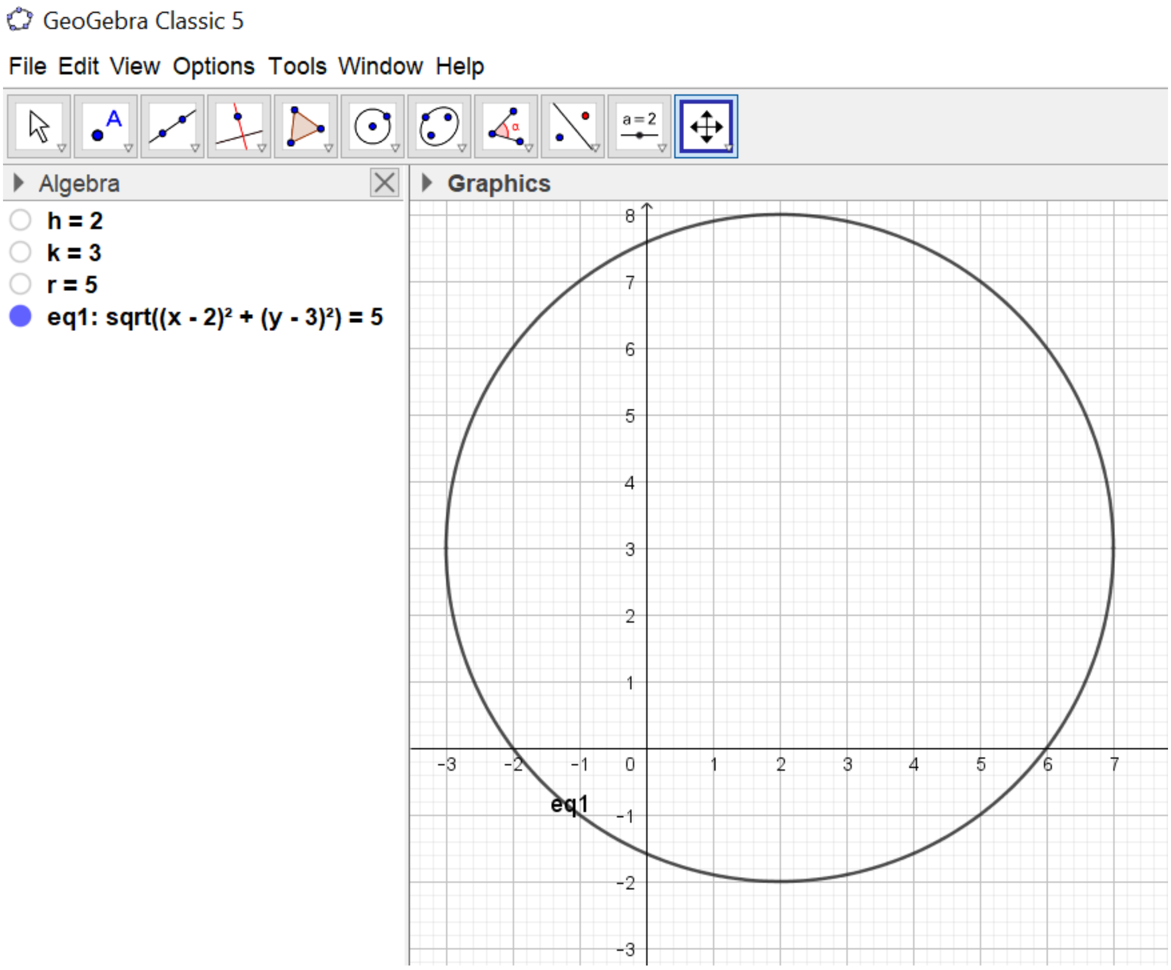
Two Definitions of a Circle

The traditional definition of a circle is “the set of all points which are the same distance from a given point, called the centre.” We make use of this definition of a circle when we take our compasses and draw, for example, a circle with a radius of 12 cm, centred on the page.

Now consider co-ordinate geometry. We can modify our definition, as follows. “A circle is the set of points P (x, y) which are the same distance from the centre Q (h, k).” In the coordinate plane, the distance between these two points is given by formula $D = \sqrt{(x-h)^2 + (y-k)^2}$. Below is the result with D = radius = 5 units and centre Q (2, 3).



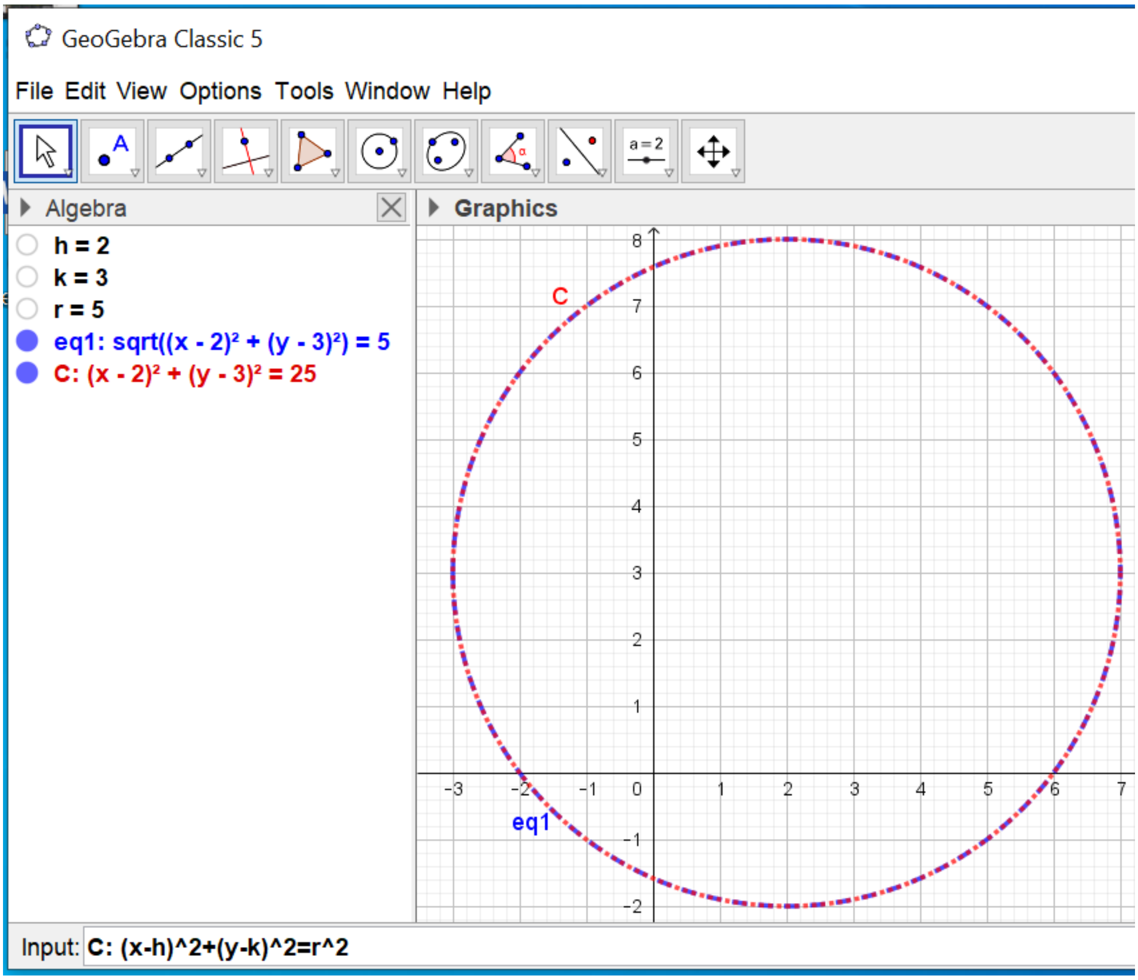
While this gives the correct result, the tradition is to work with D^2.

$$D^2 = \sqrt{(x-h)^2 + (y-k)^2}^2 = (x-h)^2 + (y-k)^2 .$$

Finally, we typically

- give the circle a name, say, C.
- change D to r, to emphasise that the relevant distance is length of the radius
- move the r^2 term to the RHS, to emphasise that the circle equation is actually the Pythagorean equation, $b^2 + a^2 = c^2$, with base, $b = x - h$, altitude, $a = y - k$, and hypotenuse, $c = r$.

Here is the result



The screenshot shows the original definition in blue and the modified definition in red. Although not a proof that the two equations represent the same object; nevertheless, I believe you will find it convincing.