NEWTON'S LAW OF COOLING

This says that the **rate** of temperature loss of an object is directly proportional to the difference between its temperature, at any time, and the temperature of its environment. For our purposes we will assume that the environment temperature is constant; this means that it is a large enough volume (usually of air) that the heat added to it as this object cools does not raise the temperature of the room.

Mathematically, this situation is expressed in what is known as a “differential equation”

$$\frac{d\left(\Delta T\right)}{dt} = -k\left(\Delta T\right) \tag{1}$$

and, using calculus, we can solve this to find the temperature as a function of time:

$$T(t) = \left(T_0 - T_{room}\right)e^{-k\,t} + T_{room} \tag{2}$$

A simple experiment was done to see if this works. A small juice can was filled with hot tap water, and its temperature was observed over a period of a few hours. This data is shown in the graph below, along with a curve that is based on Eq(2). The parameters in that equation (the initial temperature, the room temperature, and the cooling rate constant) were estimated from the data using a nonlinear regression procedure. That process led to this equation, shown in the plot:

$$T(t) = \left(121.5 - 75.25\right) e^{-0.0178\,t} + 75.25 \tag{3}$$

We can see in the plot that the agreement is pretty good, although the estimated room temperature of 75.25 degrees F is a bit higher than it really was. As the temperature drops close to the room temperature, it becomes difficult to know what time to assign to the temperature readings.

Notice that the slope of the curve, which is the rate of change of the temperature, is much steeper at the beginning than it is at the end of the data set. When the temperature difference is large, the rate of change is large, and as the water temperature approaches the room temperature, the rate of change approaches zero, so the slope is shallow, and eventually would be horizontal, or zero slope.