3\pi}{2},-1)$	$(-\infty,\infty)$	$[-1,1]$	wave starting at origin	Sine function
1	$y=\cos(x)$	$x=0$ $y=1$	No asymptote	$(0,1)(\frac{\pi}{2},0)$ $(\pi,-1)(\frac{3\pi}{2},0)$	$(-\infty,\infty)$	$[-1,1]$	wave starting at $(0,1)$	Cosine function

Slope of Tangent Line Using Secant Line and Concept of Limits
By: *Designing Team*

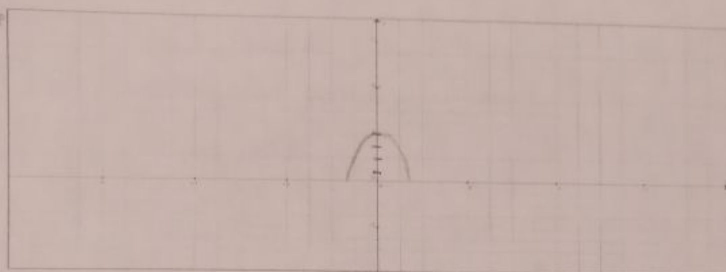
Tecnológico de Monterrey Preparatoria

Name Carlos Humberto Bikenas González Group 403 Date August 4, 2017

OK

Tangent line
Secant line

1. a) Sketch the graph of the function $f(x) = -x^2 + 4$



$\frac{4-3}{0-1} = -1$ $\frac{3.75-3}{0.5-1}$ $\frac{3.19-3}{0.9-1}$ $\frac{3.0975-3}{0.95-1}$ $\frac{3.001999-3}{.001}$ $\frac{.001999}{.001}$ $\frac{.0975}{-0.05}$

Find the slope of the secant line passing through the points P(1,3) and Q (given below)

b) Write the solpes in the following table:

$Q(x, -x^2+4)$	m
(0,4)	-1
(0.5, 3.75)	-1.5
(0.9, 3.19)	-1.9
(0.95, 3.0975)	-1.95
(0.99, 3.0199)	-1.99
(0.999, 3.001999)	-1.999

$Q(x, -x^2+4)$	m
(2,−1)	-4
(1.5, 1.75)	-2.5
(1.1, 2.79)	-2.1
(1.05, 2.8975)	-2.05
(1.01, 2.9799)	-2.01
(1.001, 2.997999)	-2.001

$\frac{-1-3}{2-1} = -4$ $\frac{1.75-3}{1.5-1}$ $\frac{2.79-3}{1.1-1}$ $\frac{2.8975-3}{1.05-1}$ $\frac{2.9799-3}{1.01-1}$ $\frac{2.997999-3}{1.001-1}$

c) Which value is being approximated by the secant line when the point Q approaches the point P(1,3)? $m = -2$

d) Based on the prevoius information find the slope of the tangent line passing through (1, 3)

$\lim_{h \to 0} \frac{[-(x+h)^2+4] - [-x^2+4]}{h}$ $[-x^2 - 2xh - h^2 + 4] + x^2 - 4$ $\frac{-2xh - h^2}{h} = \frac{h(-2x-h)}{h} = -2x - h$ $-2x - 0$ $-2x$ $-2(1) = \boxed{-2}$

e) Find the equation of the tangent line at the point (1, 3)

$m = -2$ $P(1,3)$ $3 = -2(1) + b$ $3+2 = b$ $y = -2x + 5$

2. The point (2,1) lies on the curve $f(x) = \frac{1}{x-1}$. $x - 1 = 0$ $x = 1$ Asymptote

a) If Q is the point $\left(x, \frac{1}{x-1}\right)$, find the slope of the secant line PQ (round to six decimals) for the following values of x: $P(2,1)$

i) 1.5 $\boxed{-2}$ ii) 1.75 $\boxed{-4/3}$ iii) 1.9 $\boxed{-10/9}$ iv) 1.99 $\boxed{-100/99}$ v) 1.999 $\boxed{-1000/999}$

$1.5, \frac{1}{1.5-1}$ $(1.5, \frac{1}{2})$ $\frac{\frac{1}{2}-1}{1.5-2}$ $\frac{-0.5}{-0.5} = 1$

$1.75, \frac{1}{1.75-1}$ $(1.75, 4/3)$ $\frac{4/3-1}{1.75-2}$ $\frac{1/3}{-0.25}$

$1.9, \frac{1}{1.9-1}$ $(1.9, 10/9)$ $\frac{10/9-1}{1.9-2}$ $\frac{1/9}{-0.1}$

$1.99, \frac{1}{1.99-1}$ $1.99, 100/99$ $\frac{100/99-1}{1.99-2}$ $\frac{1/99}{-0.01}$

$1.999, \frac{1}{1.999-1}$

b) Use the results of part (a) to find an estimation of the slope of the tangent line to the curve at (2,1)

m= -1

3. The point (6,2) lies on the curve $f(x)=\sqrt{x-2}$.

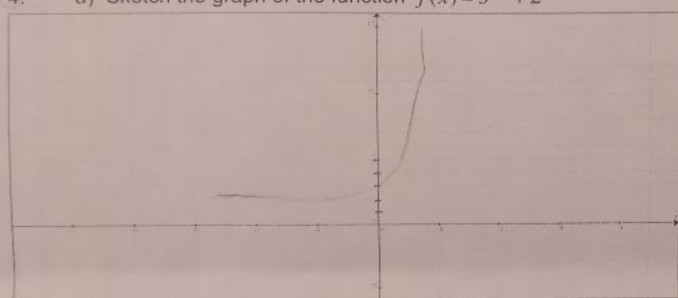
a) If Q is the point $\left(x, \sqrt{x-2}\right)$, find the slope of the secant line PQ (round to six decimals) for the following values of x:

i) 5.5 ii) 5.9 iii) 5.99 iv) 6.001 v) 6.01 vi) 6.1

b) Use the results of part (a) to find an estimation of the slope of the tangent line to the curve at (6,2)

m= 0.1

4. a) Sketch the graph of the function $f(x)=3^{x+1}+2$



b) Find the slope of the secant line passing through the points P(0,5) and Q (given below)

a) Write the solpes in the following table:

$Q(x, 3^{x+1}+2)$	m
(0,5)	6
(0.5, 7.196)	4.392
(0.25,5.948)	3.792
(0.15, 5.537)	3.58
(0.1, 5.348)	3.48
(0.01, 5.033)	3.3

$Q(x, 3^{x+1}+2)$	m
(-0.5,3.732)	2.536
(-0.25, 4.280)	2.88
(-0.15, 4.544)	3.04
(-0.1, 4.688)	3.12
(-0.01, 4.997)	0.3

b) Which value is being approximated by the secant lines when the point Q approaches the point P(0,5)?

m= 3

c) Based on the prevoius information find the slope of the tangent line passing through (0,5)

d) Find the equation of the tangent line at the point (0, 5)

40/40

Estimating a Limit Numerically

By: *Lic. Lucy Solis*

Tecnológico de Monterrey Preparatoria

ok

Name Carlos Humberto Becerra González Group 409

A01370117

Link: http://www.rootmath.org/calculus/estimating-limits-numerically

I. Instructions: Estimate the given limit using a numerical approximation

1. $\lim_{x \to 2} \frac{x-2}{x^2-4} =$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	0.25641	.2506	.2500		.2499	.2493	.2439

$\frac{1.9-2}{(1.9)^2-4} = \frac{-0.1}{-0.39}$

2. Use the table to approximate $\lim_{x \to 5} \frac{2-\sqrt{x-1}}{5-x} =$

x	4.9	4.99	4.999	5.001	5.01	5.1
$f(x)$	.251	.2501	.25	.2499	.2498	.2484

x approaches 5 from left ↑ x approaches 5 form the right ↑

3. Use the table to approximate $\lim_{x \to -3} \frac{x+3}{2x^2-18} =$ $2(x^2-9)$ $2(x+3)(x-3)$

x	−3.1	−3.01	−3.001	-3	−2.999	−2.99	−2.9
$f(x)$	0.081	−0.0831	−0.0832		0.08334	0.0834	−0.0847

4. Use the table to approximate $\underset{h \to 0}{lím} \frac{(5+h)^2-25}{h} =$

x	−0.1	−0.01	−0.001	0	0.001	0.01	0.1
$f(x)$	9.9	9.99	9.999		10.001	10.01	10.1

5. $\lim_{x \to 0} \frac{\sin(x)}{x} =$

x	−0.1	−0.01	−0.001	0	0.001	0.01	0.1
$f(x)$	0.017452	0.017453	0.017453		0.017453	0.017453	0.017453

6. $\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x}$

x	−0.1	−0.01	−0.001	0	0.001	0.01	0.1
$f(x)$	0.5131	0.5012	0.5		.4998	.4987	.488

7. Find $\lim\limits_{x \to 0} f(x)$ if $f(x) = \begin{cases} x-1 & x<0 \\ x^2 & x \geq 0 \end{cases}$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	-1.1	-1.01	-1.001		.000001	.0001	0.01

8. Find $\lim\limits_{x \to 2} f(x)$ if $f(x) = \begin{cases} x^2+1 & x<2 \\ 2x-3 & x \geq 2 \end{cases}$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	4.61	4.96	4.996	1	1.001	1.02	1.2

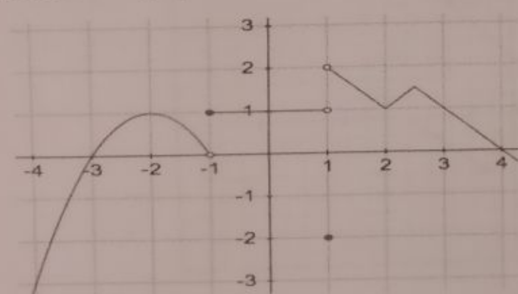
9. Find $\lim\limits_{x \to 2} \dfrac{x+1}{x-2}$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	-29	-299	-2999		3001	301	31

10. Find $\lim\limits_{x \to 1} \dfrac{x^2}{(x-1)^2}$

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$	81	9801	998001		1002001	10201	121

II. Based on the graph find the limits



a) $\lim\limits_{x \to -1^+} f(x) = 1$

b) $\lim\limits_{x \to -1^-} f(x) = 0$

c) $\lim\limits_{x \to -1} f(x) = \nexists$

d) $\lim\limits_{x \to 1^+} f(x) = 2$

e) $\lim\limits_{x \to 1} f(x) = 1$

f) $\lim\limits_{x \to 1} f(x) = \nexists$

g) $\lim\limits_{x \to 2} f(x) = 1$

h) $\lim\limits_{x \to 2} f(x) = 1$

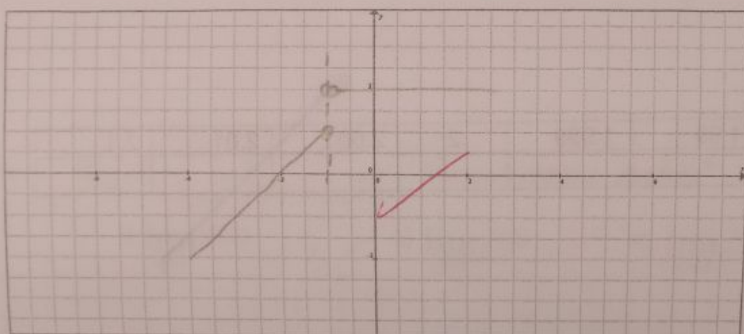
j) $\lim f(x) = 1$

2

III. Graph the following functions and find their limits

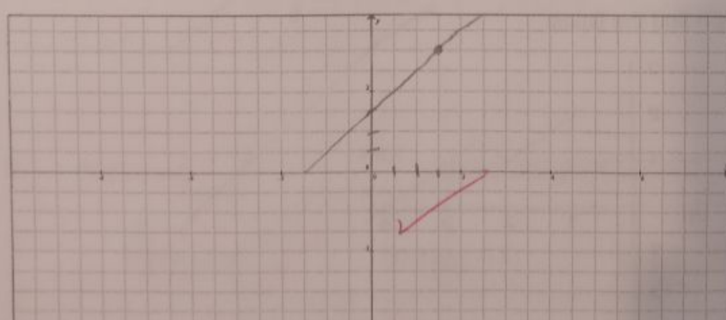
1. $f(x)=\begin{cases} x+2 & x\le -1 \\ 2 & x>-1 \end{cases}$

Find i) $\lim\limits_{x\to -1^+} f(x) = 2$ ii) $\lim\limits_{x\to -1^-} f(x) = 1$ iii) $\lim\limits_{x\to -1} f(x) = \nexists$



$(x+3)(\cancel{x-3})$

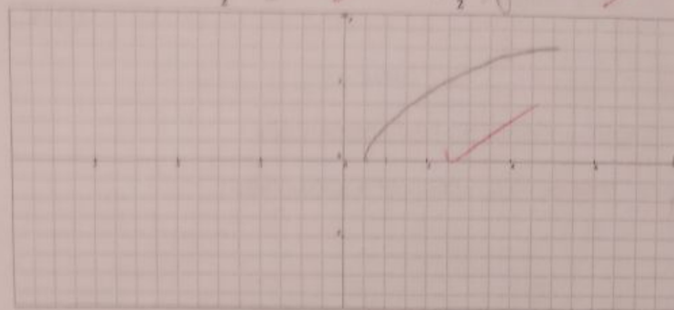
2. Graph $f(x)=\dfrac{x^2-9}{\cancel{x-3}}$ find i) $\lim\limits_{x\to 3^+} \dfrac{x^2-9}{x-3} = 6$ ii) $\lim\limits_{x\to 3^-} \dfrac{x^2-9}{x-3} = 6$ iii) $\lim\limits_{x\to 3} \dfrac{x^2-9}{x-3} = 6$



3. Graph $f(x)=\sqrt{2x-1}$ find

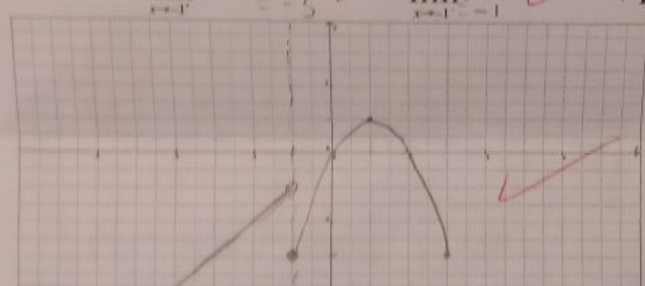
a) Domain $\left(\frac{1}{2}, \infty\right)$

b) i) $\lim\limits_{x\to\frac{1}{2}^+}\sqrt{2x-1} = 0$ ii) $\lim\limits_{x\to\frac{1}{2}}\sqrt{2x-1} = 0$



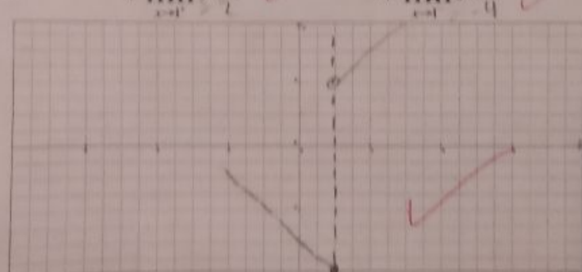
4. If $f(x)=\begin{cases} x & x<-1 \\ -x^2+2x & x\geq -1 \end{cases}$ Sketch the graph and find

i) $\lim\limits_{x\to -1^-} f(x) = -3$ ii) $\lim\limits_{x\to -1^+} f(x) = -1$ iii) $\lim\limits_{x\to -1} f(x) = \nexists$



5. If $f(x)=\begin{cases} -x-3 & x\leq 1 \\ x+1 & x>1 \end{cases}$ sketch the graph and find

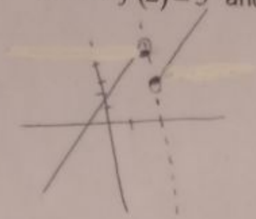
i) $\lim\limits_{x\to 1^+} f(x) = 2$ ii) $\lim\limits_{x\to 1^-} f(x) = -4$ iii) $\lim\limits_{x\to 1} f(x) = \nexists$



Give an example of a two-sided limit of a piecewise function where the limit

Ok

7. Sketch a graph so that $f(2)=5$ and $\lim_{x\to 2^-} f(x)=5$ $\lim_{x\to 2^+} f(x)=3$



8. Explain the meaning of $\lim_{x\to 1} f(x)=2$

Is it possible to obtain $\lim_{x\to 1} f(x)=2$ and $f(1)=4$? Justify the answer.

No, because both lines need to be in 2 when $x=1$

9. Explain the meaning of $\lim_{x\to 1^-} f(x)=2$ and $\lim_{x\to 1^+} f(x)=5$ when the function reaches 1 fr[om] the left should be in 2 a[nd] when it approaches from th[e] right side should be 5.

Is it possible that $\lim_{x\to 1} f(x)$ exists? Justify the answer.

If they are different results $\lim_{x\to 0} f(x)$ its not going to exist.

5

Finding *Limits numerically*
By: *Ziad Najjar*

Tecnológico de Monterrey Preparatoria

10/10

Name ______________ Group ______ Date ______

Evaluate the following limits numerically:

1) $\lim_{x \to 6} (\frac{x^2-36}{x-6}) =$

2) $\lim_{x \to 0} (\frac{x^2-2x}{x}) =$

3) $\lim_{x \to -2} (\frac{x^3+8}{x+2}) =$

4) $\lim_{x \to 2} (\frac{x^2+3x-10}{x-2}) =$

5) $\lim_{x \to 3} (\frac{\sqrt{x+1}-2}{x-3}) =$

6) $\lim_{x \to 10} (\frac{100-x^2}{x-10}) = \frac{-(x^2-100)}{x-10} = \frac{-(x-10)(x+10)}{x-10} = -x-10 = -10-10 = -20$

7) $\lim_{x \to -3} (\frac{x^2+14x+33}{2x+6}) =$

8) $\lim_{x \to -1} (\frac{2+2x^3}{x+1}) =$

9) $\lim_{x \to 25} (\frac{x-25}{\sqrt{x}-5}) =$

10) $\lim_{x \to -6} (\frac{x^3-36x}{18+3x}) = \frac{x(x^2-36)}{3x+18} = \frac{x(x-6)(x+6)}{3(x+6)}$

Answers:

1) $\lim_{x \to 6} (\frac{x^2-36}{x-6}) = 12$

2) $\lim_{x \to 0} (\frac{x^2-2x}{x}) = -2$

3) $\lim_{x \to -2} (\frac{x^3+8}{x+2}) = 12$

4) $\lim_{x \to 2} (\frac{x^2+3x-10}{x-2}) = 7$

5) $\lim_{x \to 3} (\frac{\sqrt{x+1}-2}{x-3}) = \frac{1}{4}$

6) $\lim_{x \to 10} (\frac{100-x^2}{x-10}) = -20$

7) $\lim_{x \to -3} (\frac{x^2+14x+33}{2x+6}) = 4$

8) $\lim_{x \to -1} (\frac{2+2x^3}{x+1}) = 6$

9) $\lim_{x \to 25} (\frac{x-25}{\sqrt{x}-5}) = 10$

10) $\lim_{x \to -6} (\frac{x^3-36x}{18+3x}) = 24$