Standing waves in Bohr’s atomic model

Overview

The purpose of this demonstration is to give some insight on **Bohr's atomic model** and to symbolically and visually show how a wave, following a circular closed path, can ***stand still***.

This requirement (standing waves) brings forth the **quantization** condition for the allowed orbital radii in Bohr's atomic model.

A standing wave (***n*** integer in the demonstration) interferes constructively with itself and doesn't show any apparent movement (so there's no acceleration, no E-M radiation and no loss of energy).

On the contrary, if the wave is not a standing wave (***n*** not integer in the demonstration) there's destructive interference and movement (so there's acceleration, radiation, loss of energy and no possible stable orbit).

Usage of the controls

The main controls are the *time* slider and the *n* (number of wavelengths in the circle) slider.

In both modes it's possible to interactively change the *n* value. The option “make stationary” change the *n* value to the nearest integer to make the wave a *stable* standing wave.

Details

The problem and the solution

According to **Rutherford**'s model the electron orbits around the nucleus like a planet around the sun. The problem was that an accelerated charge emits electromagnetic waves and, in doing so, loses energy. The unavoidable result would be an electron quickly collapsing in the nucleus.

To solve this problem **Bohr**'s model (with the support of **De Broglie**'s arguments) introduced the ***particle-wave*** duality: an electron behaves both as a particle and as a wave. Only specific discrete orbits are allowed.

But how does this *bold* hypothesis solve Rutherford's problem? Be the electron wave, be it particle or be it a mix of both, come what may, that doesn't actually change the fact that it must have some energy, some movement to counterbalance the attractive electric force. Even as a wave the electron should orbit around the proton and have an acceleration, and that cannot be. So it must have energy and it must stand... something like a wave (has energy) that stands still... How can it be?

... but there's a possible solution ...

A **standing wave!**

It's a wave so it has energy, yet it stands still. That should be the possible answer, the way to go.

Another way to bring forth the same underlying concept (more physically correct but more difficult to be visually represented) is that a non standing wave would **interfere with itself destructively**.

On the other hand a standing wave always **interfere constructively** with itself and that makes it exist as a physical entity (and makes it stay alive).

But how does an electron know which way to go around the nucleus?

Following **Feynman**'s way of reasoning we could think that it actually goes all ways through **all possible paths** and orbits. But along the paths where there's destructive interference there's no resulting wave, so no "real" electron left, no physical entity and no possible path/orbit. Just a faint theoretical *ghost* that cannot surface the boundary between *nothingness* and the dimension of a physical measurable reality.

Historical notes: Bohr's and De Broglie's contributions

Niels Bohr elaborated his atomic model in the years 1911-1913. He found out the ***quantization condition*** of angular momentum just because that was necessary to account for the experimental data (and for the Balmer's formula of spectral lines). He also followed, with great ingenuity, pure dimensional arguments. But he himself was aware that his model *didn't explain anything*.

It was only in 1924 that **Louis De Broglie** reached a deeper and wider perception of what was behind Bohr's quantization condition of angular momentum: the *wave nature* of the electron in a stable state and the *standing wave* condition.

The *standing wave* condition gives an elegant and deeper reason to Bohr's (unexplained) quantization formula of the electron angular momentum.

With De Broglie's contribution the Bohr's model became more meaningful: a fresh starting point towards a new paradigm and towards a better understanding of the basic principles of the early quantum mechanics theory.

Notes about the demonstration

In this demonstration the Bohr's model is applied to a single electron orbiting around a single proton (hydrogen atom). The number ***n*** (number of wavelengths in a complete circular orbit) is connected with the possible orbital radii and with the possible energy levels. The stable states are only the ones associated with an integer value of n. The standing wave condition is $n\lambda = 2\pi r$ (with n integer). In this model the main formulas for the possible orbital radii (r_n), orbital speeds (v_n), orbital periods (T_n) and energy levels (E_n) are:

$$r_n = n^2 \frac{h^2 \epsilon_0}{\pi m_e e^2};\quad v_n = \frac{1}{n}\frac{e^2}{2 h \epsilon_0};\quad T_n = n^3 \frac{4 \epsilon_0^2 h^3}{m_e e^4};\quad E_n = -\frac{1}{n^2}\frac{m_e e^4}{8 \epsilon_0^2 h^2};$$

The time unit used in the demonstration (in the main slider) is a symbolic one, only proportional to the actual physical time (τ). For convenience here t is just the angle spanned by the wave front ($t = \alpha(\tau)$) so that the orbital period (T) is just 2π. The formula to convert this time unit (t) to the real one (τ) is:

$$\tau = t \frac{1}{2\pi} n^3 \frac{4 \epsilon_0^2 h^3}{m_e e^4}$$

By changing the value of n also the orbital radius should change accordingly. This is not presented in the simulation as it's out of its scope (there is just a qualitative change in the blue disk size, symbolically representing the nucleus, to evoke a change of scale).

In this simulation the wave is traced only for a limited time span (3 periods) and it fades away with time within this interval. This is just an artifice to emphasize the wave movement in case of non standing waves.

In the case of *not standing* waves (not physically possible) occurring when n is not an integer number, the wave seems to move around with a speed that is different from the speed of the wave front. It's slower when n approaches an integer number and when n is higher. It can be shown that the absolute value of the wave speed v_w (with respect of the front speed v) goes like $v_w = \frac{|\text{Round}[n]-n|}{\text{Round}[n]} v$. Its local maximum values occur when $n = k + \frac{1}{2}$ with $k = 1, 2, 3, \ldots$ and there are discontinuities around these *special* values. The maximum wave speed occurs for $n = 1.5$ where $v_w = \frac{1}{2} v$.

Maybe there is some connection between this relationship and the *phase velocity/group velocity* relationship of wave mechanics but I didn't investigate it further.

Some useful link

http://galileo.phys.virginia.edu/classes/252/Bohr_Atom/Bohr_Atom.html

http://galileo.phys.virginia.edu/classes/252/Bohr_to_Waves/Bohr_to_Waves.html

https://en.wikipedia.org/wiki/Bohr_model

https://www.lucamoroni.it/simulations/standing-waves-in-bohrs-atomic-model/

https://youtu.be/_NFuGuVlyKc

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