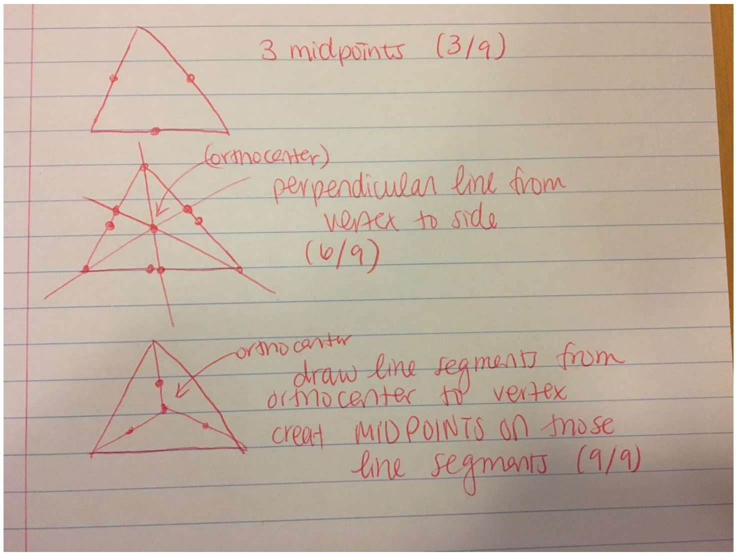
More triangle geometry

1. You should be able to construct all 4 triangle centers (Centroid, Orthocenter, Circumcenter, Incenter).
2. Describe or define Feuerbach circle.
3. You should be comfortable using the trace and locus tools in GeoGebra. Explain how you can draw a Feuerbach circle as a locus of (a) certain point(s).
4. Triangle ABC has the orthocenter O. What is the orthocenter of the triangle ABO? Prove it.
5. Triangle ABC has the orthocenter O. Describe the relationship between the Feuerbach circle of ABC and Feuerbach circle of BCO. Prove your answer.

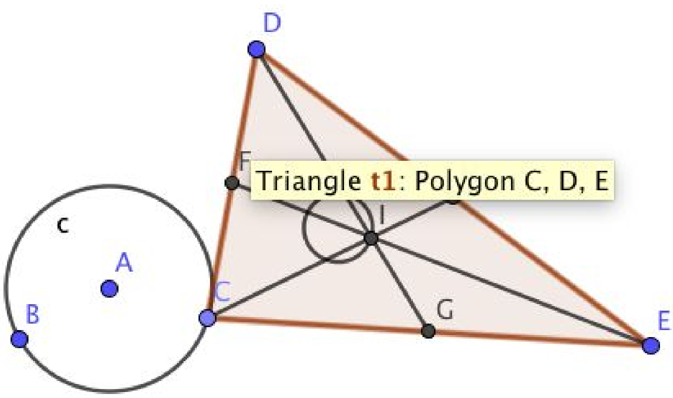
1. Construct all 4 triangle centers
   In order to create a centroid, create polygon with three points. Find midpoints of all three sides, connect midpoint to opposite vertex of triangle. medians.
   Orthocenter - where all altitudes cross - can be inside of outsider
      i. Create perpendicular line from vertex to opposite side
   Circumcenter - make midpoint on each side and create perpendicular line on midpoints - intersection of perpendicular lines - can be inside or outside
2. Feuerbach Circle - create triangle then make orthocenter and circumcircle, then find midpoint from orthocenter to circumcircle, then trace from midpoint to circumcircle. The circle that is traced is the feuerbach circle.
   9 point circle - 3 midpoints from side lengths, 3 feet of altitudes and 3 midpoints from orthocenter to vertice.
   Create triangle and midpoint of each side (3/9 points are these midpoints)
   Create perpendicular line from each vertex to opposite side (orthocenter)
      i. Create points where altitude and triangle sides intersect
   Create line from vertex to orthocenter and create midpoint of that line segment
   Extra information not discussed in class:
      ii. Create circumcenter by create perpendicular line from midpoints of triangle
      iii. Create line segment between orthocenter and circumcenter, find midpoint of that segment THIS IS center of circle

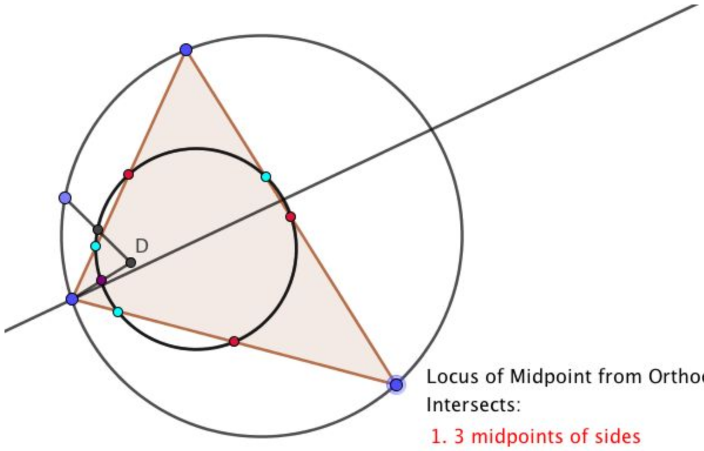


3. Explain how you can draw a Feuerbach circle as a locus of certain points

create triangle then make orthocenter and circumcircle, then find midpoint from orthocenter to circumcircle, then trace from midpoint to circumcircle. The circle that is traced is the feuerbach circle.

4. What is the orthocenter of the triangle ABO? The orthocenters are vertices





5. Describe relationship
   Geogebra File

Triangle similarity conditions

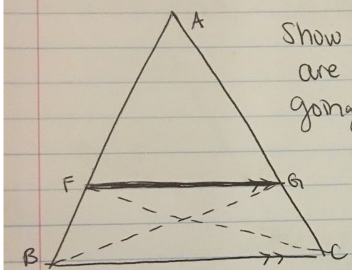
6. Discuss the difference between the definition of polygon similarity and similarity of general "figures".
7. Define similar triangles.
8. List triangle similarity conditions.
9. Formulate and prove the Side-splitter theorem. https://www.geogebra.org/m/PdGTJvDC
10. Formulate and prove corollary to the Side-splitter theorem.
11. Briefly discuss the importance of the Side-splitter theorem (and/or its corollary).
12. Prove the aa triangle similarity condition. https://www.geogebra.org/m/WT6nJZ6V
13. Using the aa similarity theorem and side-splitter theorem, outline the idea of the proof of sas similarity theorem (you don't have to prove it, just outline the idea).

6. For general blob - two shapes are similar when there is one to one correspondence for any two pairs of line segment ratios - check all such pairs in order to say that they are similar - this definition is conceptual because it gives us an idea but it is not practical
   a. When you have a blob you look at one to one ratio
7. Similar Triangles have equal corresponding angles and proportional side lengths.
8. AA, SAS and SSS, ASA?
9.

Side Splitter Theorem

$$\boxed{\text{If } FG \parallel BC \text{ then } \frac{|AF|}{|FB|} = \frac{|AG|}{|GC|}}$$

$$\frac{|AF|}{|FB|} = \frac{\text{Area } \triangle AFG}{\text{Area } \triangle FBG} \quad \text{and} \quad \frac{\text{Area } \triangle AFG}{\text{Area } \triangle FGC} = \frac{|AG|}{|GC|}$$



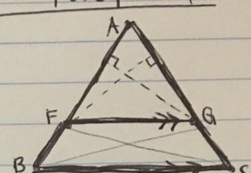
Show that the two ratios are equal. when are they going to be equal?

→ FG is the base of $\triangle FGB$ and $\triangle FGC$
→ $FG \parallel BC$ ∴ two triangles have SAME HEIGHT
→ ∴ areas in the denominator must be same which means area RATIOS are same!

*Side Splitting Proof

$$\frac{\text{Area }(\triangle AFG)}{\text{Area }(\triangle FBG)} = \frac{\cancel{\frac{1}{2}} b_1 \cancel{h_1}}{\cancel{\frac{1}{2}} b_2 \cancel{h_2}} \qquad b_1 = |AF| \quad b_2 = |FB|$$

$$\frac{\text{base}}{\text{base}} \text{(ratio)} = \boxed{\frac{|AF|}{|FB|} = \frac{|AG|}{|GC|}}$$



reciprocals must also be true

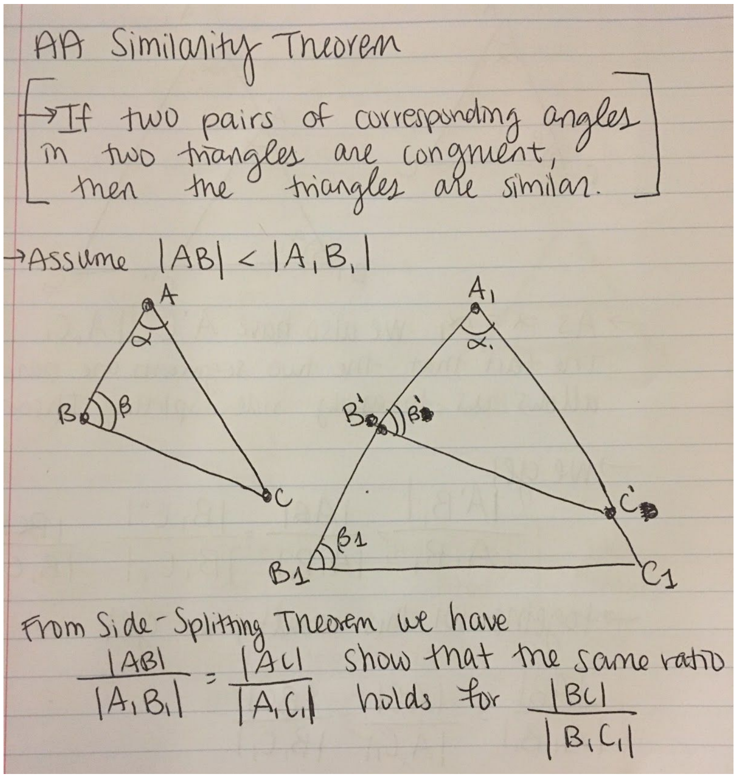
$$\frac{|FB|}{|AF|} = \frac{|GC|}{|AG|}$$

can you prove to be true as well?

WTS $\frac{|AB|}{|AF|} = \frac{|AC|}{|AG|}$

$$\frac{|AB|}{|AF|} = \left[\frac{|AF| + |FB|}{|AF|} = \frac{|AG| + |GC|}{|AG|}\right] = \frac{|AC|}{|AG|}$$

$$= \frac{|AF|}{|AF|} + \underbrace{\frac{|FB|}{|AF|} = \frac{|AG|}{|AG|} + \frac{|GC|}{|AG|}}_{\text{Already Proven by Reciprocal}}$$

10. The bottom of the above photo is the corollary
11. Helps to prove triangle similarity theorems. Ex: AA
12. 

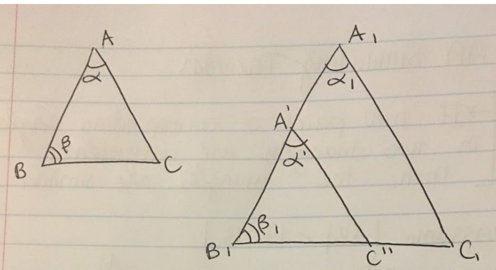
AA Similarity Theorem

→ If two pairs of corresponding angles in two triangles are congruent, then the triangles are similar.

→ Assume $|AB| < |A_1B_1|$

From Side-Splitting Theorem we have $\frac{|AB|}{|A_1B_1|} = \frac{|AC|}{|A_1C_1|}$ show that the same ratio holds for $\frac{|BC|}{|B_1C_1|}$

13.



→ As $\alpha' = \alpha_1$ we also have $A'C'' \parallel A_1C_1$
The fact that the two segments are parallel allows us to apply Side Splitting Theorem

→ we get

$$\frac{|A'B_1|}{A_1B_1} = \frac{|AB|}{|A_1B_1|} = \frac{|B_1C''|}{|B_1C_1|} = \frac{|BC|}{|B_1C_1|}$$

→ together w/ this result, we get

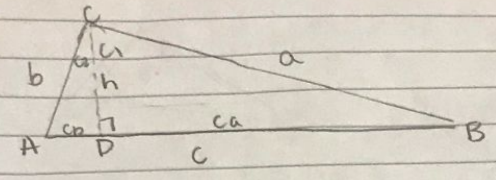
$$\frac{|AB|}{|A_1B_1|} = \frac{|AC|}{|A_1C_1|} = \frac{|BC|}{|B_1C_1|}$$

∴ all three pairs of sides are proportional

Triangle similarity applications

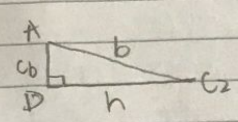
14. Formulate and prove Euclid's theorems about the height and legs in a right triangle.
    https://www.geogebra.org/m/tUYgVpRT
15. Use the previous result to prove the Pythagorean theorem.
16. The altitude to the hypotenuse of a right triangle is the geometric mean of the segments into which the altitude divides the hypotenuse. Try to make sense of what the theorem is saying. Draw a picture to explain it. Prove the theorem.

14.

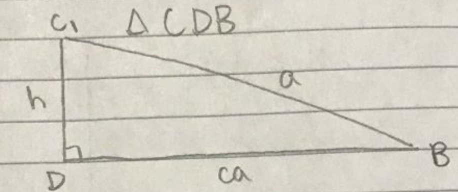


$\Delta ABC \sim \Delta CBD$	aa → share $\angle B$ & $\llcorner$
$\Delta ABC \sim \Delta ADC$	aa → share $\angle A$ & $\llcorner$
$\Delta ACD \sim \Delta CDB$	

ΔACD



ΔCDB



$\angle C_2 + \angle A = 90°$ | $\angle C_1 + \angle B = 90°$

$\angle C_1 + \angle C_2 = 90°$

$\therefore \angle C_2 + \angle A = \angle C_1 + \angle C_2$ | $\therefore \angle C_1 + \angle B = \angle C_1 + \angle C_2$

$-\angle C_2 \qquad -\angle C_2$ | $-\angle C_1 \qquad -\angle C_1$

$\angle A = \angle C_1$ | $\angle B = \angle C_2$

$\therefore \angle D = \angle D$

$\angle A = \angle C_1$

$\angle B = \angle C_2$

$\therefore$ through aa, $\Delta ADC \sim \Delta DCB$

$\boxed{h^2 = C_a \cdot C_b}$ Euclid's Theorem to Height

$\triangle ABC \sim \triangle CBD$

$\frac{a}{C_a} = \cancel{\frac{b}{h}} = \frac{c}{a}$

$\boxed{a^2 = C_a \cdot c}$

$\triangle ABC \sim \triangle ADC$

$\cancel{\frac{a}{h}} = \frac{b}{C_b} = \frac{c}{b}$

$\boxed{b^2 = C \cdot C_b}$

Use

$$\boxed{\begin{array}{l} a^2 = C \cdot C_a \\ b^2 = C \cdot C_b \\ h^2 = C_a \cdot C_b \end{array}}$$

to prove $C^2 = a^2 + b^2$

Proving Pythagoreon Theorem:

Known $C = C_a + C_b$

$$a^2+b^2 = CC_a + CC_b$$

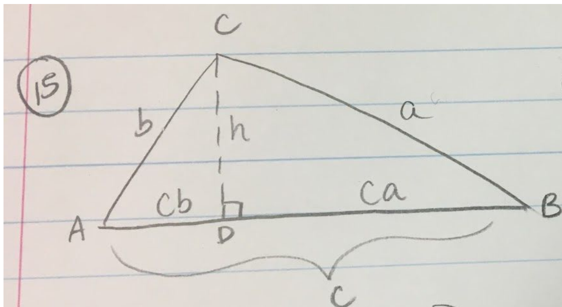
$$a^2+b^2 = C(C_a+C_b)$$

$$a^2+b^2 = C\cdot C$$

$$a^2+b^2 = C^2$$

15.

15



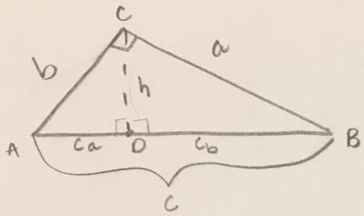
$$a^2+b^2 = C_b\cdot C + C_a\cdot C$$

$$a^2+b^2 = C(C_b+C_a)$$

$$a^2+b^2 = C^2$$

using $\left.\begin{aligned} a^2 &= C\cdot C_a \\ b^2 &= C\cdot C_b \\ h^2 &= C_a\cdot C_b \end{aligned}\right\}$ to prove $a^2+b^2=C^2$

16



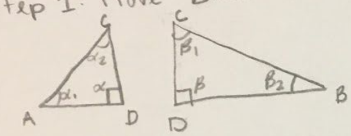
Ratios: $a^2 = C \cdot C_a$

$b^2 = C \cdot C_b$

$h^2 = C_a \cdot C_b$

$\therefore h = \sqrt{C_a \cdot C_b}$

Step 1: Prove $\triangle ACD \sim \triangle CDB$



known $\alpha = \beta$ | Right angles

$\alpha_1 + \alpha_2 = 90$ $\quad$ $\alpha_2 + \beta_1 = 90$

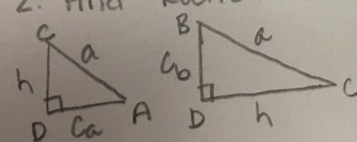
$\beta_1 + \beta_2 = 90$ $\quad$ $\alpha_2 = 90 - \beta_1$

$\therefore \alpha_1 + 90 - \beta_1 = 90$

$\therefore \alpha_1 = \beta_1$

$\therefore$ aa $\quad \therefore \triangle ACD \sim \triangle DCB$

Step 2: Find Ratios



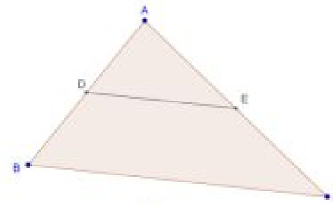
$\Rightarrow \frac{h}{C_b} = \frac{a}{a} = \frac{C_a}{h}$

$= h^2 = C_b C_a$

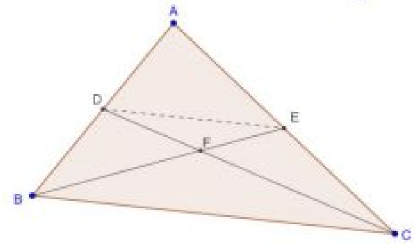
$\therefore$ h is geometric mean

16.

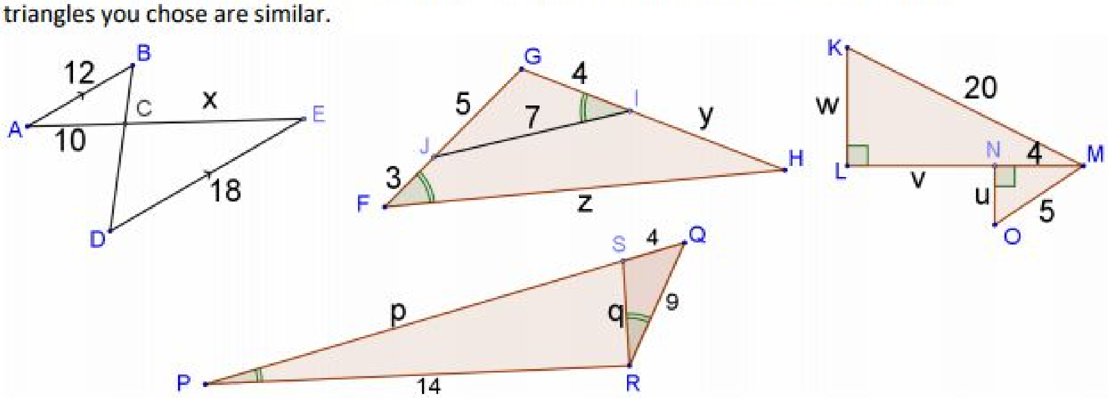
17. D is the midpoint of AB, E is the midpoint of AC. (See the picture; The segment DE is called *midsegment*.).
    a. Identify similar triangles in the picture and provide proper justification.
    b. DE || BC. Prove it.
    c. 2x|DE| = |BC| . Prove it.



If you need hints, go to: https://www.geogebra.org/m/TeXKJjVc

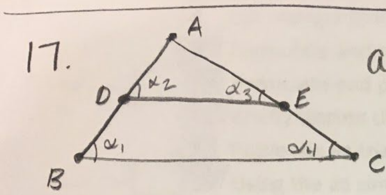


18. Using the same triangle ABC and points D, E:
    a. Show that F is the centroid of ABC.
    b. Identify similar triangles in the picture (justify your selection) that will help you prove that the Centroid splits medians in the 2:1 ratio. More specifically, prove that: 2x|DF| = |FC|. If you need hints, go to (copy-paste the link to browser if clicking does not help): https://www.geogebra.org/m/ScZJ3mqC
19. Use triangle similarity to solve for the missing dimensions. Don't forget to justify why the triangles you chose are similar.



https://www.geogebra.org/m/W3MTSFDc

17.



a) $\triangle ADE \sim \triangle ABC$

$\angle DAE = \angle BAC$	shared angle
$\frac{\lvert AB\rvert}{\lvert AD\rvert}$, $\frac{\lvert AC\rvert}{\lvert AE\rvert}$	from picture

$\therefore$ SAS $\therefore$ $\triangle ADE \sim \triangle ABC$

b) b/c 2 triangles are similar

$\alpha_2 = \alpha_1$ and $\alpha_3 = \alpha_4$ | corresponding angles

$\therefore$ $\overline{DE} \parallel \overline{BC}$ b/c of corresponding angles

c) If $\overline{DE}$ is midsegment then $2\overline{DA} = AB$, $2AE = AC$

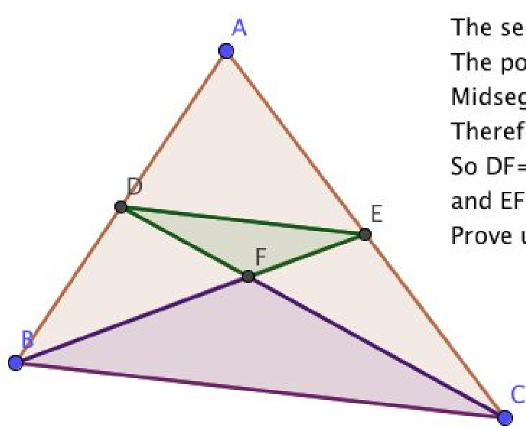
b/c similar triangles proportion is same

$\therefore 2\overline{DE} = \overline{BC}$

17.

18.

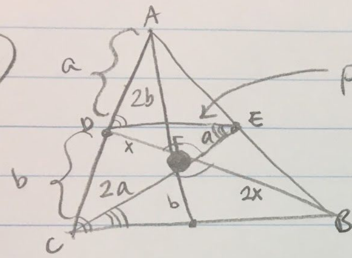
a.



The segments from vertex to midpoint have intersection F
The point of splits line into 2:1 ratio
Midsegment Theorem says that DE=1/2BC
Therefore ratio holds
So DF=1/2FC
and EF=1/2FB
Prove using similar triangles

b.

(18)



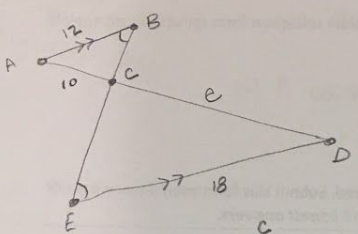
proved by (17) midsegment theorem to prove that a is half of $2a$. (works for all medians of centroids)

$\boxed{a = b}$ by corresponding angles

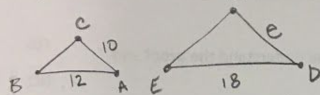
therefore $2x$ is $2/3$ the median although it is 2 times x.

Because → DE is $\frac{1}{2}$ of CB

$\Delta ABC \sim \Delta CDE$



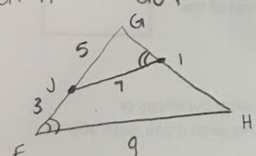
$\angle ABC = \angle CED$	AIAT
$\angle ECD = \angle ACD$	VAT
$\therefore$ aa	similarity thm
$\therefore \Delta ABC \sim \Delta CDE$	



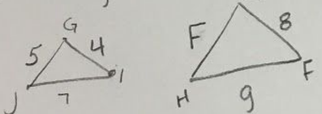
$\frac{10}{12} = \frac{e}{18} \Rightarrow 12e = 180$

$e = 15$

$\Delta GFH \sim \Delta GJI$



$\angle GIJ = \angle GFH$	Given
$\angle JGI = \angle FGH$	Same angle
$\therefore$ aa	sim. Thm
$\therefore \Delta GFH \sim \Delta GJI$	



$\frac{5}{F} = \frac{4}{8} \Rightarrow 40 = 4F$ $\quad \frac{9}{7} = \frac{8}{4} \Rightarrow 4g = 56$

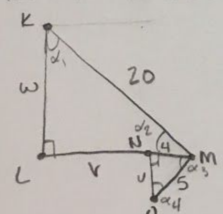
$F = 10$ $\quad g = 14$

$F = f + 4$

$10 = f + 4$

$f = 6$

$\Delta KLM \sim \Delta ONM$



Can't tell similarity b/c ambiguous.

pythagorean Theorem: $4^2 + u^2 = 5^2$

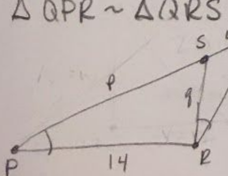
$16 + u^2 = 25$

$u^2 = 9$

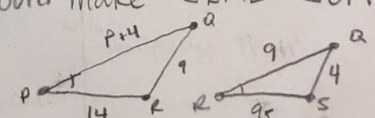
$u = 3$

To make unambiguous you could make $\angle KML = \angle OMN$

$\Delta QPR \sim \Delta QRS$



$\angle QPR = \angle SRQ$	Given
$\angle PQR = \angle SQR$	Shared
$\therefore$ aa	sim. Thm
$\therefore \Delta QPR \sim \Delta QRS$	



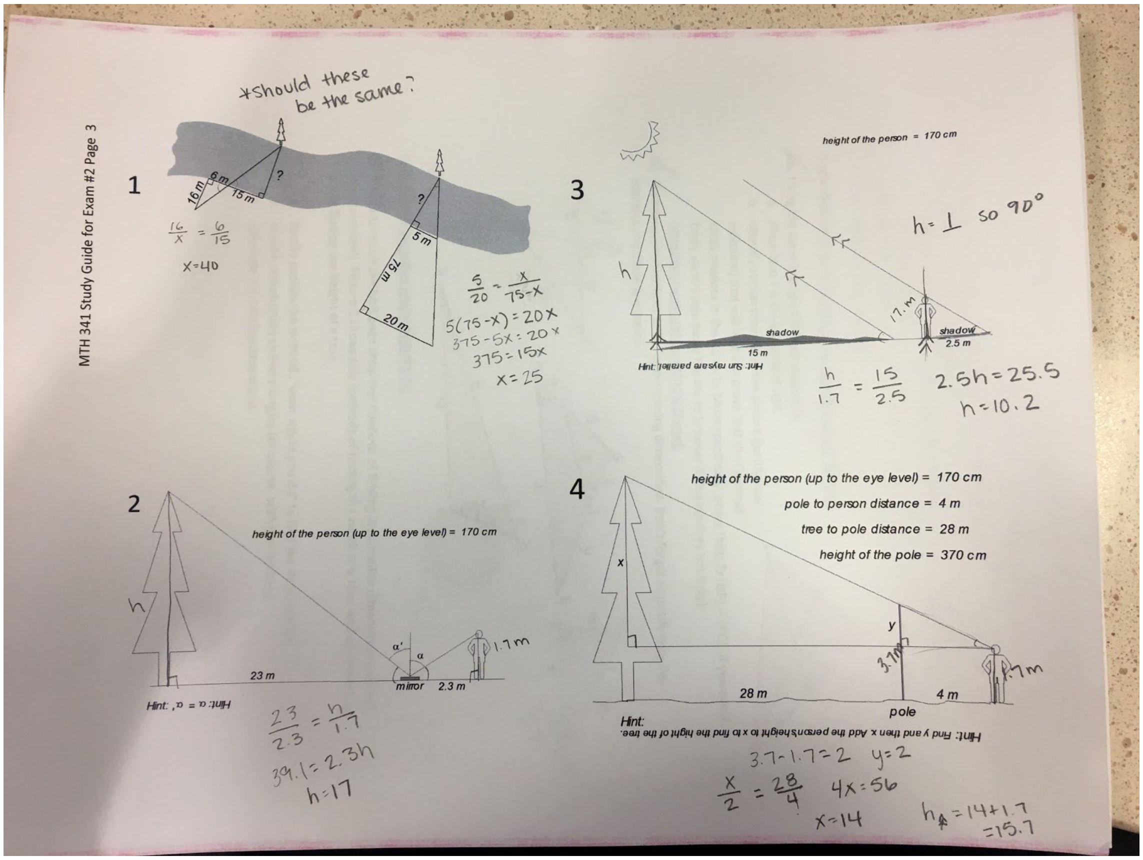
$\frac{4}{9} = \frac{9}{p+4} = 4p + 16 = 81$ $\quad \frac{q}{14} = \frac{4}{9}$

$4p = 65$ $\quad 9q = 56$

$p = 16.25$ $\quad q = 6.22$

19.

20.


MTH 341 Study Guide for Exam #2 Page 3
1
*Should these be the same?
16 m
6 m
15 m
?
5 m
75 m
20 m
16/x = 6/15
x=40
5/20 = x/(75-x)
5(75-x)=20x
375-5x=20x
375=15x
x=25
2
height of the person (up to the eye level) = 170 cm
h
23 m
mirror
2.3 m
1.7m
Hint: α = α'
23/2.3 = h/1.7
39.1=2.3h
h=17
3
height of the person = 170 cm
h=⊥ so 90°
h
shadow
15 m
1.7 m
shadow
2.5 m
Hint: Sun rays are parallel.
h/1.7 = 15/2.5
2.5h=25.5
h=10.2
4
height of the person (up to the eye level) = 170 cm
pole to person distance = 4 m
tree to pole distance = 28 m
height of the pole = 370 cm
x
y
3.7m
28 m
4 m
1.7m
pole
Hint: Find y and then x. Add the person's height to x to find the hight of the tree.
3.7-1.7=2 y=2
x/2 = 28/4 4x=56
x=14
h=14+1.7
=15.7