Gandaki Boarding School
(National School)
Lamachaur, Pokhara

THIRD TERMINAL EXAMINATIONS, 2079

Class: 10
Subject: Opt. Mathematics

Full Marks: 100
Time: 3 hours

Attempt all the questions.

Group: 'A' $[10\times1=10]$

1. a. What is the maximum value of $y = cosx$?
   b. What is the arithmetic mean between two numbers a and b?

2. a. Express in words: $\lim_{x\to a^+} f(x)$
   b. If the matrix $A = \begin{pmatrix} x & -y \\ y & x \end{pmatrix}$, find its determinant.

3. a. Write the condition of the two lines given by $ax^2 + 2hxy + by^2 = 0$ to be orthogonal.
   b. What is called a conic section formed when a cone is interested by a plane surface which is perpendicular to the axis?

4. a. Express $cos2A + cos2B$ into product from.
   b. Write the relation between $sinA$ and $sin3A$.

5. a. If $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are the position vectors of two points A and B, then express the position vender of the points P which is midpoint of line segment AB.
   b. If R_1 = reflection on x-axis and R_2 = reflection on y-axis, state the single transformation for R_1oR_2.

Group: 'B' $[13\times2=26]$

6. a. If $f(x) = 2x - 3$, find the value of $f^{-1} = (2)$.
   b. Find the remainder when a polynomial $3x^2 - 4x + 6$ is divided by $x + 2$.
   c. Find the 8th term $2 + 1 + \frac{1}{2} + \frac{1}{4} \ldots\ldots$.

7. a. If the matrix $P = \begin{pmatrix} 2 & x \\ 6 & 9 \end{pmatrix}$ has no inverse, find the value of x.
   b. Find the inverse of matrix $A = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$.

8. a. Show that the lines $y + 2x + 4 = 0$ and $x - 2y + 7 = 0$ are perpendicular to each other.
   b. Find the acute angle between the pair of lines $x^2 - 4xy + y^2 = 0$.

9. a. If $tanA = \frac{3}{4}$, find the value of $sin2A$.
   b. Prove that: $sin15^0 + cos15^0 = \sqrt{\frac{3}{2}}$
   c. Solve: $2cos\theta - 1 = 0 \quad (0^0 \le \theta \le 360^0)$

10. a. For what value of x, vectors $2\vec{i} - 3\vec{j}$ and $x\vec{i} - 2\vec{j}$ are perpendicular to each other?
    b. Point C divides the line AB internally in the ratio of 3:1. If position vectors of A and B are $\vec{i} - 3\vec{j}$ and $2\vec{i} - 5\vec{j}$ respectively, find the position vector of point C.
    c. In a data, the quartile deviation and its coefficient are 14 and $\frac{7}{22}$ respectively. Find the third quartile.

Group: 'C' $[11\times4=44]$

11. Solve: $x^3 - 9x^2 + 24x - 20 = 0$

12. Maximize P=6x+5y under the following constraints.
    $x + y \le 6,\ x - y \ge -2,\ x \ge 0,\ y \ge 0$

13. $f(x) = \begin{cases} 5x+1 & for\ x < 1 \\ 4 & for\ x = 1 \\ 6x & for\ x > 1 \end{cases}$

Is the above function continuous at x=1? ~~Find it~~. Redefine it.

14. Solve by matrix method.

$4x - \frac{9}{y} + 11 = 0$ and $\frac{6}{y} - 3x = 8$

15. Find the single equation of a pair of lines passing through the origin and perpendicular to the pair of lines given by $2x^2 - xy - 3y^3 = 0$.

16. If $A + B + C = \pi^C$, prove that $sin^2\frac{A}{2} + sin^2\frac{B}{2} - sin^2\frac{C}{2} = 1 - 2cos\frac{A}{2}.cos\frac{B}{2}.sin\frac{C}{2}$.

17. Prove that: $sin\theta\ \sin(60-\theta)\sin(60+\theta) = \frac{1}{4}sin3\theta$.

18. From the top of a vertical column of 60 m, the angles of depression of the top and the bottom of a house on the same plane are 30^0 and 60^0 respectively. Find the height of the house and its distance from the column.

19. Find 2x2 matrix which transform a unit square to a quadrilateral $\begin{bmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$.

20. Find the mean deviation from the median.

Marks	0-10	10-20	20-30	30-40	40-50
No. of students	20	40	60	50	30

21. Calculate the standard deviation.

Age	20-40	40-60	60-80	80-100
No. of persons	5	10	15	25

Group: 'D' $[4\times5 = 20]$

22. There are n GMs between 1 and 64. If the ratio of first mean to last mean is 1:16, find n.

23. Find the equation of circle passing through the points (1, 2), (3, 1) and (-3, -1).

24. Prove by vector methods that diagonals of rhombus bisect each other at right angles.

25. A(2, 5), (B(-1, 3) and C(4, 1) are the vertices of ΔABC. Find the coordinates of the vertices of the images of the ΔABC under rotation of negative 90^0 about origin followed by the enlargement E[(0, 0), 2]. Present all objects and images in the same graph.

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