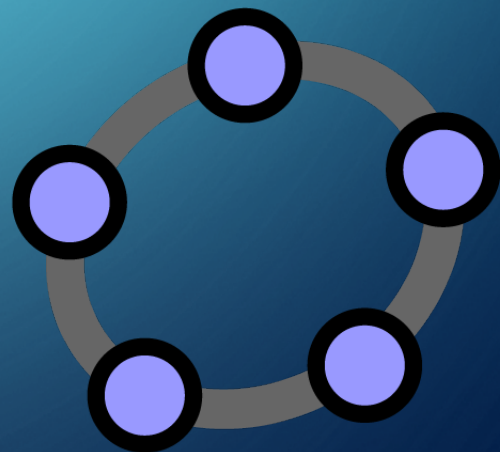


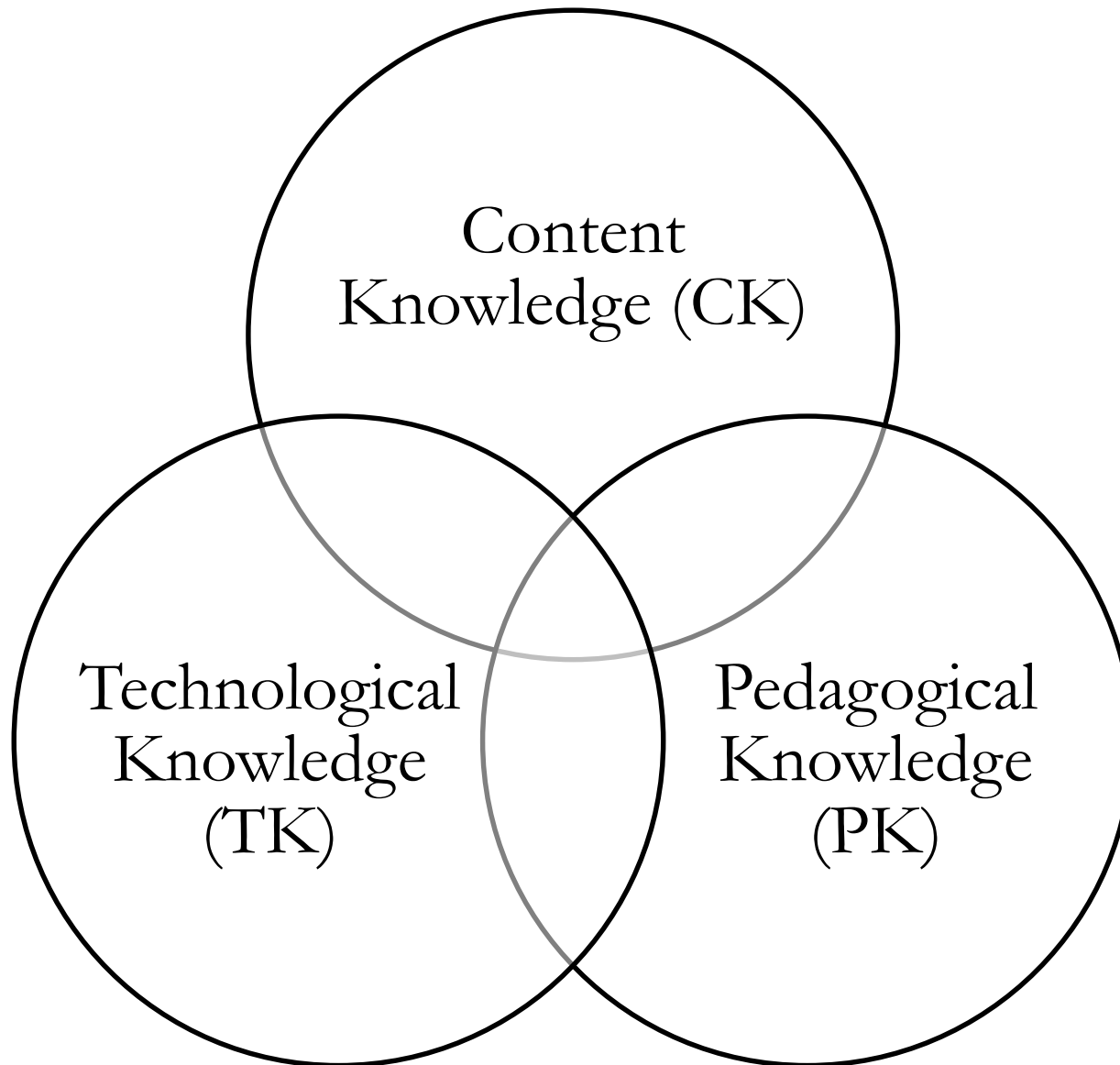
中學數學電子學習系列：(2) 在數學課堂有效運用 GEOGEBRA（進階程度）

教育局數學教育組

2022 年



術 Technology & 道 Pedagogy



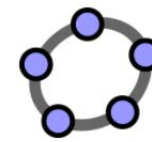


GeoGebra 簡介

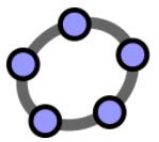
- GeoGebra 是為了小學到大學的教學而設計的開源 (open source) 動態數學軟體。
- GeoGebra 是一套結合幾何（平面+立體）、代數、統計及微積分等的免費動態幾何軟件，它是在 2001 年由 Markus Hohenwarter 在奧地利的 Salzburg 薩爾茨堡大學所設計。
 - GeoGebra 其實就是他的碩士論文。
 - 目前在奧地利 Linz 的 Johannes Kepler 大學（克卜勒大學）擔任數學教學研究所所長。
- GeoGebra 是由 Java 寫成的，因此可以跨平台使用。
- GeoGebra 的一些學與教用途：
 - 教師用於課堂演示互動幾何圖像；
 - 學生用於探索與發現幾何概念，猜想幾何定理。
- 2011: [38 developers](#) & 200 translators Celebrating 10 years of GeoGebra

*If you want to go fast, go alone.
If you want to go far, go together.*

下載與安裝

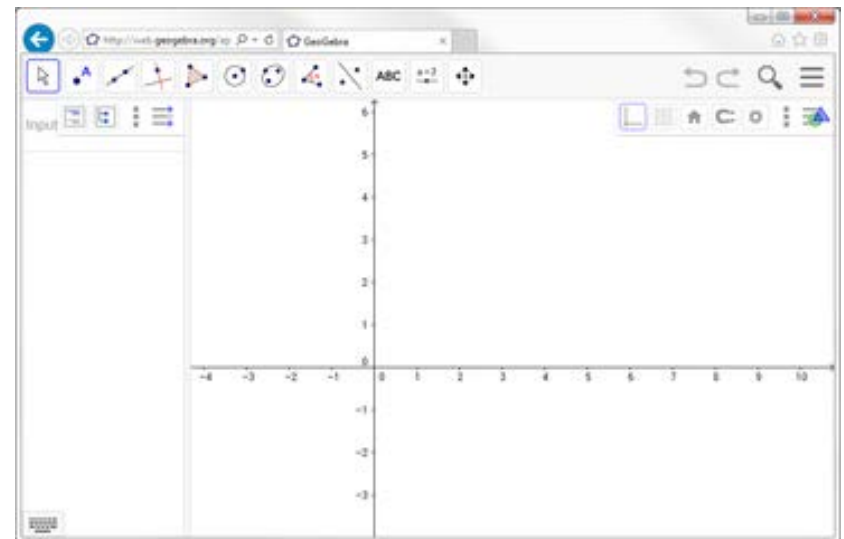
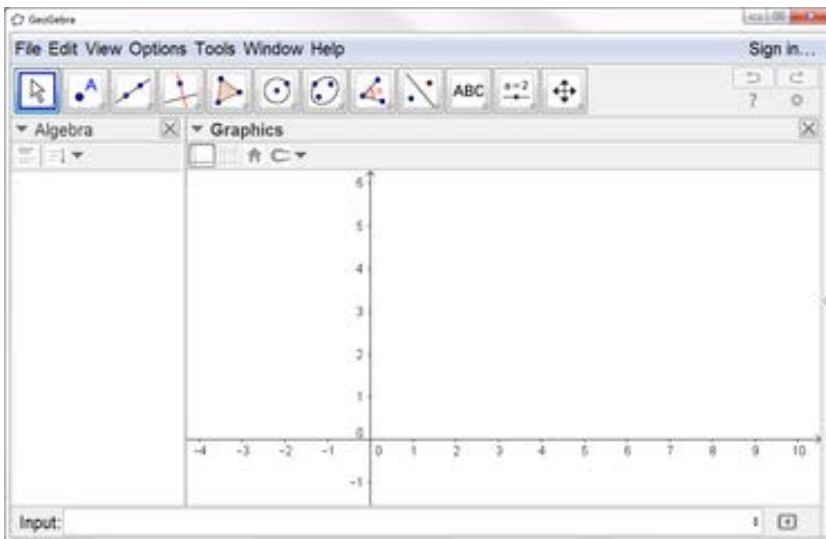
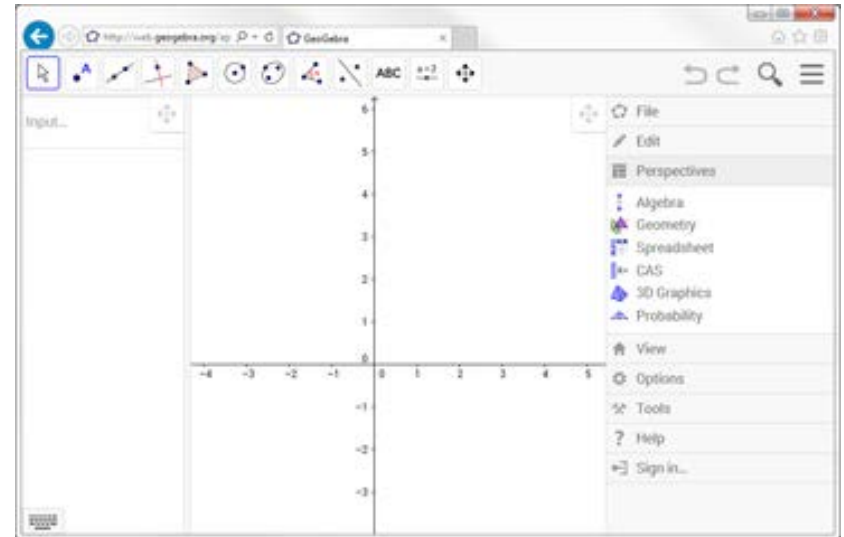
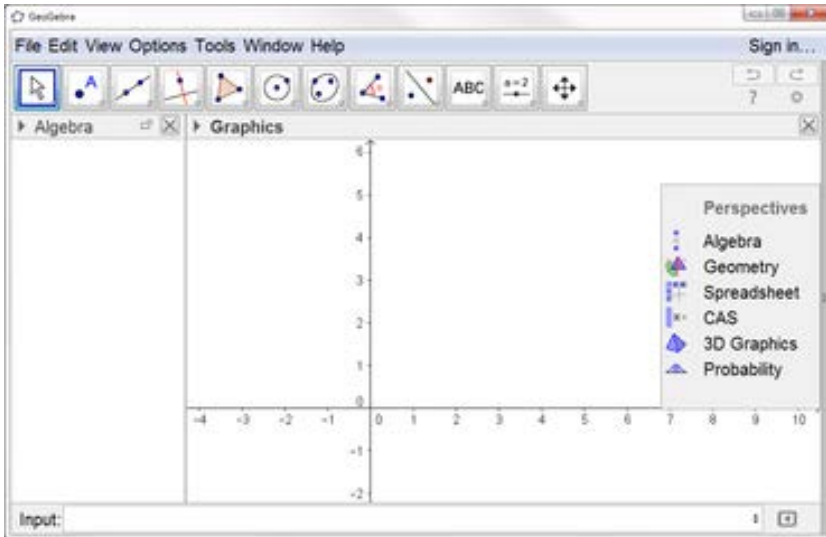


GeoGebra 經典 6










GeoGebra 經典 5

- GeoGebra Desktop 桌機版 vs. Web and Tablet App 線上版或平板版



Comparison of GeoGebra Math Apps

apps / features	 Scientific	 Graphing	 Geometry	 Suite	 3D	 CAS	 Classic
Numeric calculations	✓	✓	✓	✓	✓	✓	✓
Function operations	✓	✓	✓	✓	✓	✓	✓
Fraction operations	✓	✓	✓	✓	✓	✓	✓
Graphing		✓	✓	✓	✓	✓	✓
Sliders		✓	✓	✓	✓	✓	✓
Vectors and matrices		✓	✓	✓	✓	✓	✓
Table of values		✓		✓		✓	✓
Geometric constructions			✓	✓	✓		✓
3D graphing				✓*	✓		✓
Symbolic calculations				✓*	✓	✓	✓
Derivatives & integrals				✓	✓	✓	✓
Equation solving				✓	✓	✓	✓

*coming soon

GeoGebra: Books

- Mathematical Modeling: Applications with GeoGebra
 - Hall, J., Lingefjärd T. (2016). *Mathematical Modeling: Applications with GeoGebra*. New York: Wiley. [568 pages]
- GeoGebra - 幾何與代數的美麗邂逅
 - 羅驥韡 (2013) 。《GeoGebra 幾何與代數的美麗邂逅》。臺北市：五南。

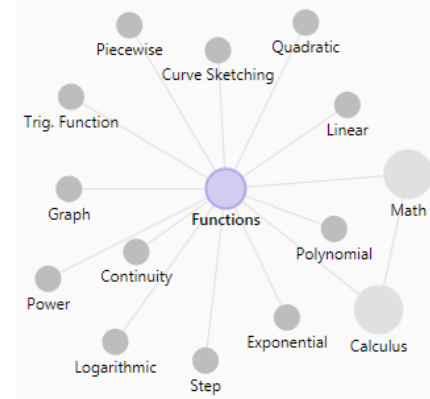
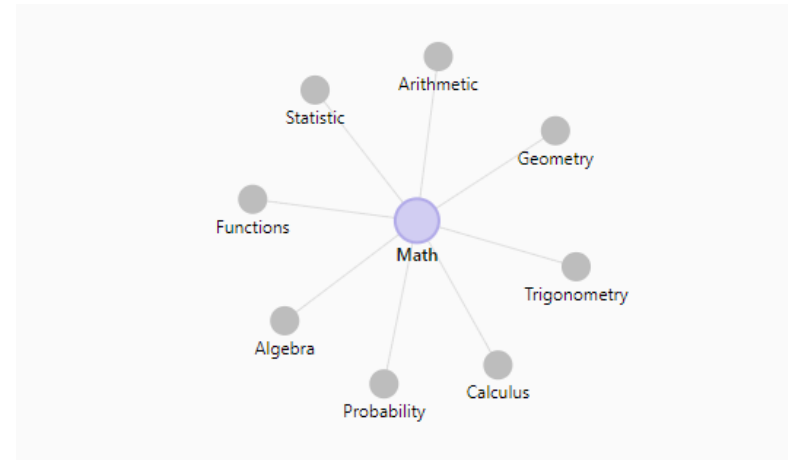
GeoGebra Classroom

GeoGebra Classroom is a virtual platform through which teachers can

- assign **interactive** and engaging tasks for students
- view **live updated progress** of students working on a specific task
- view which tasks students have (or have not) started
- ask the entire class questions and see **all student answers instantly**
- hide student names when displaying student responses to questions
- facilitate rich, interactive discussions among all students, groups of students, and individual students

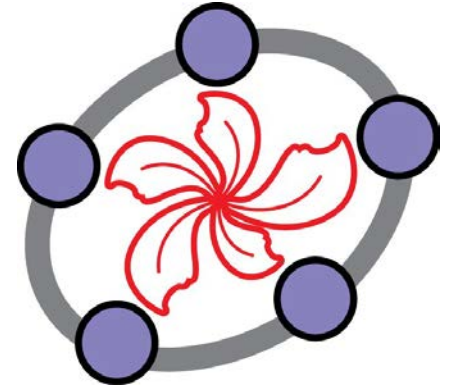
GeoGebra: Resources

- Find over 1 million free activities, simulations, exercises, lessons, and games for math & science!
 - <https://www.geogebra.org/materials>
 - <https://www.geogebra.org/t/math>
 - <https://www.geogebra.org/m/bgmn44x5>
- Tutorials:
 - <https://wiki.geogebra.org/en/Tutorials>
 - <https://www.geogebra.org/a/14>
- Learn Notes
 - <https://www.geogebra.org/m/fp7bctpr>
- Virtual Whiteboards for GeoGebra Classroom
 - <https://youtu.be/jGFyXc1qRGw>



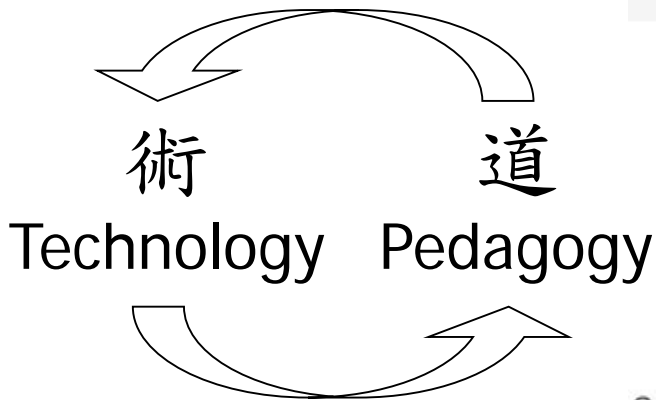
GeoGebra: Resources

- GeoGebra Institutes
 - <https://www.geogebra.org/institutes>
- GeoGebra Institute of Hong Kong (GIHK)
 - <http://www.geogebra.org.hk>
- Applets in Tablets: GeoGebra 數學電子教室
 - <https://www.gmath.hk/>
 - <http://www.geogebra.hk/>
 - <http://www.geogebra.hk/reference>
 - (Learning GeoGebra from Examples)
<https://www.geogebra.org/m/FZFjhBaa>
 - (Graph Plotter 2018)
<https://www.geogebra.org/m/wQtHUaaa>

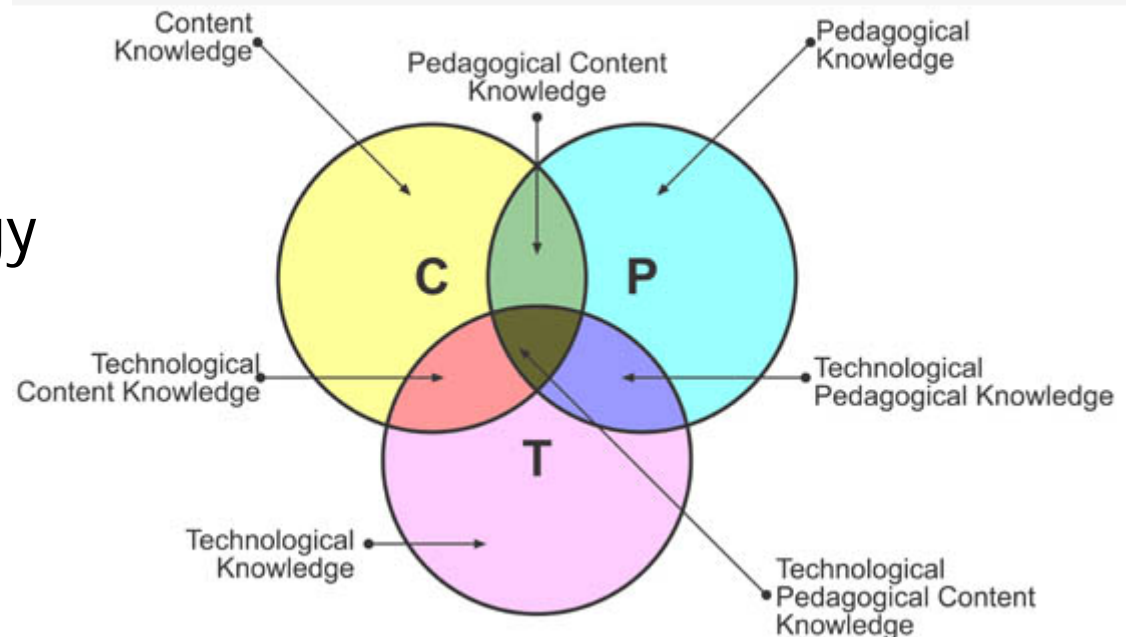


TPACK

- Technological pedagogical and content knowledge (TPACK) refers to teachers integrating technology with pedagogy (teaching methods) and content.
 - <https://julianaliebke.wordpress.com/literature-review/>

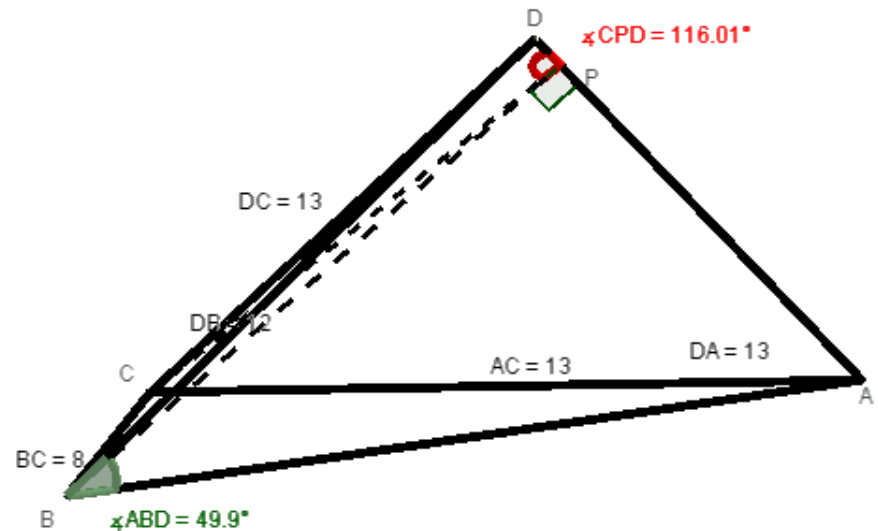
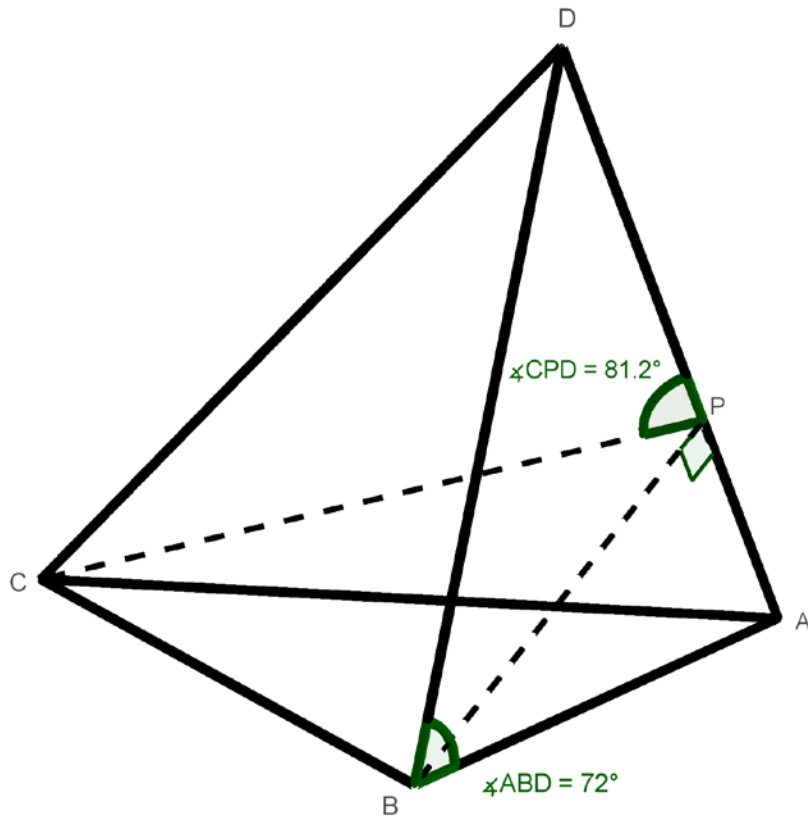


Technological Pedagogical Content Knowledge



Example: 2019 DSE Math 1 Q18

“ $\angle BPC$ is the angle between the face ABD and the face ACD ”?



*How GeoGebra (and other IT tools)
may enrich classroom L&T?*

Mathematics Education Section

Advantages of GeoGebra

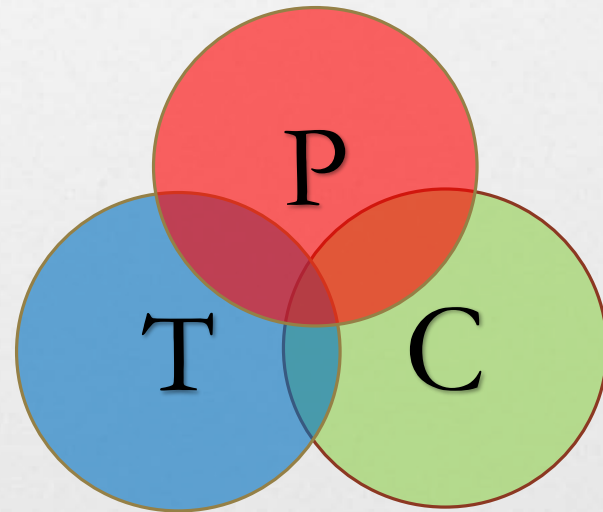
- Dynamic
- Extremely rich in functions
- Fast to observe and conclude

Disadvantages of GeoGebra

- Dynamic without knowing the mechanisms
- Too rich in functions without knowing the focus
- Too fast to observe and conclude only by superficial observations

Key question

- What to teach?
- How to use?
- How to **refine**?

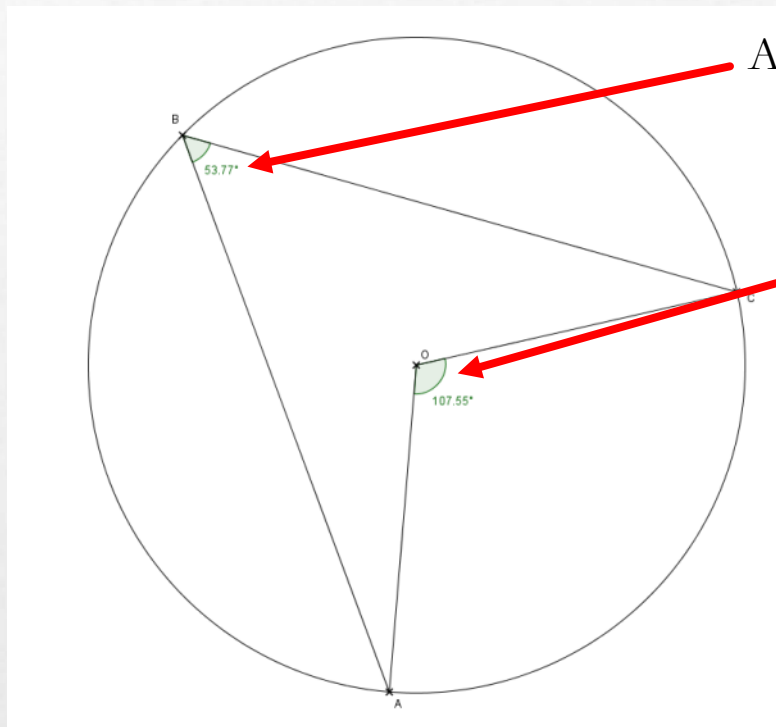


L&T in Properties of Circles

- A refinement process on L&T package using dynamic geometry software
- A theorem on the properties of circles



The theorem

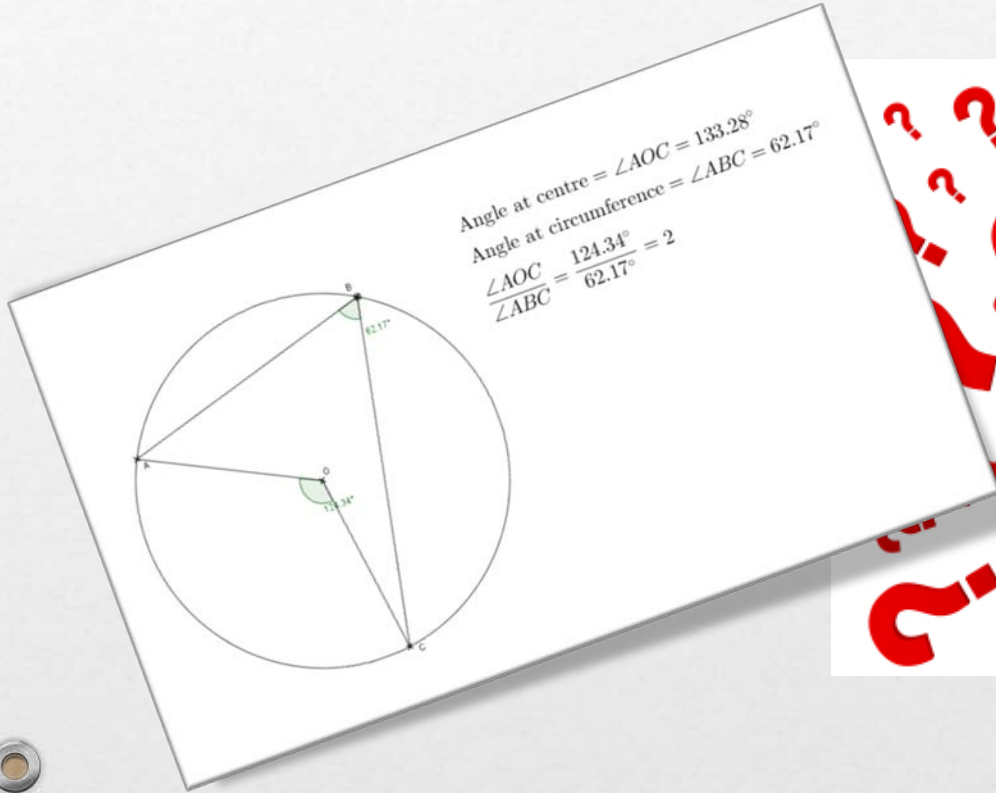


Angle at circumference

Angle at centre

“Angles at centre twice angles at circumference”

The proof



Does it work?

Pedagogical considerations?

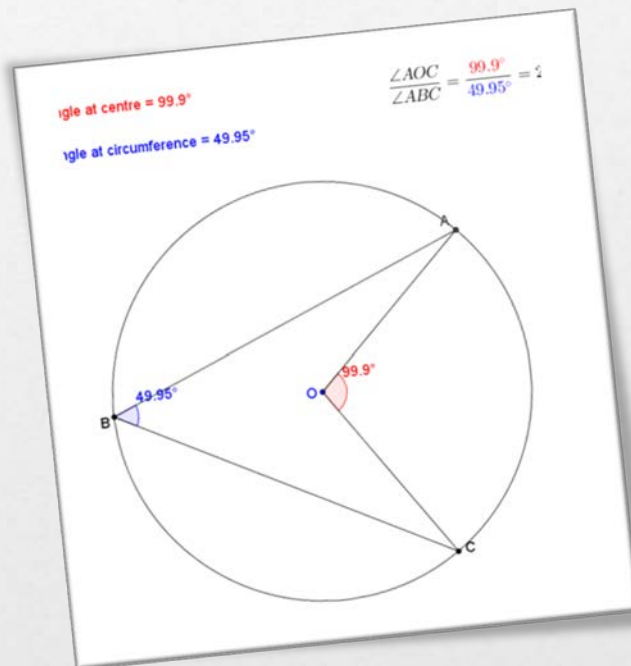
How can a task aim at
the difficulties?

What are students'
difficulties?



Refinement: From visualisation to abstraction

Understand the limitations

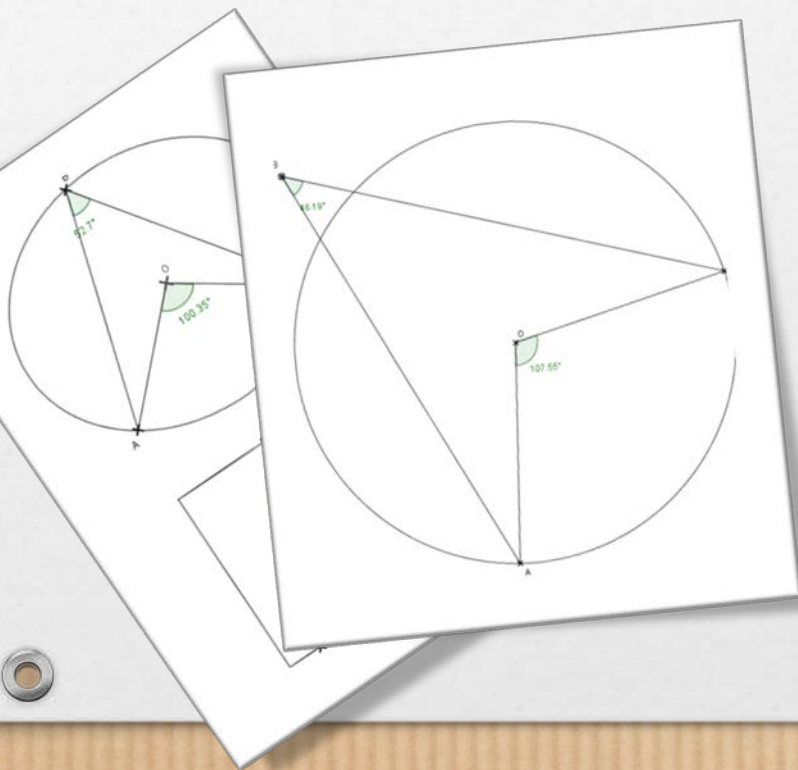


Dynamic geometry vs Euclidean Geometry?

Computation work vs Mind work

Refinement: From visualisation to abstraction

Stretch the potential



Re-reading a geometric theorem:

In a circle, an angle at any points of the circumference is half of the angle subtended by the same arc of the circle at the centre.

Conditions and consequences:

Pedagogy of variations

A complete cycle of teaching

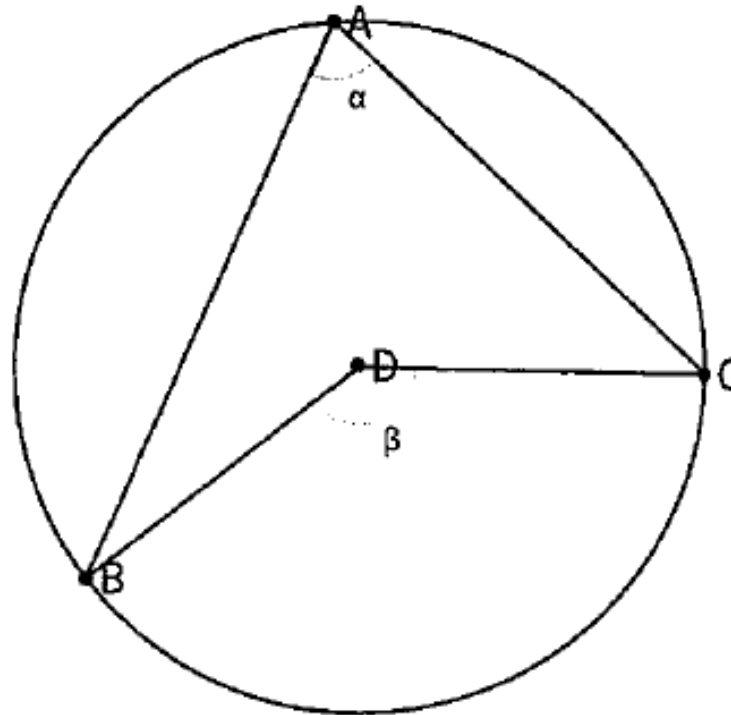
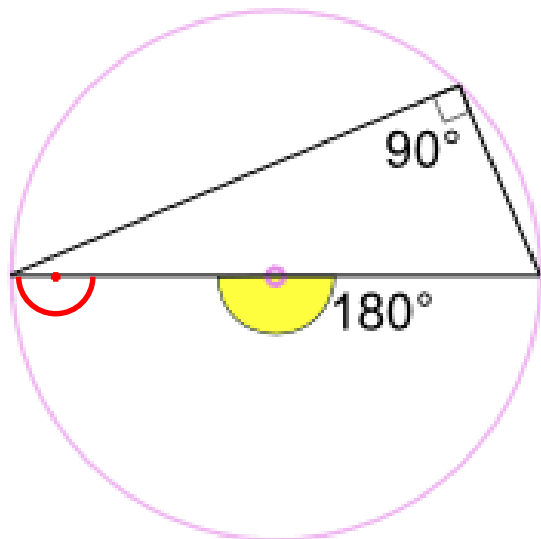
- Examples versus verifications
- Counter-examples versus constraints

Re-visit the theorem: Think about it

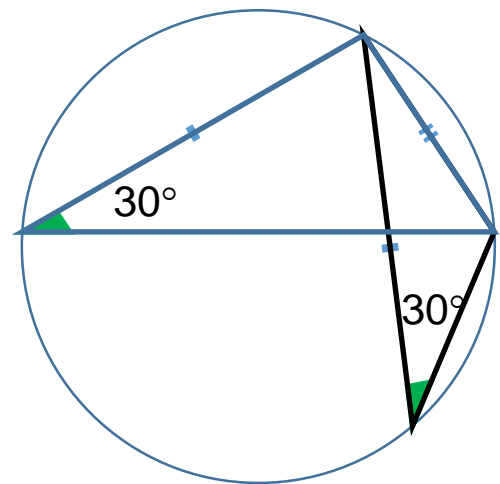
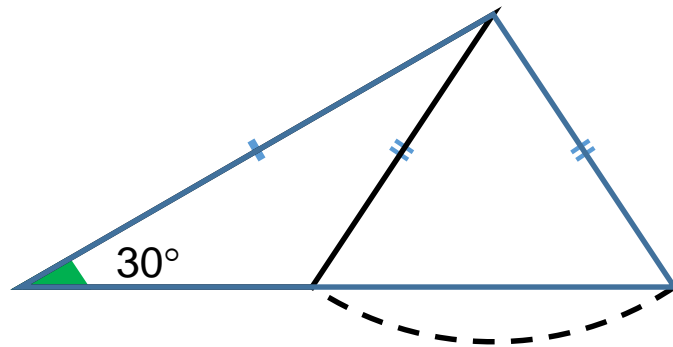
Section D: Think about it

Question 2

Cody claims that if $\beta = 2\alpha$, D must be the centre of the circle. Do you agree? Explain your answer.



學習重點	時間	注釋
<p>11.2 理解圓上角的性質</p> 		<p>圓上角的性質包括：</p> <ul style="list-style-type: none"> ➔ 一弧所對的圓心角為該弧所對的圓周角的兩倍 ➔ 同弓形內的圓周角皆相等 • 弧與所對的圓周角成正比例 ➔ 半圓內的圓周角為直角 ➔ 若圓周角是一直角，則其所對的弦是一直徑



To conclude

Technology = TOOL

Right technology at the Right time for the Right task

