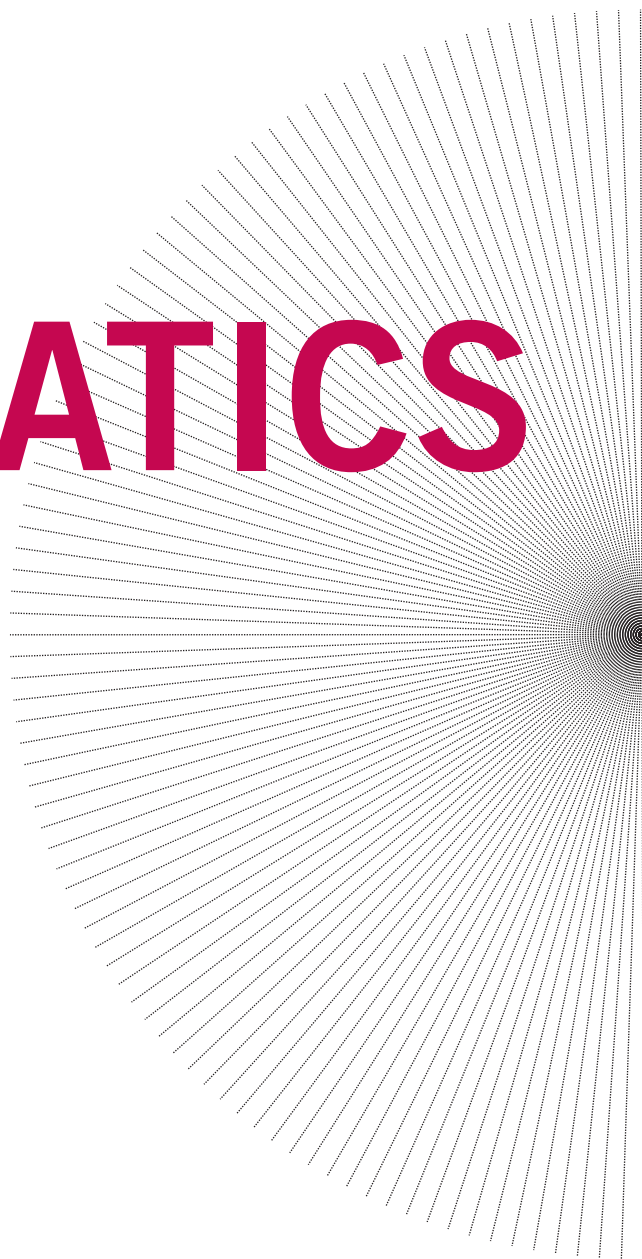


MATHEMATICS



Leaving Certificate Mathematics

Introduction and rationale

Mathematics is a wide-ranging subject with many aspects. Most people are familiar with the fact that mathematics is an intellectual discipline that deals with abstractions, logical arguments, deduction and calculation. But mathematics is also an expression of the human mind reflecting the active will, the contemplative reason and the desire for aesthetic perfection. It is also about pattern, the mathematics of which can be used to explain and control natural happenings and situations. Increasingly, mathematics is the key to opportunity. No longer simply the language of science, mathematics contributes in direct and fundamental ways to business, finance, health and defence. For students it opens doors to careers. For citizens it enables informed decisions. For nations it provides knowledge to compete in a technological community. Participating fully in the world of the future involves tapping into the power of mathematics.

Mathematical knowledge and skills are held in high esteem and are seen to have a significant role to play in the development of the knowledge society and the culture of enterprise and innovation associated with it. Mathematics education should be appropriate to the abilities, needs and interests of learners and should reflect the broad nature of the subject and its potential for enhancing their development. The elementary aspects of mathematics, use of arithmetic and the display of information by means of a graph are an everyday occurrence. Advanced mathematics is also widely used, but often in an unseen and unadvertised way. The mathematics of error-correcting codes is applied to CD players and to computers. The stunning pictures of far away planets and nebulae sent by Voyager II and Hubble could not have had their crispness and quality without such mathematics. In fact, Voyager's journey to the planets could not have been planned without the mathematics of differential equations. In ecology, mathematics is used when studying the laws of population change. Statistics not only provides the theory and methodology for the analysis of wide varieties of data but is essential in medicine, for analysing data on the causes of illness and on the utility of new drugs. Travel by aeroplane would not be possible without the mathematics

of airflow and of control systems. Body scanners are the expression of subtle mathematics discovered in the 19th century, which makes it possible to construct an image of the inside of an object from information on a number of single X-ray views of it. Thus, mathematics is often involved in matters of life and death.

Aim

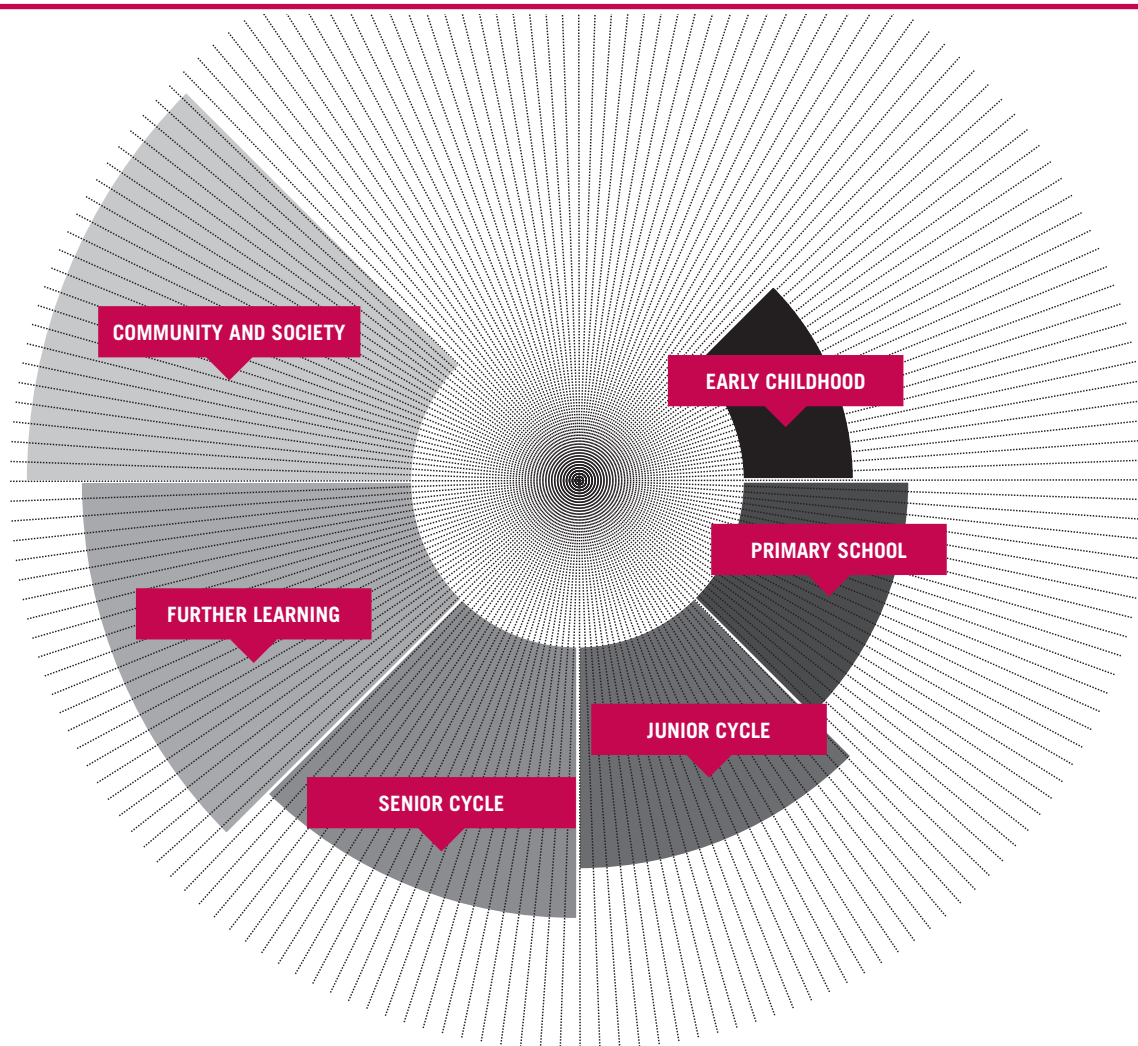
Leaving Certificate Mathematics aims to develop mathematical knowledge, skills and understanding needed for continuing education, life and work. By teaching mathematics in contexts that allow learners to see connections within mathematics, between mathematics and other subjects, and between mathematics and its applications to real life, it is envisaged that learners will develop a flexible, disciplined way of thinking and the enthusiasm to search for creative solutions.

Objectives

The objectives of Leaving Certificate Mathematics are that learners develop mathematical proficiency, characterised as

- *conceptual understanding*—comprehension of mathematical concepts, operations, and relations
- *procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence*—ability to formulate, represent, and solve mathematical problems in both familiar and unfamiliar contexts
- *adaptive reasoning*—capacity for logical thought, reflection, explanation, justification and communication
- *productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence, perseverance and one's own efficacy.

Related learning



The way in which mathematics learnt at different stages links together is very important to the overall development of mathematical understanding. The study of Leaving Certificate Mathematics encourages learners to use the numeracy and problem solving skills developed in early childhood education, primary mathematics and junior cycle mathematics. The emphasis is on building connected and integrated mathematical understanding. As learners progress through their education, mathematical skills, concepts and knowledge are developed when they work in more demanding contexts and develop more sophisticated approaches to problem solving. In this way mathematical learning is cumulative, with work at each level building on and deepening what students have learned at the previous level.

Mathematics is not learned in isolation; it has significant connections with other curriculum subjects. Many science subjects are quantitative in nature and learners are expected to be able to work with data, produce graphs and interpret patterns and trends. Design and Communication Graphics uses drawings in the analysis and solution of two- and three-dimensional problems through the

rigorous application of geometric principles. In Geography learners use ratio to determine scale. Every day, people use timetables, clocks and currency conversions to make life easier. Consumers need basic financial awareness and in Home Economics learners use mathematics when budgeting and making value for money judgements. Learners use mathematics in Economics for describing human behaviour. In Business Studies learners see how mathematics can be used by business organisations in accounting, marketing, inventory management, sales forecasting and financial analysis.

Mathematics, Music and Art have a long historical relationship. As early as the fifth century B.C., Pythagoras uncovered mathematical relationships in music, while many works of art are rich in mathematical structure. The modern mathematics of fractal geometry continues to inform composers and artists. Mathematics sharpens critical thinking skills, and by empowering learners to critically evaluate information and knowledge it promotes their development as statistically aware consumers.

Syllabus overview



Structure

The Leaving Certificate Mathematics syllabus comprises five strands:

1. Statistics and Probability
2. Geometry and Trigonometry
3. Number
4. Algebra
5. Functions

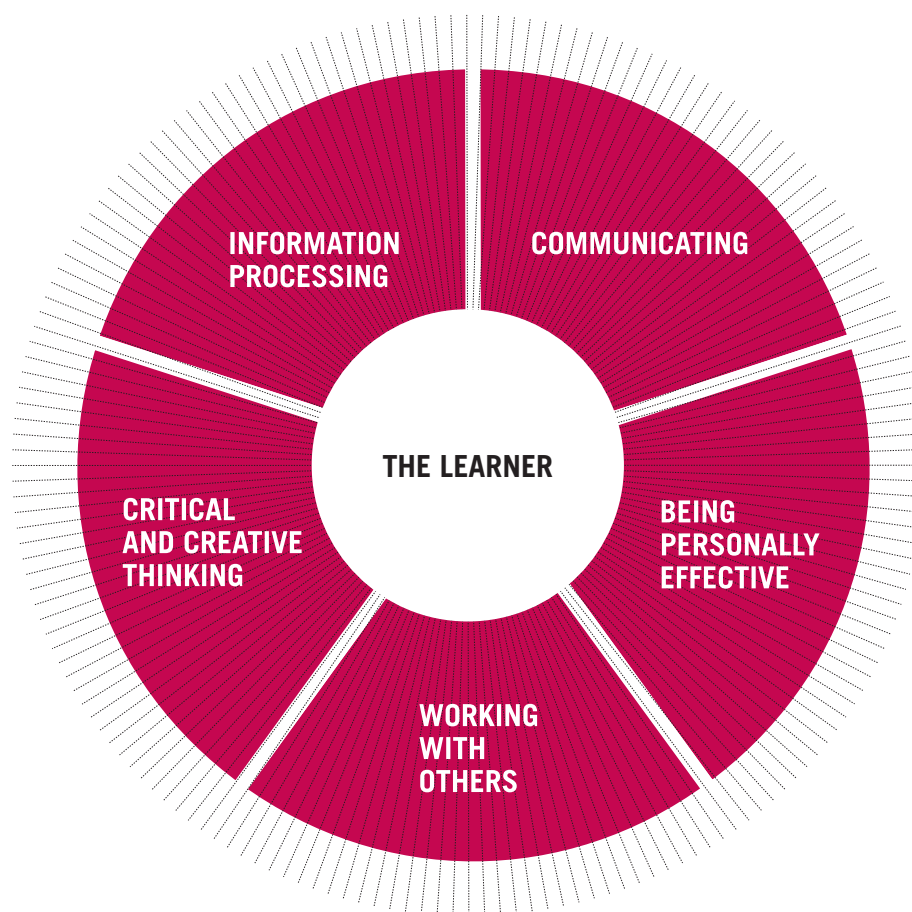
The strand structure of the syllabus should not be taken to imply that topics are to be studied in isolation. Where appropriate, connections should be made within and across the strands and with other areas of learning.

In each strand of this syllabus, learning outcomes specific to that strand are listed. The Foundation level learning outcomes are distinct from the Ordinary level and Higher level outcomes and are listed separately. The learning outcomes specified at Ordinary level are a subset of the learning outcomes for those studying at Higher level. At Ordinary level and Higher level, knowledge of the content and learning outcomes at the corresponding level in the Junior Certificate Mathematics syllabus is assumed.

Time allocation

The Leaving Certificate Mathematics syllabus is designed as a 180-hour course of study.

Key Skills



There are five key skills identified as central to teaching and learning across the curriculum at senior cycle. These are *information processing, being personally effective, communicating, critical and creative thinking* and *working with others*. These key skills are important for all learners to reach their full potential – both during their time in school and in the future – and to participate fully in society, including family life, the world of work and lifelong learning. By engaging with key skills learners enhance their ability to learn, broaden the scope of their learning and increase their capacity for learning.

Leaving Certificate Mathematics develops key skills in the following ways.

Information processing

Successful mathematics learning requires the efficient processing of the information that defines the mathematical tasks. Information is readily accessible from a variety of sources and information processing relates to the ways in which learners make sense of, or interpret, the information to which they are exposed.

Critical and creative thinking

There is a strong emphasis on investigation in mathematics and engaging in the investigative process requires learners to critically evaluate information and think creatively about it. Learners are encouraged to solve problems in a variety of ways and are required to evaluate methods and arguments and to justify their claims and results.

Communicating

In mathematics learners are encouraged to discuss approaches and solutions to problems and are expected to consider and listen to other viewpoints. Since mathematics emphasises investigation an important aspect of this is communicating findings to a variety of audiences in different ways.

Working with others

In mathematics learners are encouraged to work together in groups to generate ideas, problem solve and evaluate methods.

Being personally effective

Studying mathematics empowers learners to gain knowledge and skills that will benefit them directly in other aspects of their everyday lives. They participate in a learning environment that is open to new ideas and gain confidence in expressing their mathematical ideas and considering those of others.

While the Leaving Certificate Mathematics syllabus places particular emphasis on the development and use of information processing, logical thinking and problem-solving skills, the approach to teaching and learning involved gives prominence to learners being able to develop their skills in communicating and working with others. By adopting a variety of approaches and strategies for solving problems in mathematics, learners develop their self-confidence and personal effectiveness. The key skills are embedded within the learning outcomes and are assessed in the context of the learning outcomes.

In Leaving Certificate Mathematics students not only learn procedures and acquire reliable methods for producing correct solutions on paper-and-pencil exercises, but also learn mathematics with understanding. In particular, they should be able to explain why the procedures they apply are mathematically appropriate and justify why mathematical concepts have the properties that they do.

Problem Solving

Problem solving means engaging in a task for which the solution is not immediately obvious. Problem solving is integral to mathematical learning. In day-to-day life and in the workplace the ability to problem solve is a highly advantageous skill.

In the mathematics classroom problem solving should not be met in isolation, but should permeate all aspects of the teaching and learning experience. Problems may concern purely mathematical matters or some applied context.

In a mathematics problem-solving environment it is recognised that there are three things learners need to do:

- make sense of the problem
- make sense of the mathematics they can learn and use when doing the problem
- arrive at a correct solution to the problem.

However, in the mathematics classroom, the focus is on the mathematical knowledge and skills that can be learned in the process of obtaining an answer, rather than on the answer itself. The emphasis, therefore, is on generating discussion and on the reasoning and sense-making opportunities the problem affords the learners as they engage with the mathematics involved. They learn to analyse the problem and break it down into manageable steps, to reflect on their strategies and those of others and to adjust their own approaches where necessary.

Teachers play an important role in helping students develop these kinds of skills. By encouraging learners to share and explain their solution strategies, those that work as well as those that don't work, teachers can help learners to develop robust and deep mathematical understanding as well as confidence in their mathematical ability.

The quality of the tasks that learners engage with plays an important role in a problem-solving environment. A task must engage learners and present them with a challenge that requires exploration. Problem-solving tasks activate creative mathematical thinking processes as opposed to imitative thinking processes activated by routine tasks. Reasoning mathematically about tasks empowers learners to make connections within mathematics and to develop deep conceptual understanding.

Teaching and learning

In line with the syllabus objectives and learning outcomes, the experience of learners in the study of mathematics should contribute to the development of their problem-solving skills through the application of their mathematical knowledge and skills to appropriate contexts and situations. In each strand, at every syllabus level, emphasis is placed on appropriate contexts and applications of mathematics so that learners can appreciate its relevance to their current and future lives. The focus should be on learners understanding the concepts involved, building from the concrete to the abstract and from the informal to the formal.

Learners will build on their knowledge of mathematics constructed initially through their exploration of mathematics in the primary school and through their continuation of this exploration at junior cycle. Particular emphasis is placed on promoting learners' confidence in themselves (confidence that they can do mathematics) and in the subject (confidence that mathematics makes sense). Through the use of meaningful contexts, opportunities are presented for learners to achieve success.

Learners will integrate their knowledge and understanding of mathematics with economic and social applications of mathematics. By becoming statistically aware consumers, learners are able to critically evaluate knowledge claims and learn to interrogate and interpret data – a skill which has a value far beyond mathematics wherever data is used as evidence to support argument.

The variety of activities that learners engage in enables them to take charge of their own learning by setting goals, developing action plans and receiving and responding to assessment feedback. As well as varied teaching strategies, varied assessment strategies will provide information that can be used as feedback for teachers so that teaching and learning activities can be modified in ways which best suit individual learners. Results of assessments may also be used by teachers to reflect on their teaching practices so that instructional sequences and activities can be modified as required. Feedback to learners about their performance is critical to their learning and enables them to develop as learners. This formative assessment, when matched to the intended learning outcomes, helps to ensure consistency between the aim and objectives of the syllabus and its assessment. A wide range of assessment methods may be used, including investigations, class tests, investigation reports, oral explanation, etc.

Careful attention must be paid to learners who may still be experiencing difficulty with some of the material covered in the junior cycle. Nonetheless, they need to learn to cope with mathematics in everyday life and perhaps in further study. Their experience of Leaving Certificate Mathematics must therefore assist them in developing a clearer knowledge of and improved skills in, basic mathematics, and an awareness of its usefulness. Appropriate new material should also be introduced so that the learners can feel that they are progressing. At Leaving Certificate, the course followed should pay great attention to consolidating the foundation laid in the junior cycle and to addressing practical issues; but it should also cover new topics and lay a foundation for progress to the more traditional study of mathematics in the areas of algebra, geometry and functions.

Differentiation

Provision must be made not only for the academic student of the future, but also for the citizen of a society in which mathematics appears in, and is applied to, everyday life. The syllabus therefore focuses on material that underlies academic mathematical studies, ensuring that learners have a chance to develop their mathematical abilities and interests to a high level. It also covers the more practical and obviously applicable topics that learners meet in their lives outside school.

In each strand the learning outcomes are set out in terms of Foundation level, Ordinary level and Higher level. Ordinary level is a subset of Higher level. Therefore, learners studying at Higher level are expected to achieve the Ordinary level and Higher level learning outcomes. At Ordinary level and Higher level, knowledge of the content and learning outcomes at the corresponding level in the Junior Certificate Mathematics syllabus is assumed. In each strand, students are expected to use their mathematical knowledge and skills to solve appropriate problems, which can arise in both mathematical and applied contexts, and to make connections between topics and across strands.

Mathematics at Higher level is geared to the needs of learners who may proceed with their study of mathematics to third level. However, not all learners are future specialists or even future users of academic mathematics. Moreover, when they start to study the material, some of them are only beginning to deal with abstract concepts. For Higher level, particular emphasis can be placed on the development of powers of abstraction and generalisation and on the idea of rigorous proof, hence giving learners a feeling for the great mathematical concepts that span many centuries and cultures.

Mathematics at Ordinary level is geared to the needs of learners who are beginning to deal with abstract ideas. However, many of them may go on to use and apply mathematics in their future careers, and all of them will meet the subject to a greater or lesser degree in their daily lives. Ordinary level Mathematics, therefore, must start by offering mathematics that is meaningful and accessible to learners at their present stage of development. It should also provide for the gradual introduction of more abstract ideas, leading the learners towards the use of academic mathematics in the context of further study.

Mathematics at Foundation level places particular emphasis on the development of mathematics as a body of knowledge and skills that makes sense, and that can be used in many different ways as an efficient system for solving problems and finding answers. Alongside this, adequate attention must be paid to the acquisition and consolidation of fundamental skills, in the absence of which the learners' development and progress will be hindered. Foundation level Mathematics is intended to equip learners with the knowledge and skills required in everyday life, and it is also intended to lay the groundwork for learners who may proceed to further studies in areas in which specialist mathematics is not required.

Learners taking Foundation level mathematics are not required to deal with abstract mathematics. Thus, their experience of mathematics at Leaving Certificate should be approached in an exploratory and reflective manner, adopting a developmental and constructivist approach which allows them to make sense of their mathematical experiences to date and to solve the types of problems they may encounter in their daily lives. An appeal should be made to different interests and ways of learning, for example by paying attention to visual and spatial as well as to numerical aspects.

Differentiation will also apply in how Leaving Certificate Mathematics is assessed at Foundation, Ordinary and Higher levels. Ordinary level is a subset of Higher level and differentiation at the point of assessment will be reflected in the depth of treatment of the questions. It will be achieved also through the language level in the examination questions and the amount of structured support provided for examination candidates at different syllabus levels. Since, at Foundation level, learners have difficulty dealing with abstract ideas, at the point of assessment learners will be required to solve problems set in context relating to their daily lives. Information about the general assessment criteria applying to the examination of Leaving Certificate Mathematics is set out in the assessment section (page 44).