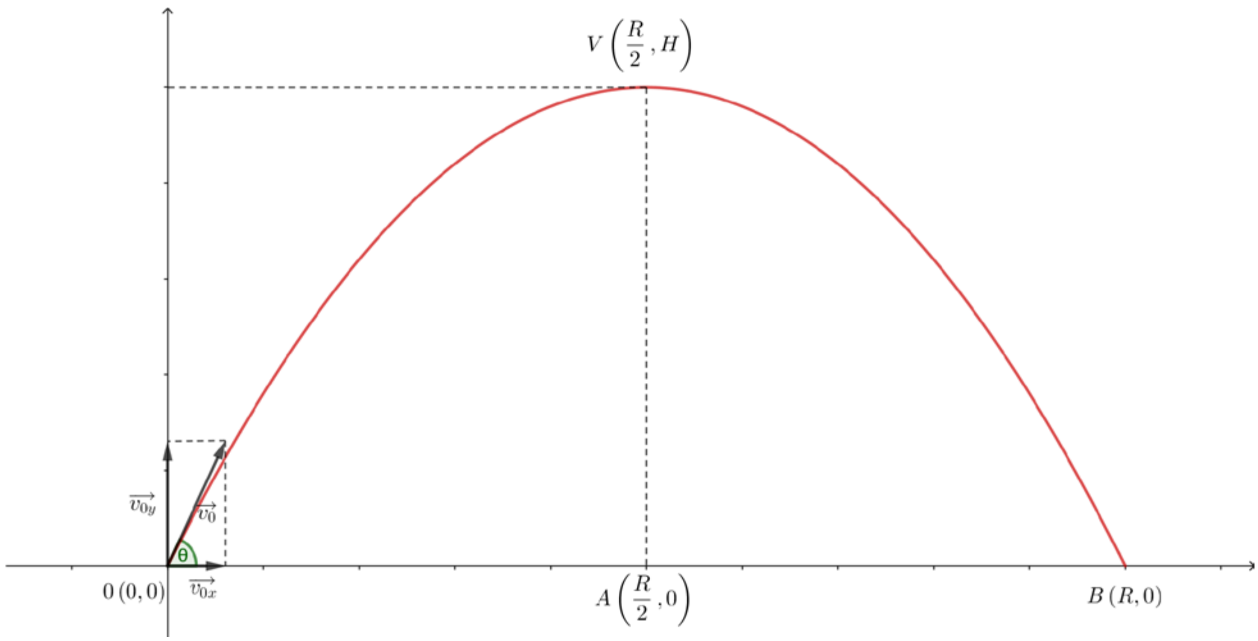
Lanching from ground level



We use notations:

$O(0,0)$ – launching point, ground level

$B(R,0)$ – arriving point, ground level

R – range (horizontally)

T – time

H – maximum height

$V\left(\frac{R}{2}, H\right)$ – vertex of parabola (turning point)

$A\left(\frac{R}{2}, 0\right)$ – projection of V on x-axis

$\overrightarrow{v_0} = \overrightarrow{v_{0x}} + \overrightarrow{v_{0y}}$, $v_0^{\ 2} = v_{0x}^{\ 2} + v_{0y}^{\ 2}$

$v_{0x} = v_0 \cos\theta,\ v_{0y} = v_0 \sin\theta$

$v_x = v_{0x},\ \forall t \in [0, T]$ constant $\Rightarrow R = v_{0x}T \Rightarrow v_{0x} = \frac{R}{T}$

$v_y = v_{0y} - gt$. In point V : $t = \frac{T}{2},\ v_y = 0 \Rightarrow v_{0y} = \frac{gT}{2}$

In point A: $x = \frac{R}{2},\ t = \frac{T}{2},\ \theta = 0$. In point B: $y = 0$

Horizontal movement: $x = v_{0x}t$ (uniform movement, constant velocity)

Vertical movement: $y = v_{0y}t - \frac{gt^2}{2}$ (uniformly varying movement, constant acceleration)

Case I: We know angle θ and range R

- Finding T

Removing t from the two movement equation: $t = \frac{x}{v_{0x}}$ we get

$y = v_{0y} \cdot \frac{x}{v_{0x}} - \frac{g}{2} \cdot \frac{x^2}{v_{0x}^{\ 2}} = \frac{v_0 \sin\theta}{v_0 \cos\theta} \cdot x - \frac{g}{2} \cdot \frac{T^2}{R^2} \cdot x^2$, hence $y = x \operatorname{tg}\theta - \frac{gT^2}{2R^2} \cdot x^2$ (parabola).

In points O and B, there is $y = 0$ so $x_1 = 0,\ x_2 = R$ are roots of parabola equation.

But then, $y = 0 \Leftrightarrow x \operatorname{tg}\theta - \frac{gT^2}{2R^2} x^2 = 0$, hence $x_1 = 0,\ x_2 = \frac{2R^2 \operatorname{tg}\theta}{gT^2}$.

From last two expressions of $x_2 \Rightarrow \frac{2R \operatorname{tg}\theta}{gT^2} = 1$ hence $T = \frac{\sqrt{2Rg \operatorname{tg}\theta}}{g}$ (1)

- Finding H

In vertex V: $t = \frac{T}{2}$ și $v_y = 0$, hence $v_{0y} = gt$.

$$y = H \text{ și } t = \frac{T}{2}, \text{ hence } H = v_{0y} \cdot \frac{T}{2} - \frac{g}{2} \cdot \frac{T^2}{4} = g \cdot \frac{T}{2} \cdot \frac{T}{2} - \frac{g}{2} \cdot \frac{T^2}{4} = \frac{gT^2}{8}$$

From (1) $\Rightarrow H = \frac{R \operatorname{tg}\theta}{4}$ (2)

- Finding v_0

We have $v_{0x} = v_0 \cos\theta$. On the other hand $v_{0x} = \frac{R}{T}$. From last two we get $v_0 = \frac{R}{T\cos\theta}$.

From (1) we get $T = \frac{\sqrt{2Rg \operatorname{tg}\theta}}{g}$ and replacing we get $v_0 = \frac{R}{\frac{\sqrt{2Rg \operatorname{tg}\theta}}{g} \cdot \cos\theta} = \frac{\sqrt{Rg}}{\sqrt{2\frac{\sin\theta}{\cos\theta} \cdot \cos^2\theta}}$,

hence $v_0 = \sqrt{\frac{Rg}{\sin 2\theta}}$, or $v_0 = \frac{\sqrt{Rg \sin 2\theta}}{\sin 2\theta}$ or $v_0^{\ 2} = \frac{Rg}{\sin 2\theta}$ (3).

So, when we know angle θ and range R we can find T, H, v_0 from formulas (1), (2), (3).

Case II: We know angle θ and initial velocity v_0

From $v_0^{\ 2} = \frac{Rg}{\sin 2\theta}$ (3) we get $R = \frac{v_0^{\ 2} \sin 2\theta}{g}$ (4)

From $H = \frac{R \operatorname{tg}\theta}{4}$ (2) we get $H = \frac{v_0^{\ 2} \sin^2\theta}{g}$ (5)

From $T = \frac{\sqrt{2Rg \operatorname{tg}\theta}}{g}$ (1) we get $T = \frac{\sqrt{2 \cdot \frac{v_0^{\ 2} \sin 2\theta}{g} \cdot g \operatorname{tg}\theta}}{g} \Rightarrow T = \frac{2v_0 \sin\theta}{g}$ (6)