CHAPTER

2

Linear Equations

In this chapter, we will learn to solve linear equations and investigate applications of this type of equation. The chapter also discusses applications involving percents and proportions. The chapter concludes with a section on linear inequalities.

STUDY STRATEGY

Using Your Textbook In this chapter, we will focus on how to get the most out of your textbook. Students who treat their books solely as a source of homework exercises are turning their backs on one of their best resources.

Throughout this chapter, we will revisit this study strategy and help you incorporate it into your study habits.

2.1 Introduction to Linear Equations

OBJECTIVES

1. **Identify linear equations.**
2. **Determine whether a value is a solution of an equation.**
3. **Solve linear equations using the multiplication property of equality.**
4. **Solve linear equations using the addition property of equality.**
5. **Solve applied problems using the multiplication property of equality or the addition property of equality.**

Linear Equations

Objective 1 Identify linear equations. An **equation** is a mathematical statement of equality between two expressions. It is a statement that asserts that the value of the expression on the left side of the equation is equal to the value of the expression on the right side. Here are a few examples of equations.

$$2x = 8 \qquad x + 17 = 20 \qquad 3x - 8 = 2x + 6 \qquad 5(2x - 9) + 3 = 7(3x + 16)$$

All of these are examples of linear equations. A **linear equation** in one variable has a single variable, and the exponent for that variable is 1. For example, if the variable in a linear equation is x, the equation cannot have terms containing x^2 or x^3. The variable in a linear equation cannot appear in a denominator either. Here are some examples of equations that are not linear equations.

$x^2 - 5x - 6 = 0$ Variable is squared.
$m^3 - m^2 + 7m = 7$ Variable has exponents greater than 1.
$\frac{5x + 3}{x - 2} = -9$ Variable is in denominator.

Solutions of Equations

Objective 2 Determine whether a value is a solution of an equation. A **solution** of an equation is a value that when substituted for the variable in the equation, produces a true statement, such as $5 = 5$.

EXAMPLE 1 Is $x = 3$ a solution of $9x - 7 = 20$?

SOLUTION To check whether a particular value is a solution of an equation, we substitute that value for the variable in the equation. If after simplifying both sides of the equation we have a true mathematical statement, the value is a solution.

$$
\begin{aligned}
9x - 7 &= 20 \\
9(3) - 7 &= 20 && \text{Substitute 3 for } x. \\
27 - 7 &= 20 && \text{Multiply.} \\
20 &= 20 && \text{Subtract.}
\end{aligned}
$$

Because 20 is equal to 20, we know that $x = 3$ is a solution.

EXAMPLE 2 Is $x = -2$ a solution of $3 - 4x = -5$?

SOLUTION Again, substitute for x and simplify both sides of the equation.

$$
\begin{aligned}
3 - 4x &= -5 \\
3 - 4(-2) &= -5 && \text{Substitute } -2 \text{ for } x. \\
3 + 8 &= -5 && \text{Multiply.} \\
11 &= -5 && \text{Add.}
\end{aligned}
$$

This statement, $11 = -5$, is not true because 11 is not equal to -5. Therefore, $x = -2$ is not a solution of the equation.

Quick Check 1

Is $x = -7$ a solution of $4x + 23 = -5$?

The set of all solutions of an equation is called its **solution set**. The process of finding an equation's solution set is called **solving the equation**. When we find all of the solutions to an equation, we write those values using set notation inside braces { }.

When solving a linear equation, our goal is to convert it to an equivalent equation that has the variable isolated on one side with a number on the other side (for example, $x = 3$). The value on the opposite side of the equation from the variable after it has been isolated is the solution of the equation.

Multiplication Property of Equality

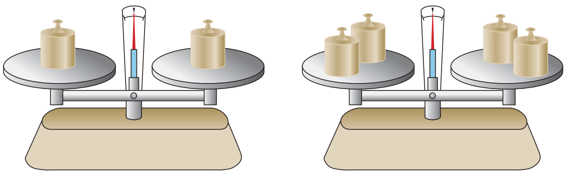
Objective 3 Solve linear equations using the multiplication property of equality. The first tool for solving linear equations is the **multiplication property of equality**. It says that for any equation, if we multiply both sides of the equation by the same nonzero number, both sides remain equal to each other.

Multiplication Property of Equality

For any algebraic expressions A and B and any nonzero number n,

$$\text{if } A = B, \text{ then } n \cdot A = n \cdot B.$$

Think of a scale that holds two weights in balance. If we double the amount of weight on each side of the scale, will the scale still be balanced? Of course it will.



Think of an equation as a scale and the expressions on each side as the weights that are balanced. Multiplying both sides by the same nonzero number leaves both sides still balanced and equal to each other.

Why is it important to multiply both sides of an equation by a *nonzero* number? Multiplying both sides of an equation by 0 can take an equation that is false and make it true. For example, we know that $5 = 3$ is false. If we multiply both sides of that equation by 0, the resulting equation is $0 = 0$, which is true.

EXAMPLE 3 Solve $\frac{x}{3} = 4$ using the multiplication property of equality.

SOLUTION The goal when solving this linear equation is to isolate the variable x on one side of the equation. We begin by multiplying both sides of the equation by 3. The expression $\frac{x}{3}$ is equivalent to $\frac{1}{3}x$; so when we multiply by 3, we are multiplying by the reciprocal of $\frac{1}{3}$. The product of reciprocals is equal to 1, so the resulting expression on the left side of the equation is $1x$, or x.

Another way to think of $\frac{x}{3}$ is as $x \div 3$. Division and multiplication are inverse operations, and we solve the equation by multiplying both sides of the equation by 3. This will "undo" dividing by 3.

$$\frac{x}{3} = 4$$

$$3 \cdot \frac{x}{3} = 3 \cdot 4 \quad \text{Multiply both sides by 3.}$$

$$\overset{1}{\cancel{3}} \cdot \frac{x}{\underset{1}{\cancel{3}}} = 3 \cdot 4 \quad \text{Divide out common factors.}$$

$$x = 12 \quad \text{Multiply.}$$

The solution that we found is $x = 12$. Before moving on, we must check this value to ensure that we made no mistakes and that it is a solution.

Check

$$\frac{x}{3} = 4$$

$$\frac{12}{3} = 4 \quad \text{Substitute 12 for } x.$$

$$4 = 4 \quad \text{Divide.}$$

This is a true statement; so $x = 12$ is a solution, and the solution set is $\{12\}$.

Quick Check 2

Solve $\frac{x}{8} = -2$ using the multiplication property of equality.

The multiplication property of equality also allows us to divide both sides of an equation by the same nonzero number without affecting the equality of the two sides. This is because dividing both sides of an equation by a nonzero number, n, is equivalent to multiplying both sides of the equation by the reciprocal of the number, $\frac{1}{n}$.

EXAMPLE 4 Solve $5y = 40$ using the multiplication property of equality.

SOLUTION Begin by dividing both sides of the equation by 5, which will isolate the variable y.

$$5y = 40$$
$$\frac{5y}{5} = \frac{40}{5} \quad \text{Divide both sides by 5.}$$
$$\frac{\overset{1}{\cancel{5}}y}{\underset{1}{\cancel{5}}} = \frac{40}{5} \quad \text{Divide out common factors on the left side.}$$
$$y = 8 \quad \text{Simplify.}$$

Again, check the solution before writing it in solution set notation.

Check

$$5y = 40$$
$$5(8) = 40 \quad \text{Substitute 8 for } y.$$
$$40 = 40 \quad \text{Multiply.}$$

$y = 8$ is indeed a solution, and the solution set is $\{8\}$.

Quick Check 3

Solve $4a = 56$ using the multiplication property of equality.

If the coefficient of the variable term is negative, we must divide both sides of the equation by that negative number.

EXAMPLE 5 Solve $-7n = -56$ using the multiplication property of equality.

SOLUTION In this example, the coefficient of the variable term is -7. To solve this equation, we need to divide both sides by -7.

$$-7n = -56$$
$$\frac{-7n}{-7} = \frac{-56}{-7} \quad \text{Divide both sides by } -7.$$
$$n = 8 \quad \text{Simplify.}$$

The check of this solution is left to the reader. The solution set is $\{8\}$.

Quick Check 4

Solve $-9a = 144$ using the multiplication property of equality.

A WORD OF CAUTION When dividing both sides of an equation by a negative number, keep in mind that this will change the sign of the number on the other side of the equation.

Consider the equation $-x = 16$. The coefficient of the variable term is -1. To solve this equation, we can multiply both sides of the equation by -1 or divide both sides by -1. Either way, the solution is $x = -16$. We also could have solved the equation by inspection by reading the equation $-x = 16$ as *the opposite of x is 16.* If the opposite of x is 16, we know that x must be equal to -16.

We will now learn how to solve an equation in which the coefficient of the variable term is a fraction.

EXAMPLE 6 Solve $\frac{3}{8}a = -\frac{5}{2}$ using the multiplication property of equality.

SOLUTION In this example, we have a variable multiplied by a fraction. In such a case, we can multiply both sides of the equation by the reciprocal of the fraction. When we multiply a fraction by its reciprocal, the result is 1. This will leave the variable isolated.

$$\frac{3}{8}a = -\frac{5}{2}$$

$$\frac{\cancel{8}^{1}}{\cancel{3}_{1}} \cdot \frac{\cancel{3}^{1}}{\cancel{8}_{1}}a = \frac{\cancel{8}^{4}}{3}\left(-\frac{5}{\cancel{2}_{1}}\right) \quad \text{Multiply by the reciprocal of the fraction } \tfrac{3}{8} \text{ and divide out common factors.}$$

$$a = -\frac{20}{3} \quad \text{Simplify.}$$

The check of this solution is left to the reader. The solution set is $\{-\frac{20}{3}\}$.

Quick Check 5

Solve $\frac{9}{16}x = \frac{21}{8}$ using the multiplication property of equality.

Addition Property of Equality

Objective 4 Solve linear equations using the addition property of equality.

The **addition property of equality** says that we can add the same number to both sides of an equation or subtract the same number from both sides of an equation without affecting the equality of the two sides.

Addition Property of Equality

For any algebraic expressions A and B and any number n,

$$\text{if} \quad A = B, \quad \text{then} \quad A + n = B + n$$
$$\text{and} \quad A - n = B - n.$$

This property helps us solve equations in which a number is added to or subtracted from a variable on one side of an equation.

EXAMPLE 7 Solve $x + 4 = 11$ using the addition property of equality.

SOLUTION In this example, the number 4 is being added to the variable x. To isolate the variable, use the addition property of equality to subtract 4 from both sides of the equation.

$$x + 4 = 11$$
$$x + 4 - 4 = 11 - 4 \quad \text{Subtract 4 from both sides.}$$
$$x = 7 \quad \text{Simplify.}$$

To check this solution, substitute 7 for x in the original equation.

Check

$$x + 4 = 11$$
$$7 + 4 = 11 \quad \text{Substitute 7 for } x.$$
$$11 = 11 \quad \text{Add.}$$

The statement is true, so the solution set is $\{7\}$.

Quick Check 6

Solve $a + 22 = -8$ using the addition property of equality.

EXAMPLE 8 Solve $13 = y - 9$ using the addition property of equality.

SOLUTION When a value is subtracted from a variable, isolate the variable by adding the value to both sides of the equation. In this example, add 9 to both sides.

$$\begin{aligned} 13 &= y - 9 \\ 13 + 9 &= y - 9 + 9 && \text{Add 9 to both sides.} \\ 22 &= y && \text{Add.} \end{aligned}$$

Check

$$\begin{aligned} 13 &= y - 9 \\ 13 &= 22 - 9 && \text{Substitute 22 for } y. \\ 13 &= 13 && \text{Subtract.} \end{aligned}$$

This is a true statement, so the solution set is $\{22\}$.

Quick Check 7

Solve $-13 = x - 28$ using the addition property of equality.

In the next section, we will learn how to solve equations requiring us to use both the multiplication and addition properties of equality.

Applications

Objective 5 Solve applied problems using the multiplication property of equality or the addition property of equality. This section concludes with an example of an applied problem that can be solved with a linear equation.

EXAMPLE 9 Admission to the county fair is \$8 per person; so the admission price for a group of x people can be represented by $8x$. If a Cub Scout group paid a total of \$208 for admission to the county fair, how many people were in the group?

SOLUTION We can express this relationship in an equation using the idea that the total cost of admission is equal to \$208. Because the total cost for x people can be represented by $8x$, the equation is $8x = 208$.

$$\begin{aligned} 8x &= 208 \\ \frac{8x}{8} &= \frac{208}{8} && \text{Divide both sides by 8.} \\ x &= 26 && \text{Divide.} \end{aligned}$$

We can verify that this value checks as a solution. Whenever we work on an applied problem, we should write the solution as a complete sentence. There were 26 people in the Cub Scout group at the county fair.

Quick Check 8

Josh spent a total of \$26.50 to take his date to a movie. This left him with only \$38.50 in his pocket. How much money did he have with him before going to the movie?

BUILDING YOUR STUDY STRATEGY

Using Your Textbook, 1 Reading Ahead One effective way to use your textbook is to read a section in the text before it is covered in class. This will give you an idea about the main concepts covered in the section, and your instructor can clarify these concepts in class.

When reading ahead, you should scan the section. Look for definitions that are introduced; as well as procedures that are developed. Pay close attention to the examples. If you find a step in the examples that you do not understand, ask your instructor about it in class.

Exercises 2.1

Vocabulary

1. A(n) ________ is a mathematical statement of equality between two expressions.
2. A(n) ________ of an equation is a value that when substituted for the variable in the equation, produces a true statement.
3. The ________ of an equation is the set of all solutions to that equation.
4. State the multiplication property of equality.
5. State the addition property of equality.
6. Freebird's Pizza charges \$12 for a pizza. If the bill for an office pizza party is \$168, which equation can be used to determine the number of pizzas that were ordered?

   a) $x + 12 = 168$ **b)** $x - 12 = 168$

   c) $12x = 168$ **d)** $\frac{x}{12} = 168$

Is the given equation a linear equation? If not, explain.

7. $5x^2 - 7x = 3x + 8$

8. $4x - 9 = 17$

9. $3x - 5(2x + 3) = 8 - x$

10. $\frac{7}{x^2} - \frac{5}{x} - 13 = 0$

11. $y = 5$

12. $x^4 - 1 = 0$

13. $\frac{3x}{11} - \frac{5}{4} = \frac{2x}{7}$

14. $x \cdot 4 - 1 = 0$

15. $x + \frac{3}{x} + 18 = 0$

16. $3(4x - 9) + 7(2x + 5) = 15$

Check to determine whether the given value is a solution of the equation.

17. $x = 7,\quad 5x - 9 = 26$

18. $x = 3,\quad 2x - 11 = x + 2$

19. $a = 8,\quad 3 - 2a = a + 2a - 11$

20. $m = 4,\quad 15 - 8m = 3m - 29$

21. $z = \frac{1}{4},\quad \frac{2}{3}z + \frac{11}{6} = 2$

22. $t = \frac{5}{3},\quad \frac{1}{10}t + \frac{1}{3} = \frac{1}{2}$

23. $m = 3.4,\quad 3m - 2 = 2m + 0.4$

24. $a = -2.5,\quad 4a - 6 = 9 + 10a$

Solve using the multiplication property of equality.

25. $7x = -91$ **26.** $9a = 72$

27. $6y = 84$ **28.** $11x = -1331$

29. $8b = 22$ **30.** $20z = -35$

31. $5a = 0$ **32.** $0 = 12x$

33. $-5t = 35$ **34.** $-11x = -44$

35. $-2x = -28$ **36.** $-9h = 54$

37. $-t = 45$ **38.** $-y = -31$

39. $\frac{x}{3} = 7$ **40.** $\frac{x}{4} = 3$

41. $-\frac{t}{8} = 12$ **42.** $-\frac{g}{13} = -7$

43. $\frac{7}{12}x = \frac{14}{3}$ **44.** $\frac{3}{4}x = -\frac{9}{2}$

45. $-\frac{2}{5}x = 4$ **46.** $-\frac{5}{6}x = 15$

47. $3.2x = 6.944$ **48.** $-4.7x = 15.04$

Solve using the addition property of equality.

49. $a + 9 = 16$ **50.** $b + 4 = 28$

51. $x + 11 = 3$ **52.** $x + 5 = -18$

53. $n - 13 = 30$ **54.** $n - 9 = -23$

55. $a + 3.2 = 5.7$

56. $x - 4.9 = -11.2$

57. $12 = x + 3$

58. $4 = m + 10$ **59.** $b - 7 = 13$

60. $a - 9 = 99$ **61.** $t - 7 = -4$

62. $n - 3 = -18$ **63.** $-4 + x = 19$

64. $-15 + x = -7$ **65.** $x + 9 = 0$

66. $b - 17 = 0$ **67.** $9 + a = 5$

68. $7 + b = -22$

69. $a + 5 + 6 = 7$

70. $x + 4 - 9 = -3$

Mixed Practice, 71–88

Solve the equation.

71. $60 = 5a$

72. $-m = -15$

73. $x - 27 = -11$

74. $\frac{3}{14}x = \frac{5}{2}$

75. $-\frac{b}{10} = -3$

76. $b + 39 = 30$

77. $x + 24 = -17$

78. $\frac{x}{5} = 13$

79. $11 = -\frac{n}{7}$

80. $47 = y + 35$

81. $0 = x + 56$

82. $x - 38 = -57$

83. $-t = 18$

84. $\frac{t}{15} = -7$

85. $\frac{4}{9}x = -\frac{14}{15}$

86. $x - 3 = -9$

87. $0 = 45m$

88. $40 = -6m$

Provide an equation that has the given solution.

89. $x = 7$

90. $x = -13$

91. $n = \frac{5}{2}$

92. $m = -\frac{3}{10}$

Set up a linear equation and solve it for the following problems.

93. Zoe has only nickels in her pocket. If she has \$1.35 in her pocket, how many nickels does she have?

94. Fruit smoothies were sold at a campus fund-raiser for \$3.25. If total sales were \$217.75, how many smoothies were sold?

95. A local garage band, the Grease Monkeys, held a rent party. They charged \$3 per person for admission, with the proceeds used to pay the rent. If the rent is \$425 and they ended up with an extra \$52 after paying the rent, how many people came to see them play?

96. Ross organized a tour of a local winery. Attendees paid Ross \$12 to go on the tour, plus another \$5 for lunch. If Ross collected \$493, how many people came on the tour?

97. An insurance company hired 8 new employees. This brought the total to 174 employees. How many employees did the company have before these 8 people were hired?

98. Geena scored 17 points lower than Jared on the last math exam. If Geena's score was 65, find Jared's score.

99. As a cold front was moving in, the temperature in Visalia dropped by 19°F in a two-hour period. If the temperature dropped to 37°F, what was the temperature before the cold front moved in?

100. A company was forced to lay off 37 workers due to an economic downturn. If the company now has 144 employees, how many workers were employed before the layoffs?

Writing in Mathematics

Answer in complete sentences.

101. Explain why the equation $0x = 15$ cannot be solved.

102. Find a value for x such that the expression $x + 21$ is less than -39.

2.2 Solving Linear Equations: A General Strategy

OBJECTIVES

1. Solve linear equations using both the multiplication property of equality and the addition property of equality.
2. Solve linear equations containing fractions.
3. Solve linear equations using the five-step general strategy.
4. Identify linear equations with no solution.
5. Identify linear equations with infinitely many solutions.
6. Solve literal equations for a specified variable.

In the previous section, we solved equations that required only one operation to isolate the variable. In this section, we will learn how to solve equations requiring the use of both the multiplication and addition properties of equality.

Solving Linear Equations

Objective 1 Solve linear equations using both the multiplication property of equality and the addition property of equality. Suppose we needed to solve the equation $4x - 7 = 17$. Should we divide both sides by 4 first? Should we add 7 to both sides first? We refer to the order of operations. This says that in the expression $4x - 7$, we multiply 4 by x and then subtract 7 from the result. To isolate the variable x, we undo these operations in the opposite order. We first add 7 to both sides to undo the subtraction and then divide both sides by 4 to undo the multiplication.

Solution

$$\begin{aligned} 4x - 7 &= 17 \\ 4x - 7 + 7 &= 17 + 7 \\ 4x &= 24 \\ \frac{4x}{4} &= \frac{24}{4} \\ x &= 6 \end{aligned}$$

Check

$$\begin{aligned} 4(6) - 7 &= 17 \\ 24 - 7 &= 17 \\ 17 &= 17 \end{aligned}$$

Because the solution $x = 6$ checks, the solution set is $\{6\}$.

EXAMPLE 1 Solve the equation $3x + 41 = 8$.

SOLUTION To solve this equation, begin by subtracting 41 from both sides. This will isolate the term $3x$. Then divide both sides of the equation by 3 to isolate the variable x.

$$\begin{aligned} 3x + 41 &= 8 && \\ 3x + 41 - 41 &= 8 - 41 && \text{Subtract 41 from both sides to isolate } 3x. \\ 3x &= -33 && \text{Subtract.} \\ \frac{3x}{3} &= -\frac{33}{3} && \text{Divide both sides by 3 to isolate } x. \\ x &= -11 && \text{Simplify.} \end{aligned}$$

Now check the solution.

$$\begin{aligned} 3x + 41 &= 8 && \\ 3(-11) + 41 &= 8 && \text{Substitute } -11 \text{ for } x. \\ -33 + 41 &= 8 && \text{Multiply.} \\ 8 &= 8 && \text{Simplify.} \end{aligned}$$

Because $x = -11$ produced a true statement, the solution set is $\{-11\}$.

Quick Check 1

Solve the equation $5x - 2 = 33$.

EXAMPLE 2 Solve the equation $-8x - 19 = 13$.

SOLUTION

$$\begin{aligned} -8x - 19 &= 13 && \\ -8x - 19 + 19 &= 13 + 19 && \text{Add 19 to both sides to isolate } -8x. \\ -8x &= 32 && \text{Add.} \\ \frac{-8x}{-8} &= \frac{32}{-8} && \text{Divide by } -8 \text{ to isolate } x. \\ x &= -4 && \text{Simplify.} \end{aligned}$$

Now check the solution.

$$\begin{aligned} -8x - 19 &= 13 \\ -8(-4) - 19 &= 13 && \text{Substitute } -4 \text{ for } x. \\ 32 - 19 &= 13 && \text{Multiply.} \\ 13 &= 13 && \text{Simplify.} \end{aligned}$$

The solution set is $\{-4\}$.

Quick Check 2

Solve the equation $6 - 4x = 38$.

Solving Linear Equations Containing Fractions

Objective 2 Solve linear equations containing fractions. If an equation contains fractions, we may find it helpful to convert it to an equivalent equation that does not contain fractions before we solve it. This can be done by multiplying both sides of the equation by the LCM of the denominators.

EXAMPLE 3 Solve the equation $\frac{2}{3}x - \frac{5}{6} = \frac{1}{2}$.

SOLUTION This equation contains three fractions, and the denominators are 3, 6, and 2. The LCM of these denominators is 6, so begin by multiplying both sides of the equation by 6.

$$\begin{aligned} \frac{2}{3}x - \frac{5}{6} &= \frac{1}{2} \\ 6\left(\frac{2}{3}x - \frac{5}{6}\right) &= 6\left(\frac{1}{2}\right) && \text{Multiply both sides by the LCM 6.} \\ \overset{2}{\cancel{6}} \cdot \frac{2}{\underset{1}{\cancel{3}}}x - \overset{1}{\cancel{6}} \cdot \frac{5}{\underset{1}{\cancel{6}}} &= \overset{3}{\cancel{6}} \cdot \frac{1}{\underset{1}{\cancel{2}}} && \text{Distribute and divide out common factors.} \\ 4x - 5 &= 3 && \text{Multiply.} \\ 4x - 5 + 5 &= 3 + 5 && \text{Add 5 to both sides to isolate } 4x. \\ 4x &= 8 && \text{Add.} \\ \frac{4x}{4} &= \frac{8}{4} && \text{Divide by 4 to isolate } x. \\ x &= 2 && \text{Simplify.} \end{aligned}$$

Now check the solution.

$$\begin{aligned} \frac{2}{3}x - \frac{5}{6} &= \frac{1}{2} \\ \frac{2}{3}(2) - \frac{5}{6} &= \frac{1}{2} && \text{Substitute 2 for } x. \\ \frac{4}{3} - \frac{5}{6} &= \frac{1}{2} && \text{Simplify.} \\ \frac{8}{6} - \frac{5}{6} &= \frac{1}{2} && \text{Rewrite } \frac{4}{3} \text{ as } \frac{8}{6}. \\ \frac{3}{6} &= \frac{1}{2} && \text{Subtract.} \\ \frac{1}{2} &= \frac{1}{2} && \text{Simplify.} \end{aligned}$$

The solution set is $\{2\}$.

Quick Check 3

Solve the equation $\frac{2}{7}x + \frac{1}{2} = \frac{4}{3}$.

A WORD OF CAUTION When multiplying both sides of an equation by the LCM of the denominators, make sure you multiply each term by the LCM, including any terms that do not contain fractions.

A General Strategy for Solving Linear Equations

Objective 3 Solve linear equations using the five-step general strategy. Now we will examine a process that can be used to solve any linear equation. This process works not only for types of equations we have already learned to solve, but also for more complicated equations such as $7x + 4 = 3x - 20$ and $5(2x - 9) + 3x = 7(3 - 8x)$.

Solving Linear Equations

1. **Simplify each side of the equation completely.**
   - Use the distributive property to clear any parentheses.
   - If there are fractions in the equation, multiply both sides of the equation by the LCM of the denominators to clear the fractions from the equation.
   - Combine any like terms that are on the same side of the equation. After you have completed this step, the equation should contain, at most, one variable term and one constant term on each side.
2. **Collect all variable terms on one side of the equation.** If there is a variable term on each side of the equation, use the addition property of equality to place both variable terms on the same side of the equation.
3. **Collect all constant terms on the other side of the equation.** If there is a constant term on each side of the equation, use the addition property of equality to isolate the variable term.
4. **Divide both sides of the equation by the coefficient of the variable term.** At this point the equation should be of the form $ax = b$. Use the multiplication property of equality to find the solution.
5. **Check your solution.** Check that the value creates a true equation when substituted for the variable in the original equation.

In the next example, we will solve equations that have variable terms and constant terms on both sides of the equation.

EXAMPLE 4 Solve the equation $3x + 8 = 7x - 6$.

SOLUTION In this equation, there are no parentheses or fractions to clear and there are no like terms to be combined. Begin by gathering the variable terms on one side of the equation.

$$3x + 8 = 7x - 6$$

$$3x + 8 - 3x = 7x - 6 - 3x$$ Subtract $3x$ from both sides to gather the variable terms on the right side of the equation.

$$8 = 4x - 6$$ Subtract.

$$8 + 6 = 4x - 6 + 6$$ Add 6 to both sides to isolate $4x$.

$$14 = 4x$$ Add.

$$\frac{14}{4} = \frac{4x}{4}$$ Divide both sides by 4 to isolate x.

$$\frac{7}{2} = x$$ Simplify.

The check of this solution is left to the reader. The solution set is $\left\{\frac{7}{2}\right\}$.

Quick Check 4

Solve the equation $6x + 19 = 3x - 8$.

Using Your Calculator You can use a calculator to check the solutions to an equation. Substitute the value for the variable and simplify the expressions on each side of the equation. Here is how the check of the solution $x = \frac{7}{2}$ would look on the TI-84.

```
3(7/2)+8
                18.5
7(7/2)-6
                18.5
```

Because each expression is equal to 18.5 when $x = \frac{7}{2}$, this solution checks.

In the next example, we will solve an equation containing like terms on the same side of the equation.

EXAMPLE 5 Solve the equation $13x - 43 - 9x = 6x + 37 + 5x - 66$.

SOLUTION This equation has like terms on each side of the equation, so begin by combining these like terms.

$13x - 43 - 9x = 6x + 37 + 5x - 66$	
$4x - 43 = 11x - 29$	Combine like terms on each side.
$4x - 43 - 4x = 11x - 29 - 4x$	Subtract $4x$ from both sides to gather variable terms on the right side of the equation.
$-43 = 7x - 29$	Simplify.
$-43 + 29 = 7x - 29 + 29$	Add 29 to both sides to isolate $7x$.
$-14 = 7x$	Simplify.
$-\frac{14}{7} = \frac{7x}{7}$	Divide both sides by 7 to isolate x.
$-2 = x$	Divide.

The check of this solution is left to the reader. The solution set is $\{-2\}$.

Quick Check 5

Solve the equation $6x - 9 + 4x = 3x + 13 - 8$.

EXAMPLE 6 Solve the equation $6(3x - 8) + 14 = x - 3(x - 5) + 1$.

SOLUTION Begin by distributing the 6 on the left side of the equation and the -3 on the right side of the equation. Then combine like terms before solving.

$6(3x - 8) + 14 = x - 3(x - 5) + 1$	
$18x - 48 + 14 = x - 3x + 15 + 1$	Distribute 6 and -3.
$18x - 34 = -2x + 16$	Combine like terms.
$18x - 34 + 2x = -2x + 16 + 2x$	Add $2x$ to both sides to gather variable terms on the left side of the equation.
$20x - 34 = 16$	Simplify.
$20x - 34 + 34 = 16 + 34$	Add 34 to both sides to isolate $20x$.
$20x = 50$	Simplify.
$\frac{20x}{20} = \frac{50}{20}$	Divide by 20 to isolate x.
$x = \frac{5}{2}$	Simplify.

The check of this solution is left to the reader. The solution set is $\{\frac{5}{2}\}$.

Quick Check 6

Solve the equation $3(2x - 7) + x = 2(x - 9) - 18$.

In the next example, we will clear the equation of fractions before solving.

EXAMPLE 7 Solve the equation $\frac{3}{5}x - \frac{2}{3} = 2x + \frac{5}{6}$.

SOLUTION Begin by finding the LCM of the three denominators (5, 3, and 6), which is 30. Then multiply both sides of the equation by 30 to clear the equation of fractions.

$$\frac{3}{5}x - \frac{2}{3} = 2x + \frac{5}{6}$$

$$30\left(\frac{3}{5}x - \frac{2}{3}\right) = 30\left(2x + \frac{5}{6}\right) \quad \text{Multiply both sides by LCM (30).}$$

$$\overset{6}{\cancel{30}} \cdot \frac{3}{\underset{1}{\cancel{5}}}x - \overset{10}{\cancel{30}} \cdot \frac{2}{\underset{1}{\cancel{3}}} = 30 \cdot 2x + \overset{5}{\cancel{30}} \cdot \frac{5}{\underset{1}{\cancel{6}}} \quad \text{Distribute and divide out common factors.}$$

$$18x - 20 = 60x + 25 \quad \text{Multiply.}$$

$$18x - 20 - 18x = 60x + 25 - 18x \quad \text{Subtract } 18x \text{ from both sides to gather variable terms on the right side of the equation.}$$

$$-20 = 42x + 25 \quad \text{Simplify.}$$

$$-20 - 25 = 42x + 25 - 25 \quad \text{Subtract 25 from both sides to isolate } 42x.$$

$$-45 = 42x \quad \text{Simplify.}$$

$$-\frac{45}{42} = \frac{42x}{42} \quad \text{Divide both sides by 42 to isolate } x.$$

$$-\frac{15}{14} = x \quad \text{Simplify.}$$

The check of this solution is left to the reader. The solution set is $\left\{-\frac{15}{14}\right\}$.

Quick Check 7

Solve the equation $\frac{1}{10}x - \frac{2}{5} = \frac{1}{20}x - \frac{7}{10}$.

Contradictions and Identities

Objective 4 Identify linear equations with no solution. Each equation we have solved to this point has had exactly one solution. This will not always be the case. Now we consider two special types of equations: contradictions and identities.

A **contradiction** is an equation that is always false regardless of the value substituted for the variable. A contradiction has no solution, so its solution set is the empty set { }. The empty set also is known as the **null set** and is denoted by the symbol $\emptyset$.

EXAMPLE 8 Solve the equation $3(x + 1) + 2(x + 4) = 5x + 6$.

SOLUTION Begin solving this equation by distributing on the left side of the equation.

$$3(x + 1) + 2(x + 4) = 5x + 6$$

$$3x + 3 + 2x + 8 = 5x + 6 \quad \text{Distribute.}$$

$$5x + 11 = 5x + 6 \quad \text{Combine like terms.}$$

$$5x + 11 - 5x = 5x + 6 - 5x \quad \text{Subtract } 5x \text{ from both sides.}$$

$$11 = 6 \quad \text{Simplify.}$$

We are left with an equation that is a false statement because 11 is never equal to 6. This equation is a contradiction and has no solutions. Its solution set is $\emptyset$.

Quick Check 8

Solve the equation $3x - 4 = 3x + 4$.

Objective 5 Identify linear equations with infinitely many solutions. An **identity** is an equation that is always true. If we substitute any real number for the variable in an identity, it will produce a true statement. The solution set for an identity is the set of all real numbers. We denote the set of all real numbers as $\mathbb{R}$. An identity has infinitely many solutions rather than a single solution.

EXAMPLE 9 Solve the equation $9x - 5 = 2(4x - 1) + x - 3$.

SOLUTION Begin to solve this equation by distributing on the right side of the equation.

$$\begin{aligned} 9x - 5 &= 2(4x - 1) + x - 3 & \\ 9x - 5 &= 8x - 2 + x - 3 & \text{Distribute.} \\ 9x - 5 &= 9x - 5 & \text{Combine like terms.} \\ 9x - 5 - 9x &= 9x - 5 - 9x & \text{Subtract } 9x \text{ from both sides.} \\ -5 &= -5 & \text{Simplify.} \end{aligned}$$

Because -5 is always equal to -5 regardless of the value of x, this equation is an identity. Its solution set is the set of all real numbers $\mathbb{R}$.

Quick Check 9

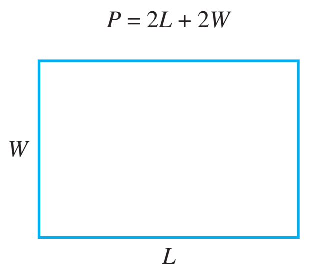
Solve the equation $5a + 4 = 4 + 5a$.

Literal Equations

Objective 6 Solve literal equations for a specified variable. A **literal equation** is an equation that contains two or more variables. Literal equations are often used in real-world applications involving many unknowns, such as geometry problems.

Perimeter of a Rectangle

The **perimeter of a rectangle** is a measure of the distance around the rectangle. The formula for the perimeter (P) of a rectangle with length L and width W is $P = 2L + 2W$.



The equation $P = 2L + 2W$ is a literal equation with three variables. We may be asked to solve literal equations for one variable in terms of the other variables in the equation. The equation $P = 2L + 2W$ is solved for the variable P. If we solved for the width W in terms of the length L and the perimeter P, we would have a formula for the width of a rectangle if we knew its length and perimeter.

We solve literal equations by isolating the specified variable. We use the same general strategy that we used to solve linear equations. We treat the other variables in the equation as if they were constants.

EXAMPLE 10 Solve the literal equation $P = 2L + 2W$ (perimeter of a rectangle) for W.

SOLUTION Gather all terms containing the variable W on one side of the equation and gather all other terms on the other side. This can be done by subtracting $2L$ from both sides.

$$\begin{aligned} P &= 2L + 2W & \\ P - 2L &= 2L + 2W - 2L & \text{Subtract } 2L \text{ to isolate } 2W. \\ P - 2L &= 2W & \text{Simplify.} \\ \frac{P - 2L}{2} &= \frac{2W}{2} & \text{Divide by 2 to isolate } W. \\ \frac{P - 2L}{2} &= W & \end{aligned}$$

We usually rewrite the equation so that the variable we solved for appears on the left side: $W = \frac{P - 2L}{2}$.

Quick Check 10

Solve the literal equation $x + 2y = 5$ for y.

EXAMPLE 11 Solve the literal equation $A = \frac{1}{2}bh$ (area of a triangle) for h.

SOLUTION Because this equation has a fraction, begin by multiplying both sides by 2.

$$A = \frac{1}{2}bh$$

$$2 \cdot A = 2 \cdot \frac{1}{2}bh \qquad \text{Multiply both sides by 2 to clear fractions.}$$

$$2 \cdot A = \overset{1}{\cancel{2}} \cdot \frac{1}{\underset{1}{\cancel{2}}}bh \qquad \text{Divide out common factors.}$$

$$2A = bh \qquad \text{Multiply.}$$

$$\frac{2A}{b} = \frac{bh}{b} \qquad \text{Divide both sides by } b \text{ to isolate } h.$$

$$\frac{2A}{b} = h \quad \text{or} \quad h = \frac{2A}{b}$$

Quick Check 11

Solve the literal equation $\frac{1}{5}xy = z$ for x.

Some geometry formulas contain the symbol π, such as $C = 2\pi r$ and $A = \pi r^2$. The symbol π is the Greek letter pi. It is used to represent an irrational number that is approximately equal to 3.14. When solving literal equations containing π, do not replace it with its approximate value.

BUILDING YOUR STUDY STRATEGY

Using Your Textbook, 2 Quick Check Exercises The Quick Check exercises following most examples in this text are similar to the examples in the book. After reading through and possibly reworking an example, try the corresponding Quick Check exercise. You can then decide whether you need more practice.

Exercises 2.2

MyMathLab — PRACTICE — WATCH — DOWNLOAD — READ — REVIEW

Vocabulary

1. To clear fractions from an equation, multiply both sides of the equation by the ______ of the denominators.
2. An equation that is never true is a(n) ______.
3. The solution set to a contradiction can be denoted $\varnothing$, which represents ______.
4. An equation that is always true is a(n) ______.
5. To solve the equation $2x - 9 = 7$, the best first step is to ______.
   a) divide both sides of the equation by 2
   b) add 9 to both sides of the equation
   c) subtract 7 from both sides of the equation
6. To solve the equation $3(4x + 5) = 11$, the best first step is to ______.
   a) distribute 3 on the left side of the equation
   b) divide both sides of the equation by 4
   c) subtract 5 from both sides of the equation

Solve.

7. $5x + 31 = 16$
8. $2x + 9 = 31$
9. $23 - 4x = 9$
10. $33 - 6x = -24$
11. $6 = 2a + 20$
12. $-29 = 6b - 17$
13. $9x + 24 = 24$
14. $-4x + 30 = -34$
15. $16.2x - 43.8 = 48.54$
16. $-9.5x - 72.35 = 130$
17. $7a + 11 = 5a - 9$

18. $3m - 11 = 9m + 5$

19. $16 - 4x = 2x + 61$

20. $6n + 14 = 27 + 6n$

21. $5x - 9 = -9 + 5x$

22. $x - 7 = 11 - 3x$

23. $16 - 5x = 2x - 5$

24. $-31 + 11n = 4n + 60$

25. $3.2x + 8.3 = 1.3x + 19.7$

26. $4x - 29.2 = 7.5x - 6.8$

27. $3x + 8 + x = 2x - 9 + 13$

28. $3x + 14 + 8x - 90 = 4x + 11 + x + 27$

29. $x + (x + 1) + (x + 2) = 378$

30. $x + (x + 2) + (x + 4) = 447$

31. $2L + 2(L - 5) = 94$

32. $2(w + 7) + 2w = 74$

33. $2(2w - 3) + 2w = 102$

34. $2L + 2(3L - 50) = 68$

35. $4(2x - 3) - 5x = 3(x + 4)$

36. $2(5x - 3) + 4(x - 8) = 11$

37. $13(3x + 4) - 7(2x - 5) = 5x - 13$

38. $3(2x - 8) - 5(15 - 6x) = 9(4x - 11)$

39. $0.06x + 0.03(4000 - x) = 156$

40. $0.11x + 0.05(7500 - x) = 675$

41. $0.17x - 0.4(x + 1700) = -2566$

42. $0.09x - 0.02(x - 2100) = 322$

43. $0.48x + 0.72(120 - x) = 0.66(120)$

44. $0.3x + 0.55(95 - x) = 0.45(95)$

45. $\frac{1}{4}x - \frac{1}{3} = \frac{5}{12}$

46. $\frac{3}{11}x + \frac{5}{2} = 8$

47. $\frac{x}{12} - \frac{11}{6} = \frac{5}{4}$

48. $\frac{2}{5} - \frac{3}{8}x = -\frac{11}{10}$

49. $\frac{1}{2}x - 3 = \frac{11}{5} - \frac{3}{4}x$

50. $\frac{3}{5} - \frac{5}{6}x = 2x + \frac{4}{3}$

51. $\frac{2}{9}x + \frac{3}{4} = \frac{1}{6}x - \frac{5}{3}$

52. $\frac{4}{7}x - \frac{3}{2} = x - \frac{5}{4}$

53. Write a linear equation that is a contradiction.

54. Write a linear equation that is an identity.

55. Write an equation that has infinitely many solutions.

56. Write an equation that has no solutions.

57. Write an equation whose single solution is negative.

58. Write an equation whose single solution is a fraction.

Solve the following literal equations for the specified variable.

59. $5x + y = -2$ for y

60. $-6x + y = 9$ for y

61. $7x + 2y = 4$ for y

62. $9x + 4y = 20$ for y

63. $-4x + 3y = 10$ for y

64. $-8x + 5y = -6$ for y

65. $P = a + b + c$ for b

66. $P = a + b + 2c$ for c

67. $d = r \cdot t$ for t

68. $d = r \cdot t$ for r

69. $C = 2\pi r$ for r

70. $S = 2\pi rh$ for r

To convert a Celsius temperature (C) to a Fahrenheit temperature (F), use the formula $F = \frac{9}{5}C + 32$.

71. If the temperature outside is 68°F, find the Celsius temperature.

72. If the normal body temperature for a person is 98.6°F, find the Celsius equivalent.

73. A number is tripled and then added to 64. If the result is 325, find the number.

74. If two-thirds of a number is added to 48, the result is 74. Find the number.

75. It costs \$200 to rent a booth at a craft fair. Tina wants to sell homemade kites at the fair. It costs Tina \$4 in material to make each kite.

a) If she has \$520 available to buy material and pay expenses, how many kites can she make to sell at the craft fair after paying the \$200 rental charge?

b) If she is able to sell all of the kites, how much will she need to charge for each kite in order to break even?

c) How much should she charge for each kite in order to make a profit of \$500?

76. A churro is a Mexican dessert pastry. Dan is able to buy churros for 75 cents each, which he plans to sell at a high school baseball game. Dan must donate \$50 to the high school team to be allowed to sell the churros at the game.

a) If Dan has \$110 to invest in this venture, how many churros will he be able to buy after paying \$50 to the team?

b) If Dan charges \$2 for each churro, how many must he sell to break even?

c) If Dan charges \$2 for each churro and he is able to sell all of his churros, what will his profit be?

77. Find the missing value such that $x = 3$ is a solution of $5x - ? = 11$.

78. Find the missing value such that $x = -2$ is a solution of $3(2x - 5) + ? = 5(3 - x)$.

79. Find the missing value such that $x = -\frac{3}{2}$ is a solution of $2x + 9 = 6x + ?$.

80. Find the missing value such that $x = \frac{2}{5}$ is a solution of $4(2x + 7) = 7(3x + ?)$.

Mixed Practice, 81–92

Solve. (If the equation is a literal equation, solve for x.)

81. $14x = -105$

82. $\frac{1}{6}x + \frac{3}{4} = \frac{5}{9}$

83. $27x - 16 - 6x = 14 + 21x - 30$

84. $2x - 3y + 4z = 0$

85. $-5n + 11 - 4n + 9 = n - 45$

86. $a + 43 = 16$

87. $11x - 8y = 34$

88. $2(3w + 7) + 2w = 86$

89. $\frac{2}{5}x - \frac{3}{8} = 3x - \frac{7}{10}$

90. $5t + 72 + 8t = 17t - 28 + 11t$

91. $0.16x + 0.07(3000 - x) = 372$

92. $9n - 17 - 5n - 32 = 23 + 4n + 5$

Writing in Mathematics

Answer in complete sentences.

93. Solve the linear equation $14 = 2x - 9$, showing all of the steps. Next to this, show all of the steps necessary to solve the literal equation $y = 2x - 9$ for x. Write a paragraph explaining how these two processes are similar and how they are different.

94. Write a word problem whose solution can be found from the equation $4.75x + 225 = 795$.

95. ***Newsletter*** Write a newsletter explaining the steps for solving linear equations.

2.3 Problem Solving; Applications of Linear Equations

OBJECTIVES

1. Understand the six steps for solving applied problems.
2. Solve problems involving an unknown number.
3. Solve problems involving geometric formulas.
4. Solve problems involving consecutive integers.
5. Solve problems involving motion.
6. Solve other applied problems.

Introduction to Problem Solving

Although the ability to perform mathematical computations and abstract algebraic manipulations is valuable, one of the most important skills developed in math classes is the skill of problem solving. Every day we are faced with making important decisions and predictions, and the thought process required in decision making is similar to the process of solving mathematical problems.

When faced with a problem to solve in the real world, we first take inventory of the facts that we know. We also determine exactly what it is we are trying to figure out. Then we develop a plan for taking what we already know and using it to figure out how to solve the problem. Once we have solved the problem, we reflect on the route we took to solve it to make sure that route was a logical way to solve the problem. After examining the solution for correctness and practicality, we may finish by presenting the solution to others for their consideration, information, or approval.

An example of one such real-world problem is determining how early to leave for school on the first day of classes. Suppose your first class is at 9 A.M. The problem to solve is figuring out what time to leave your house so that you will not be late. Think about the facts you know: it is normally a 20-minute drive to school, and the classroom is a 10-minute walk from the parking lot. Add information specific to the first day of classes: traffic near the school will be more congested than normal, and it will take longer to find a parking space in the parking lot. Based on past experience, you figure an extra 15 minutes for traffic and another 10 minutes for parking. Adding up all of these times tells you that you need 55 minutes to get to your class. Adding an extra 15 minutes just to be sure, you decide that you need to leave home by 7:50 A.M.

Objective 1 Understand the six steps for solving applied problems. In the previous example, we identified the problem to be solved, gathered the facts, and used them to find a solution to the problem. Solving applied math problems requires that we follow a similar procedure. George Pólya, this chapter's Mathematician in History, was a leader in the study of problem solving. Here is a general plan for solving applied math problems that is based on the work of Pólya's text *How to Solve It.*

Solving Applied Problems

1. **Read the problem.** This step is often overlooked, but misreading or misinterpreting the problem essentially guarantees an incorrect solution. Quickly read the problem once to get a rough idea of what is going on, then read it more carefully a second time to gather all of the important information.
2. **List all of the important information.** Identify all known quantities presented in the problem and determine which quantities you need to find. Creating a table to hold this information or a drawing to represent the problem is a good idea.
3. **Assign a variable to the unknown quantity.** If there is more than one unknown quantity, express each one in terms of the same variable, if possible.
4. **Find an equation relating the known values to the variable expressions representing the unknown quantities.** Sometimes this equation can be translated directly from the wording of the problem. Other times the equation will depend on general facts that are known outside the statement of the problem, such as geometry formulas.
5. **Solve the equation.** Solving the equation is not the end of the problem, as the value of the variable often is not what you were originally looking for. If you were asked for the length and width of a rectangle and you replied "x equals 4," you would not have answered the question. Check your solution to the equation.
6. **Present the solution.** Take the value of the variable from the solution to the equation and use it to figure out all unknown quantities. Check these values to ensure that they make sense in the context of the problem. For example, the length of a rectangle cannot be negative. Finally, present your solution in a complete sentence using the proper units.

Now we will put this strategy to use. All of the applied problems in this section lead to linear equations.

Objective 2 **Solve problems involving an unknown number.**

EXAMPLE 1 Seven more than twice a number is 39. Find the number.

SOLUTION This is not a very exciting problem, but it provides a good opportunity to practice setting up applied problems.

There is one unknown in this problem. Let's use n to represent the *unknown number.*

Unknown
Number: n

Now translate the sentence *Seven more than twice a number is 39* into an equation. In this case, twice a number is $2n$; so 7 more than twice a number is $2n + 7$.

$$\underbrace{\text{Seven more than twice a number}}_{2n + 7} \quad \underbrace{\text{is}}_{=} \quad \underbrace{39}_{39}$$

Now solve the equation.

$$2n + 7 = 39$$
$$2n = 32 \quad \text{Subtract 7 from both sides.}$$
$$n = 16 \quad \text{Divide both sides by 2.}$$

The solution of the equation is $n = 16$. Now refer back to the table containing the unknown quantity. Because the unknown number is n, the solution to the problem is the same as the solution of the equation. The number is 16.

It is a good idea to check the solution. Twice the number is 32, and 7 more than that is 39.

Quick Check 1

Three less than 4 times a number is 29. Find the number.

Geometry Problems

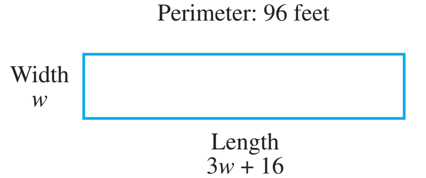
Objective 3 **Solve problems involving geometric formulas.** The next example involves the perimeter of a rectangle, which is a measure of the distance around the outside of the rectangle. The perimeter P of a rectangle whose length is L and whose width is W is given by the formula $P = 2L + 2W$.

EXAMPLE 2 Mark's vegetable garden is in the shape of a rectangle, and he can enclose it with 96 feet of fencing. If the length of his garden is 16 feet more than 3 times the width, find the dimensions of his garden.

SOLUTION The unknown quantities are the length and the width. Because the length is given in terms of the width, pick a variable to represent the width and write the length in terms of this variable. Let w represent the width of the rectangle. The length is 16 feet more than 3 times the width, so express the length as $3w + 16$. One piece of information provided is that the perimeter is 96 feet. This information can be recorded in the following table or drawing:

Unknowns
Length: $3w + 16$
Width: w

Known
Perimeter: 96 feet



Start with the formula for perimeter and substitute the appropriate values and expressions.

$$
\begin{aligned}
P &= 2L + 2W & & \\
96 &= 2(3w + 16) + 2w & & \text{Substitute 96 for } P,\ 3w + 16 \text{ for } L, \text{ and } w \text{ for } W. \\
96 &= 6w + 32 + 2w & & \text{Distribute.} \\
96 &= 8w + 32 & & \text{Combine like terms.} \\
64 &= 8w & & \text{Subtract 32 from both sides.} \\
8 &= w & & \text{Divide both sides by 8.}
\end{aligned}
$$

Now take this solution and substitute 8 for w in the expressions for length and width, which can be found in the table that lists the unknowns.

Length: $3w + 16 = 3(8) + 16 = 40$
Width: $w = 8$

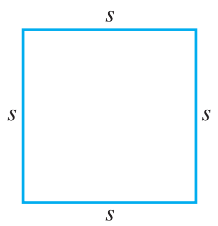
The length of Mark's garden is 40 feet, and the width is 8 feet. It checks that the perimeter of this rectangle is 96 feet.

Quick Check 2

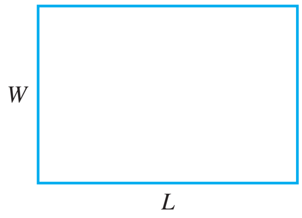
A rectangular living room has a perimeter of 80 feet. The length of the room is 8 feet less than twice the width of the room. Find the dimensions of the room.

Here is a summary of useful formulas and definitions from geometry.

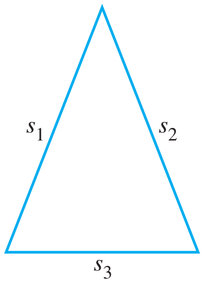
Geometry Formulas and Definitions



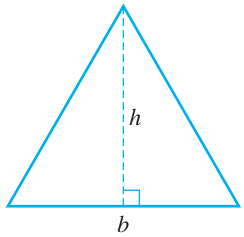
- Perimeter of a square with side s: $P = 4s$
- Area of a square with side s: $A = s^2$



- Perimeter of a rectangle with length L and width W: $P = 2L + 2W$
- Area of a rectangle with length L and width W: $A = LW$

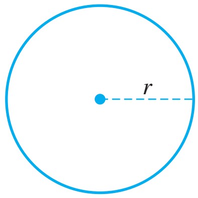


- Perimeter of a triangle with sides s_1, s_2, and s_3: $P = s_1 + s_2 + s_3$



- Area of a triangle with base b and height h: $A = \frac{1}{2}bh$

- An **equilateral triangle** is a triangle that has three equal sides.
- An **isosceles triangle** is a triangle that has at least two equal sides.



- **Circumference** of a circle with radius r: $C = 2\pi r$
- Area of a circle with radius r: $A = \pi r^2$

EXAMPLE 3 An isosceles triangle has a perimeter of 30 centimeters. The third side is 3 centimeters shorter than each of the sides that have equal lengths. Find the lengths of the three sides.

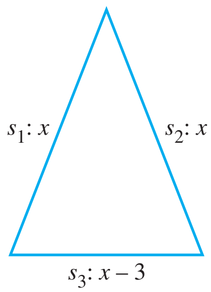
SOLUTION In this problem, there are three unknowns, which are the lengths of the three sides. Because the triangle is an isosceles triangle, the first two sides have equal lengths. Represent the length of each side using the variable x. The third side is 3 centimeters shorter than the other two sides, and its length can be represented by $x - 3$. Here is a summary of this information, together with the known perimeter.

Unknowns

s_1: x
s_2: x
s_3: $x - 3$

Known

Perimeter: 30 cm



Now set up the equation and solve for x.

$P = s_1 + s_2 + s_3$

$30 = x + x + (x - 3)$ Substitute 30 for P, x for s_1 and s_2, and $x - 3$ for s_3.

$30 = 3x - 3$ Combine like terms.

$33 = 3x$ Add 3 to both sides.

$11 = x$ Divide both sides by 3.

Looking back to the table, substitute 11 for x.

s_1: $x = 11$
s_2: $x = 11$
s_3: $x - 3 = 11 - 3 = 8$

The three sides are 11 centimeters, 11 centimeters, and 8 centimeters. The perimeter of this triangle checks to be 30 centimeters.

Quick Check 3

The perimeter of a triangle is 112 inches. The longest side is 8 inches longer than 3 times the shortest side. The other side of the triangle is 20 inches longer than twice the shortest side. Find the lengths of the three sides.

Consecutive Integers

Objective 4 Solve problems involving consecutive integers. The following example involves consecutive integers, which are integers that are 1 unit apart from each other on the number line.

EXAMPLE 4 The sum of three consecutive integers is 87. Find them.

SOLUTION In this problem, the three consecutive integers are the three unknowns. Let x represent the first integer. Because consecutive integers are 1 unit apart from each other, let $x + 1$ represent the second integer. Adding another 1, let $x + 2$ represent the third integer. Here is a table of the unknowns.

Unknowns

First: x
Second: $x + 1$
Third: $x + 2$

We know that the sum of these three integers is 87, which leads to the equation.

$$x + (x + 1) + (x + 2) = 87$$
$$3x + 3 = 87 \quad \text{Combine like terms.}$$
$$3x = 84 \quad \text{Subtract 3 from both sides.}$$
$$x = 28 \quad \text{Divide both sides by 3.}$$

Now substitute this solution to find the three unknowns.

First: $x = 28$
Second: $x + 1 = 28 + 1 = 29$
Third: $x + 2 = 28 + 2 = 30$

The three integers are 28, 29, and 30. The three integers add up to 87.

Quick Check 4

The sum of three consecutive integers is 213. Find them.

Suppose a problem involves consecutive *odd* integers rather than consecutive integers. Consider any string of consecutive odd integers (for example, 15, 17, 19, 21, 23, . . .). How far apart are these consecutive odd integers? Each odd integer is 2 away from the previous one. When working with consecutive odd integers (or consecutive even integers for that matter), let x represent the first integer and add 2 to find the next consecutive integer of that type. Here is a table showing the pattern of unknowns for consecutive integers, consecutive odd integers, and consecutive even integers.

Consecutive Integers	Consecutive Odd Integers	Consecutive Even Integers
First: x	First: x	First: x
Second: $x + 1$	Second: $x + 2$	Second: $x + 2$
Third: $x + 2$	Third: $x + 4$	Third: $x + 4$
⋮	⋮	⋮

Motion Problems

Objective 5 Solve problems involving motion.

Distance Formula

When an object such as a car moves at a constant rate of speed, r, for an amount of time, t, the distance traveled, d, is given by the formula $d = r \cdot t$.

Suppose a person drove 238 miles in 3.5 hours. To determine the car's rate of speed for this trip, we can substitute 238 for d and 3.5 for t in the equation $d = r \cdot t$.

$$
\begin{aligned}
d &= r \cdot t \\
238 &= r \cdot 3.5 && \text{Substitute 238 for } d \text{ and 3.5 for } t. \\
\frac{238}{3.5} &= \frac{r \cdot 3.5}{3.5} && \text{Divide both sides by 3.5.} \\
68 &= r && \text{Simplify.}
\end{aligned}
$$

The car's rate of speed for this trip was 68 mph.

EXAMPLE 5 Tina drove her car at a rate of 60 mph from her home to Rochester. Her mother, Linda, made the same trip at a rate of 80 mph, and it took her 2 hours less than it took Tina to make the trip. How far is Tina's home from Rochester?

SOLUTION For this problem, begin by setting up a table showing the relevant information. Let t represent the time it took Tina to make the trip. Because it took Linda 2 hours less to make the trip, represent her time by $t - 2$. Multiply each person's rate of speed by the time she traveled to find the distance she traveled.

	Rate	Time	Distance ($d = r \cdot t$)
Tina	60	t	$60t$
Linda	80	$t - 2$	$80(t - 2)$

Because both trips were exactly the same distance, we get the equation by setting the expression for Tina's distance equal to the expression for Linda's distance and then solve for t.

$$
\begin{aligned}
60t &= 80(t - 2) \\
60t &= 80t - 160 && \text{Distribute.} \\
-20t &= -160 && \text{Subtract } 80t \text{ from both sides.} \\
t &= 8 && \text{Divide both sides by } -20.
\end{aligned}
$$

At this point, we must be careful to answer the appropriate question. We were asked to find the distance traveled. We can substitute 8 for t in either of the expressions for distance. Using the expression for the distance traveled by Tina, $60t = 60(8) = 480$. Using the expression for the distance traveled by Linda, $80(t - 2)$, produces the same result. Tina's home is 480 miles from Rochester.

Quick Check 5

Jake drove at a rate of 85 mph for a certain amount of time. His brother Elwood then took over as the driver. Elwood drove at a rate of 90 mph for 3 hours longer than Jake had driven. If the two brothers drove a total of 970 miles, how long did Elwood drive?

Other Problems

Objective 6 Solve other applied problems.

EXAMPLE 6 One number is 5 more than twice another number. If the sum of the two numbers is 80, find the two numbers.

SOLUTION In this problem, there are two unknown numbers. Let one of the numbers be x. Because one number is 5 more than twice the other number, we can represent the other number as $2x + 5$.

Unknown

First: x
Second: $2x + 5$

Finally, we know that the sum of these two numbers is 80, which leads to the equation $x + 2x + 5 = 80$.

$$
\begin{aligned}
x + 2x + 5 &= 80 \\
3x + 5 &= 80 && \text{Combine like terms.} \\
3x &= 75 && \text{Subtract 5 from both sides.} \\
x &= 25 && \text{Divide both sides by 3.}
\end{aligned}
$$

Now substitute this solution to find the two unknown numbers.

First: $x = 25$
Second: $2x + 5 = 2(25) + 5 = 50 + 5 = 55$

The two numbers are 25 and 55. The sum of these two numbers is 80.

Quick Check 6

A number is 1 more than five times another number. If the sum of the two numbers is 103, find the two numbers.

This section concludes with a problem involving coins.

EXAMPLE 7 Ernie has nickels and dimes in his pocket. He has 5 more dimes than nickels. If Ernie has \$2.30 in his pocket, how many nickels and dimes does he have?

SOLUTION We know that the amount of money that Ernie has in nickels and dimes totals \$2.30. Because the number of dimes is given in terms of the number of nickels, let n represent the number of nickels. Because we know that Ernie has 5 more dimes than nickels, we can represent the number of dimes by $n + 5$. A table can be quite helpful in organizing this information, as well as in finding the equation.

	Number of Coins	Value of Coin	Amount of Money
Nickels	n	0.05	$0.05n$
Dimes	$n + 5$	0.10	$0.10(n + 5)$

The amount of money that Ernie has in nickels and dimes is equal to $2.30.

$$
\begin{aligned}
0.05n + 0.10(n + 5) &= 2.30 & &\\
0.05n + 0.10n + 0.50 &= 2.30 & &\text{Distribute.}\\
0.15n + 0.50 &= 2.30 & &\text{Combine like terms.}\\
0.15n &= 1.80 & &\text{Subtract 0.50 from both sides.}\\
n &= 12 & &\text{Divide both sides by 0.15.}
\end{aligned}
$$

The number of nickels is 12. The number of dimes $(n + 5)$ is $12 + 5$, or 17. Ernie has 12 nickels and 17 dimes. You are responsible for verifying that these coins are worth a total of $2.30.

Quick Check 7

Bert has a pocketful of nickels and quarters. He has 13 more nickels than quarters. If Bert has $3.35 in his pocket, how many nickels and quarters does he have?

BUILDING YOUR STUDY STRATEGY

Using Your Textbook, 3 Supplementing Notes and Using Examples When you are rewriting your classroom notes, your textbook can be helpful in supplementing those notes. If your instructor gave you a set of definitions in class, look up the corresponding definitions in the textbook.

You also should keep the textbook handy while you are working on your homework. If you are stuck on a particular problem, look through the section for an example. It may give you an idea about how to proceed.

Exercises 2.3

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Vocabulary

1. Which equation can be used to solve *7 less than twice a number is 23?*

a) $2(n - 7) = 23$ **b)** $2n - 7 = 23$

c) $7 - 2n = 23$

2. State the formula for the perimeter of a rectangle.

3. If a rectangle has a perimeter of 80 feet and its length is 8 feet less that 3 times its width, which equation can be used to find the dimensions of the rectangle?

a) $2(W - 8) + 2(3W) = 80$

b) $2(3W - 8) + 2W = 80$

c) $2(L) + 2(3L - 8) = 80$

4. Consecutive integers are integers that are ______ unit(s) apart on a number line.

5. If you let x represent the first of three consecutive integers, you can represent the other two integers by ________ and ________.

6. If you let x represent the first of three consecutive even integers, you can represent the other two integers by ________ and ________.

7. If you let x represent the first of three consecutive odd integers, you can represent the other two integers by ________ and ________.

8. Charles has 7 fewer $5 bills than $10 bills. The total value of these bills is $265. Which equation can be used to determine how many of each type of bill Charles has?

a) $5(n - 7) + 10n = 265$

b) $5n - 7 + 10n = 265$

c) $5n + 10(n - 7) = 265$

d) $5n + 10n - 7 = 265$

9. Five more than twice a number is 97. Find the number.

10. Eighteen more than 7 times a number is 109. Find the number.

11. Eight less than 5 times a number is 717. Find the number.

12. Nine less than 3 times a number is 42. Find the number.

13. When 3.2 is added to 6 times a number, the result is 56.6. Find the number.

14. When 17.2 is subtracted from twice a number, the result is 11.4. Find the number.

15. The width of a rectangle is 5 meters less than its length, and the perimeter is 26 meters. Find the length and width of the rectangle.

16. A rectangle has a perimeter of 62 feet. If its length is 9 feet longer than its width, find the dimensions of the rectangle.

17. A rectangular patio has a perimeter of 56 feet. If the length of the patio is 2 feet less than twice the width, find the dimensions of the patio.

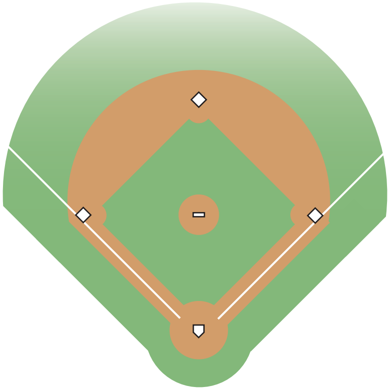
18. A rectangular sheet of paper has a perimeter of 39 inches. The length of the paper is 6 inches less than twice its width. Find the dimensions of the sheet of paper.

19. A rectangle has a perimeter of 130 centimeters. If the length of the rectangle is 4 times its width, find the dimensions of the rectangle.

20. A homeowner has a rectangular garden in her backyard. The length of the garden is 9 times as long as the width, and the perimeter is 160 feet. Find the dimensions of the garden.

21. A square has a perimeter of 220 feet. What is the length of one of its sides?

22. The bases in a Major League Baseball infield are set out in the shape of a square. When Kevin Youkilis hits a home run, he has to run 360 feet to make it all the way around the bases. How far is the distance from home plate to first base?



23. An equilateral triangle has a perimeter of 135 centimeters. Find the length of each side of this triangle.

24. The third side of an isosceles triangle is 2 inches shorter than each of the other two sides. If the perimeter of this triangle is 67 inches, find the length of each side.

25. One side of a triangle is 5 inches longer than the side that is the base of the triangle. The third side is 7 inches longer than the base. If the perimeter is 36 inches, find the length of each side.

26. Two sides of a triangle are 3 feet and 6 feet longer than the third side of the triangle. If the perimeter of the triangle is 36 feet, find the length of each side.

27. A circle has a circumference of 69.08 inches. Use $\pi \approx 3.14$ to find the radius of the circle.

28. A circle has a circumference of 48π inches. Find its radius.

Two positive angles are said to be **complementary** if their measures add up to 90°.

29. Angles A and B are complementary angles. If the measure of angle A is 30° more than the measure of angle B, find the measures of the two angles.

30. Angles A and B are complementary angles. Angle A is 15° less than angle B. Find the measures of the two angles.

31. An angle is 10° more than 3 times its complementary angle. Find the measures of the two angles.

32. An angle is 6° less than twice its complementary angle. Find the measures of the two angles.

Two positive angles are said to be **supplementary** if their measures add up to 180°.

33. An angle is 12° more than 3 times its supplementary angle. Find the measures of the two angles.

34. An angle is 39° less than twice its supplementary angle. Find the measures of the two angles.

The measures of the three angles inside a triangle total 180°.

35. One angle in a triangle is 10° more than the smallest angle in the triangle, while the other angle is 20° more than the smallest angle. Find the measures of the three angles.

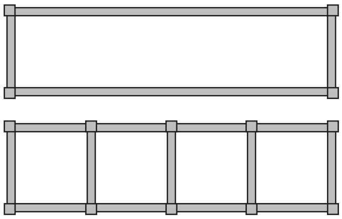
36. A triangle contains three angles: A, B, and C. Angle B is twice angle A, and angle C is 20° more than angle A. Find the measures of the three angles.

37. If the perimeter of a rectangle is 104 inches and the length of the rectangle is 6 inches more than its width, find the area of the rectangle.

38. If the perimeter of a rectangle is 76 inches and the width of the rectangle is 4 inches less than its length, find the area of the rectangle.

39. A rectangle has a perimeter of 46 inches. The length of the rectangle is 7 inches shorter than twice its width. If a square's side is as long as the length of this rectangle, find the perimeter of the square.

40. A rectangular pen with a perimeter of 200 feet is divided into four square pens by building three fences inside the rectangular pen as follows:



a) What are the original dimensions of the rectangular pen?

b) What is the total length of the three sections of fencing that were added inside the rectangular pen?

c) If the additional fencing cost $2.85 per foot, find the cost of the fencing required to make the four rectangular pens.

41. The sum of three consecutive integers is 459. Find the three integers.

42. The sum of three consecutive integers is 186. Find the three integers.

43. The sum of three consecutive even integers is 246. Find the integers.

44. The sum of five consecutive odd integers is 755. Find the integers.

45. The sum of four consecutive odd integers is 304. Find the integers.

46. The sum of four consecutive even integers is 508. Find the integers.

47. The smallest of three consecutive integers is 18 less than the sum of the two larger integers. Find the three integers.

48. There are three consecutive odd integers, and the sum of the smaller two integers is 49 less than 3 times the largest integer. Find the three odd integers.

49. A. J. drove his race car at a rate of 125 miles per hour for 4 hours. How far did he drive?

50. Angela made the 300-mile drive to Las Vegas in 6 hours. What was her average rate for the trip?

51. If Mario drives at a rate of 68 miles per hour, how long will it take him to drive 374 miles?

52. A bullet train travels 240 kilometers per hour. How far can it travel in 45 minutes?

53. Janet swam a 50-meter race in 28 seconds. What was her speed in meters per second?

54. In 1927, Charles Lindbergh flew his airplane, *Spirit of St. Louis,* 3500 miles from New York City to Paris in 33.5 hours. What was his speed for the trip, to the nearest tenth of a mile per hour?

55. Susan averaged 80 miles per hour on the way to Phoenix to make a sales call. On the way home, she averaged 70 miles per hour and it took her 1 hour longer to drive home than it did to drive to Phoenix.

a) What was the total driving time for Susan's trip?

b) How far does Susan live from Phoenix?

56. Geoffrey drove 60 miles per hour on the way to visit his parents. On the way back to school, he drove 75 miles per hour and it took him 1 hour less than it did to drive to his parents' home.

a) What was the total driving time for Geoffrey's trip?

b) How far is Geoffrey's school from his parents' home?

57. Don started driving east at a rate of 60 miles per hour. If Dennis leaves the same place 2 hours later heading east at a rate of 80 miles per hour, how long will it take him to catch up to Don?

58. Maxine started driving south at a rate of 50 miles per hour. One hour later Claudia started riding her motorcycle south at a rate of 70 miles per hour. How long will it take Claudia to catch Maxine?

59. Lance heads directly east at a constant rate of 25 miles per hour. One hour later, Levi leaves the same spot heading directly west at a constant rate of 20 miles per hour. How long after Levi leaves will the two cyclists be 250 miles apart?

60. This morning a train left the station heading directly north at 50 miles per hour. Two hours later a bus left the same station heading directly south at 60 miles per hour. How long after the bus leaves the station will the bus and the train be 650 miles apart?

61. One number is 17 more than another. If the sum of the two numbers is 71, find the two numbers.

62. One number is 20 more than another. If the sum of the two numbers is 106, find the two numbers.

63. One number is 8 less than another number. If the sum of the two numbers is 96, find the two numbers.

64. One number is 27 less than another number. If the sum of the two numbers is 153, find the two numbers.

65. One number is 5 more than another number. If the smaller number is doubled and added to 3 times the larger number, the sum is 80. Find the two numbers.

66. One number is 11 more than another number. If 3 times the smaller number is added to 5 times the larger number, the sum is 191. Find the two numbers.

67. Chris has a jar with dimes and quarters in it. The jar has 7 more quarters in it than dimes. If the total value of the coins is \$5.60, how many quarters are in the jar?

68. Kay's change purse has nickels and dimes in it. The number of nickels is 5 more than twice the number of dimes. If the total value of the coins is \$4.05, how many nickels are in Kay's purse?

69. The film department held a fund-raiser by showing the movie π. Admission was \$4 for students and \$7 for nonstudents. The number of students who attended was 10 more than 4 times the number of nonstudents who attended. If the department raised \$500, how many students attended the movie?

70. For a matinee, a movie theater charges \$4.50 for children and \$6.75 for adults. At today's matinee, there are 20 more children than adults and the total receipts are \$405. How many children are at today's matinee?

Writing in Mathematics

Answer in complete sentences.

71. Write a word problem involving a rectangle whose length is 50 feet and whose width is 20 feet. Explain how you created your problem.

72. Write a word problem whose solution is *There were 70 children and 40 adults in attendance.* Explain how you created your problem.

2.4 Applications Involving Percents; Ratio and Proportion

OBJECTIVES

1. Use the basic percent equation to find an unknown amount, base, or percent.
2. Solve applied problems using the basic percent equation.
3. Solve applied problems involving percent increase or percent decrease.
4. Solve problems involving interest.
5. Solve mixture problems.
6. Solve for variables in proportions.
7. Solve applied problems involving proportions.

The Basic Percent Equation

Objective 1 Use the basic percent equation to find an unknown amount, base, or percent. We know that 40 is one-half of 80. Because the fraction $\frac{1}{2}$ is the same as 50%, we also can say that 40 is 50% of 80. In this example, the number 40 is referred to as the **amount** and the number 80 is referred to as the **base**. The basic equation relating these two quantities reflects that the amount is a percentage of the base, or

$$\text{Amount} = \text{Percent} \cdot \text{Base}.$$

In this equation, it is important to express the percent as a decimal or a fraction rather than a percent. Obviously, it is crucial to identify the amount, the percent, and the base correctly. It is a good idea to write the information in the following form:

$$\underset{\text{(Amount)}}{\underline{\hspace{2cm}}} \text{ is } \underset{\text{(Percent)}}{\underline{\hspace{2cm}}} \text{ \% of } \underset{\text{(Base)}}{\underline{\hspace{2cm}}}$$

EXAMPLE 1 What number is 40% of 45?

SOLUTION Letting n represent the unknown number, write the information as follows:

$$\underset{\text{(Amount)}}{\underline{\quad n \quad}} \text{ is } \underset{\text{(Percent)}}{\underline{\quad 40 \quad}} \text{ \% of } \underset{\text{(Base)}}{\underline{\quad 45 \quad}}$$

We see that the amount is n; the percent is 40%, or 0.4; and the base is 45.

Now translate this information to an equation, making sure the percent is written as a decimal rather than a percent.

$$n = 0.4(45)$$
$$n = 18 \qquad \text{Multiply.}$$

We find that 18 is 40% of 45.

Quick Check 1

25% of 44 is what number?

A WORD OF CAUTION When using the basic percent equation, make sure you rewrite the percent as a decimal or a fraction before solving for an unknown amount or base.

EXAMPLE 2 Eight percent of what number is 12?

SOLUTION Letting n represent the unknown number, rewrite the information as follows:

$$\underset{\text{(Amount)}}{\underline{\quad 12 \quad}} \text{ is } \underset{\text{(Percent)}}{\underline{\quad 8 \quad}} \text{ \% of } \underset{\text{(Base)}}{\underline{\quad n \quad}}$$

The amount is 12, the percent is 8%, and the base is unknown. Make sure you write the percent as a decimal before working with the equation.

$$12 = 0.08n$$
$$\frac{12}{0.08} = \frac{0.08n}{0.08} \qquad \text{Divide both sides by 0.08.}$$
$$150 = n \qquad \text{Simplify.}$$

Eight percent of 150 is 12.

Quick Check 2

39 is 60% of what number?

EXAMPLE 3 What percent of 75 is 39?

SOLUTION In this problem, the percent is unknown. Letting p represent the percent, write the information as follows:

$$\underset{\text{(Amount)}}{\underline{\quad 39 \quad}} \text{ is } \underset{\text{(Percent)}}{\underline{\quad p \quad}} \text{ \% of } \underset{\text{(Base)}}{\underline{\quad 75 \quad}}$$

Now translate directly to the basic percent equation and solve it.

$$39 = p \cdot 75$$
$$\frac{39}{75} = \frac{p \cdot 75}{75} \qquad \text{Divide both sides by 75.}$$
$$0.52 = p \qquad \text{Simplify.}$$

0.52 is equivalent to 52%, so 39 is 52% of 75.

Quick Check 3

56 is what percent of 80?

A WORD OF CAUTION After using the basic percent equation to solve for an unknown percent, make sure you rewrite the solution as a percent by multiplying by 100%.

When using the basic percent equation to solve for an unknown percent, keep in mind that the base is not necessarily the larger of the two given numbers. For example, if we are trying to determine what percent of 80 is 200, the base is the smaller number (80).

Applications

Objective 2 Solve applied problems using the basic percent equation. Now we will use the basic percent equation to solve applied problems. We will continue to use the strategy for solving applied problems developed in Section 2.3.

EXAMPLE 4 There are 7107 female students at a certain community college. If 60% of all students at the college are female, what is the total enrollment of the college?

SOLUTION We know that 60% of the students are female, so the number of female students (7107) is 60% of the total enrollment. Letting n represent the unknown total enrollment, write the information as follows:

7107	is	60	% of	n
(Amount)		(Percent)		(Base)

Translate this sentence to an equation and solve.

$$7107 = 0.6n$$
$$11{,}845 = n \qquad \text{Divide both sides by 0.6.}$$

There are 11,845 students at the college.

Quick Check 4

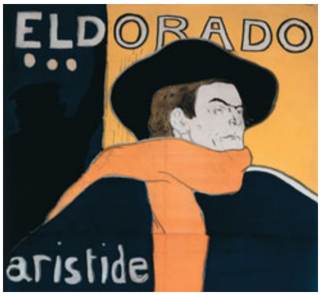
Of the 500 children who attend an elementary school, 28% buy their lunch at school. How many children buy their lunch at this school?

Percent Increase and Percent Decrease

Objective 3 Solve applied problems involving percent increase or percent decrease. **Percent increase** is a measure of how much a quantity has increased from its original value. The amount of increase is the amount in the basic percent equation, while the original value is the base.

$$\text{Amount of Increase} = \text{Percent} \cdot \text{Original}$$

Percent decrease is a similar measure of how much a quantity has decreased from its original value.



EXAMPLE 5 An art collector bought a lithograph for $2500. After three years, the lithograph was valued at $3800. Find the percent increase in the value of the lithograph over the three years.

SOLUTION The amount of increase in value is $1300 because $3800 - 2500 = 1300$. We need to determine what percent of the original value is the increase in value. Letting p represent the unknown percent, write the information as follows:

1300	is	p	% of	2500
(Amount)		(Percent)		(Base)

This translates to the equation $1300 = p \cdot 2500$, which we can solve.

$$1300 = p \cdot 2500$$
$$0.52 = p \qquad \text{Divide both sides by 2500.}$$

Converting this decimal result to a percent, we see that the lithograph increased in value by 52%.

Quick Check 5

Lee bought a new car for $30,000 three years ago, and now it is worth $13,800. Find the percent decrease in the value of the car.

A WORD OF CAUTION When solving problems involving percent increase or percent decrease, remember that the base is always the *original amount*.

Percents are involved in many business applications. One such problem is determining the sale price of an item after a percent discount has been applied.

EXAMPLE 6 A department store is having a "20% off" sale, so all prices have been reduced by 20% of the original price. If a robe was originally priced at \$37.50, what is the sale price after the 20% discount?

SOLUTION Begin by finding the amount by which the original price has been discounted. Let d represent the amount of the discount. Because the amount of the discount is 20% of the original price (\$37.50), we can write the information as follows:

$$\underset{\text{(Amount)}}{\underline{\quad d \quad}} \text{ is } \underset{\text{(Percent)}}{\underline{\quad 20 \quad}} \text{ \% of } \underset{\text{(Base)}}{\underline{\quad \$37.50 \quad}}$$

This translates to the equation $d = 0.2(37.50)$, which we solve for d.

$$d = 0.2(37.50)$$
$$d = 7.50 \qquad \text{Multiply.}$$

The discount is \$7.50, so the sale price is \$37.50 − \$7.50, or \$30.00.

Quick Check 6

A department store bought a shipment of MP3 players for \$42 each wholesale. If the store marks the MP3 players up by 45%, what is the selling price of an MP3 player?

Interest

Objective 4 Solve problems involving interest. **Interest** is a fee a borrower pays for the privilege of borrowing a sum of money. Banks also pay interest to customers who deposit money in their institutions. Simple interest is calculated as a percentage of the **principal**, which is the amount borrowed or deposited. If r is the annual interest rate, the simple interest (I) owed on a principal (P) is given by the formula $I = P \cdot r \cdot t$, where t is time in years.

If an investor deposited \$3000 in an account that paid 4% annual interest for one year, we could determine how much money would be in the account at the end of one year by substituting 3000 for P, 0.04 for r, and 1 for t in the formula $I = P \cdot r \cdot t$.

$$I = P \cdot r \cdot t$$
$$I = 3000(0.04)(1) \qquad \text{Substitute 3000 for } P\text{, 0.04 for } r\text{, and 1 for } t.$$
$$I = 120 \qquad \text{Multiply.}$$

The interest earned in one year is \$120. There is \$3000 plus the \$120 in interest, or \$3120, in the account after one year.

EXAMPLE 7 Martha invested some money in an account that paid 6% annual interest. Her friend Stuart found an account that paid 7% interest, and he invested \$5000 more in this account than Martha did in her account. If the pair earned a total of \$1650 in interest from the two accounts in one year, how much did Martha invest? How much did Stuart invest?

SOLUTION Letting x represent the amount Martha invested at 6%, use $x + 5000$ to represent the amount Stuart invested at 7%. The equation for this problem comes from the fact that the interest earned in each account must add up to \$1650. Use a table to display the important information for this problem.

Investor	Principal (P)	Rate (r)	Time (t)	Interest $I = P \cdot r \cdot t$
Martha	x	0.06	1	$0.06x$
Stuart	$x + 5000$	0.07	1	$0.07(x + 5000)$
Total				1650

The interest from the first account is $0.06x$, and the interest from the second account is $0.07(x + 5000)$; so their sum must be \$1650. Essentially, we can find this equation in the final column in the table.

$$\begin{aligned} 0.06x + 0.07(x + 5000) &= 1650 \\ 0.06x + 0.07x + 350 &= 1650 && \text{Distribute.} \\ 0.13x + 350 &= 1650 && \text{Combine like terms.} \\ 0.13x &= 1300 && \text{Subtract 350 from both sides.} \\ x &= 10{,}000 && \text{Divide both sides by 0.13.} \end{aligned}$$

Because $x = 10{,}000$, the amount Martha invested at 6% interest (x) is \$10,000 and the amount Stuart invested at 7% interest $(x + 5000)$ is \$15,000. You may verify that Martha earned \$600 in interest and Stuart earned \$1050 in interest, which totals \$1650.

Quick Check 7

Mark invested money in two accounts. One account paid 3% annual interest, and the other account paid 5% annual interest. He invested \$2000 more in the account that paid 5% interest than in the account that paid 3% interest. If Mark earned a total of \$500 in interest from the two accounts in one year, how much did he invest in each account?

EXAMPLE 8 Donald loaned a total of \$13,500 to two borrowers, Carolyn and George. Carolyn paid 8% annual interest, while George paid 12%. If Donald earned a total of \$1320 in interest from the two borrowers in one year, how much did Carolyn borrow? How much did George borrow?

SOLUTION Letting x represent the amount that Carolyn borrowed, use $13{,}500 - x$ to represent the amount George borrowed. (By subtracting the amount Carolyn borrowed from \$13,500, we are left with the amount George borrowed.) The equation for this problem comes from the fact that the interest earned by Donald must add up to \$1320. Again, use a table to display the important information for this problem.

Borrower	Principal (P)	Rate (r)	Time (t)	Interest $I = P \cdot r$
Carolyn	x	0.08	1	$0.08x$
George	$13{,}500 - x$	0.12	1	$0.12(13{,}500 - x)$
Total	13,500			1320

The interest paid by Carolyn is $0.08x$, and the interest paid by George is $0.12(13{,}500 - x)$; so their sum must be \$1320. Again, we can find this equation in the final column in the table.

$$\begin{aligned} 0.08x + 0.12(13{,}500 - x) &= 1320 \\ 0.08x + 1620 - 0.12x &= 1320 && \text{Distribute.} \\ -0.04x + 1620 &= 1320 && \text{Combine like terms.} \\ -0.04x &= -300 && \text{Subtract 1620 from both sides.} \\ x &= 7500 && \text{Divide both sides by } -0.04. \end{aligned}$$

Because $x = 7500$, the amount Carolyn borrowed is \$7500 and the amount George borrowed $(13{,}500 - x)$ is \$6000. You may verify that Carolyn paid \$600 in interest and George paid \$720 in interest, which totals \$1320.

Quick Check 8

Patsy invested a total of \$1900 in two accounts. One account paid 4% annual interest, and the other account paid 5% annual interest. If Patsy earned a total of \$86.50 in interest from the two accounts, how much did she invest in each account?

Mixture Problems

Objective 5 Solve mixture problems. The next example is a **mixture problem**. In this problem, we will be mixing two solutions that have different concentrations of alcohol to produce a mixture whose concentration of alcohol is somewhere between the two individual solutions. We will determine how much of each solution should be used to produce a mixture of the desired specifications.

EXAMPLE 9 A chemist has two solutions. The first is 20% alcohol, and the second is 30% alcohol. How many milliliters (mL) of each solution should she use if she wants to make 40 mL of a solution that is 24% alcohol?

SOLUTION Let x represent the volume of the 20% alcohol solution in milliliters. We need to express the volume of the 30% solution in terms of x as well. Because the two quantities must add up to 40 mL, the volume of the 30% solution can be represented by $40 - x$.

The equation that solves this problem is based on the volume of alcohol in each solution as well as the volume of alcohol in the mixture. When we add the volume of alcohol that comes from the 20% solution to the volume of alcohol that comes from the 30% solution, it should equal the volume of alcohol in the mixture.

Because we want to end up with 40 mL of a solution that is 24% alcohol, this solution should contain $0.24(40) = 9.6$ mL of alcohol. The following table, which is similar to the one used in the previous example involving interest, presents all of the information:

Solution	Volume of Solution (mL)	% Alcohol	Volume of Alcohol (mL)
Solution 1 (20%)	x	0.2	$0.2x$
Solution 2 (30%)	$40 - x$	0.3	$0.3(40 - x)$
Mixture	40	0.24	$0.24(40) = 9.6$

Alcohol from Solution 1	+	Alcohol from Solution 2	=	Alcohol in Mixture
$0.2x$	+	$0.3(40 - x)$	=	9.6

Now solve the equation for x.

$$
\begin{aligned}
0.2x + 0.3(40 - x) &= 9.6 \\
0.2x + 12 - 0.3x &= 9.6 && \text{Distribute.} \\
-0.1x + 12 &= 9.6 && \text{Combine like terms.} \\
-0.1x &= -2.4 && \text{Subtract 12 from both sides.} \\
x &= 24 && \text{Divide both sides by } -0.1.
\end{aligned}
$$

Because x represented the volume of the 20% alcohol solution that the chemist should use, she should use 24 mL of the 20% solution. Subtracting this amount from 40 mL tells us that she should use 16 mL of the 30% alcohol solution.

Quick Check 9

Marie had one solution that was 30% alcohol and a second solution that was 42% alcohol. How many milliliters of each solution should be mixed to make 80 milliliters of a solution that is 39% alcohol?

Ratio and Proportion

A **ratio** is a comparison of two quantities using division and is usually written as a fraction. Suppose a first-grade student has 20 minutes to eat her lunch, and this is followed by a 30-minute recess. The ratio of the time for lunch to the time for recess is $\frac{20}{30}$, which can be simplified to $\frac{2}{3}$. This means that for every 2 minutes of eating time, there are 3 minutes of recess time.

Objective 6 Solve for variables in proportions. Ratios are important tools for solving some applied problems involving proportions. A **proportion** is a statement of equality between two ratios. In other words, a proportion is an equation in which two ratios are equal to each other. Here are a few examples of proportions.

$$\frac{5}{8} = \frac{n}{56} \qquad \frac{8}{11} = \frac{5}{n} \qquad \frac{9}{2} = \frac{4x + 5}{x}$$

The **cross products** of a proportion are the two products obtained when we multiply the numerator of one fraction by the denominator of the other fraction. The two cross products for the proportion $\frac{a}{b} = \frac{c}{d}$ are $a \cdot d$ and $b \cdot c$.

When we solve a proportion, we are looking for the value of the variable that produces a true statement. We use the fact that the cross products of a proportion are equal if the proportion is indeed true.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } a \cdot d = b \cdot c.$$

Consider the two equal fractions $\frac{3}{6}$ and $\frac{5}{10}$. If we write these two fractions in the form of an equation $\frac{3}{6} = \frac{5}{10}$, both cross products $(3 \cdot 10$ and $6 \cdot 5)$ are equal to 30. This holds true for any proportion.

To solve a proportion, we multiply each numerator by the denominator on the other side of the equation and set the products equal to each other. We then solve the resulting equation for the variable in the problem. This process is called **cross multiplying** and will be demonstrated in the next example.

EXAMPLE 10 Solve $\frac{3}{4} = \frac{n}{68}$.

SOLUTION Begin by cross multiplying. Multiply the numerator on the left by the denominator on the right $(3 \cdot 68)$ and multiply the denominator on the left by the numerator on the right $(4 \cdot n)$.

Then write an equation that states that the two products are equal to each other and solve the equation.

$3 \cdot 68$ $\quad$ $4 \cdot n$

$$\frac{3}{4} = \frac{n}{68}$$

$$\begin{aligned} \frac{3}{4} &= \frac{n}{68} & \\ 3 \cdot 68 &= 4 \cdot n & \text{Cross multiply.} \\ 204 &= 4n & \text{Multiply.} \\ 51 &= n & \text{Divide both sides by 4.} \end{aligned}$$

The solution set is $\{51\}$.

Quick Check 10

Solve $\frac{3}{17} = \frac{n}{187}$.

A WORD OF CAUTION When solving a proportion, cross multiply. *Do not "cross-cancel."*

Applications of Proportions

Objective 7 Solve applied problems involving proportions. Now we turn our attention to applied problems involving proportions.

EXAMPLE 11 Studies show that 1 out of every 9 people is left-handed. In a sample of 216 people, how many would be left-handed according to these studies?

SOLUTION Begin by setting up a ratio based on the known information. Because 1 person out of every 9 is left-handed, the ratio is $\frac{1}{9}$. Notice that the numerator represents the number of left-handed people while the denominator represents the number of all people. When setting up a proportion, keep this ordering consistent. This given ratio will be the left side of the proportion. To set up the right side of the proportion, write a second ratio relating the unknown quantity to the given quantity. The unknown quantity is the number of left-handed people in the group of 216 people. Let n represent the number of left-handed people. The proportion then is $\frac{1}{9} = \frac{n}{216}$. Again, make sure the ratio on the left side $\left(\frac{\text{left-handed}}{\text{total}}\right)$ is consistent with the ratio on the right side. Thus,

$$\frac{1}{9} = \frac{n}{216}$$

$$216 = 9n \quad \text{Cross multiply.}$$

$$24 = n \quad \text{Divide both sides by 9.}$$

According to these studies, there should be 24 left-handed people in the group of 216 people.

Quick Check 11

Studies show that 4 out of 5 dentists recommend sugarless gum for their patients who chew gum. In a group of 85 dentists, according to these studies, how many would recommend sugarless gum to their patients?

Another use of proportions is for **dimensional analysis**: the process of converting from one unit to another. Here is a set of unit conversions that will allow us to convert from the English system of measurement to the metric system of measurement.

Conversions

Length

1 kilometer (km) $\approx$ 0.62 mile (mi)	1 mi $\approx$ 1.61 km
1 meter (m) $\approx$ 3.28 feet (ft)	1 ft $\approx$ 0.305 m
1 centimeter (cm) $\approx$ 0.39 inch (in.)	1 in. $\approx$ 2.54 cm

Volume

1 liter (L) $\approx$ 0.264 gallons (gal)	1 gal $\approx$ 3.785 L

Mass

1 kilogram (kg) $\approx$ 2.2 pounds (lb)	1 lb $\approx$ 0.454 kg
1 gram (g) $\approx$ 0.035 ounce (oz)	1 oz $\approx$ 28.35 g

EXAMPLE 12 Convert 80 meters to feet.

SOLUTION To convert from meters to feet, we have a choice of two conversion factors: 1 m $\approx$ 3.28 ft or 1 ft $\approx$ 0.305 m. (Note that depending on which conversion factor is used, answers may vary slightly.) Use 1 m $\approx$ 3.28 ft to set up a proportion, letting n represent the number of feet in 80 m.

$$\frac{1}{3.28} = \frac{80}{n} \quad \text{Set up the proportion with meters in the numerator and feet in the denominator.}$$

$$n = 262.4 \quad \text{Cross multiply.}$$

There are 262.4 ft in 80 m. (If we had used 1 ft $\approx$ 0.305 m as the conversion factor, the answer would have been 262.3 feet.)

Quick Check 12

Use the fact that 1 lb $\approx$ 0.454 kg to convert 175 pounds to kilograms.

BUILDING YOUR STUDY STRATEGY

Using Your Textbook, 4 Creating Note Cards Your textbook can be used to create a series of note cards for studying new terms and procedures or difficult problems. Each time you find a new term in the textbook, write the term on one side of a note card and the definition on the other side. Collect all of these cards and cycle through them, reading the term and trying to recite the definition by memory. You can do the same thing for each new procedure introduced in the textbook.

If you are struggling with a particular type of problem, find an example that has been worked out in the text. Write the problem on the front of the card and write the complete solution provided on the back of the card. As you cycle through the cards, try to solve the problem and then check the solution provided on the back of the card.

Some students have trouble recalling only the first step for solving particular problems. If you are in this group, you can write a series of problems on the front of some note cards and the first step on the back of the cards. You can then cycle through these cards in the same way.

Exercises 2.4

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Vocabulary

1. State the basic percent equation.
2. In a percent problem, the ______ is a percentage of the base.
3. Percent increase is a measure of how much a quantity has increased from its ____________.
4. Simple interest is calculated as a percentage of the ________.
5. State the formula for calculating simple interest.
6. A(n) ______ is a comparison of two quantities using division and is usually written as a fraction.
7. A(n) _________ is a statement of equality between two ratios.
8. If $\frac{a}{b} = \frac{c}{d}$, then $a \cdot d =$ ____.

9. Forty-five percent of 80 is what number?
10. What percent of 130 is 91?
11. Forty percent of what number is 92?
12. Fifty-seven is 6% of what number?
13. What percent of 256 is 224?
14. What number is 37% of 94?
15. What is $37\frac{1}{2}$% of 104?
16. Fifteen is what percent of 36?
17. What is 240% of 68?
18. What percent of 24 is 108?
19. Fifty-five is 125% of what number?
20. What is 600% of 53?
21. Thirty percent of the M&Ms in a bowl are brown. If there are 280 M&Ms in the bowl, how many are brown?
22. A doctor helped 320 women deliver their babies last year. Sixteen of these women had twins. What percent of the doctor's patients had twins?
23. Forty percent of the registered voters in a certain precinct are registered as Democrats. If 410 of the registered voters in the precinct are registered as Democrats, how many registered voters are in the precinct?
24. An online retailer adds a 15% charge to all items for shipping and handling. If a shipping-and-handling fee of $96 is added to the cost of a computer, what was the original price of the computer?
25. A certain brand of rum is 40% alcohol. How many milliliters of alcohol are there in a 750-milliliter bottle of rum?
26. A certificate of deposit (CD) pays 3.08% interest. Find the amount of interest earned in one year on a $5000 CD.
27. Marlana invested $3500 in the stock market. After one year, her portfolio had increased in value by 17%. What is the new value of her portfolio?
28. Kristy bought shares of a stock valued at $48.24. If the stock price dropped to $30.15, find the percent decrease in the value of the stock.

29. The price of a gallon of milk increased by 26 cents. This represented a price increase of 8%. What was the original price of the milk?

30. A bookstore charges its customers 32% over the wholesale cost of the book. How much does the bookstore charge for a book that has a wholesale price of $64?

31. A community college has 1600 parking spots on campus. When it builds its new library, the college will lose 120 of its parking spots. What will be the percent decrease in the number of available parking spots?

32. A community college had its budget slashed by $3,101,000. This represents a cut of 7% of its total budget from last year. What was last year's budget?

33. Last year the average SAT math score for students at a high school was 500.

a) This year the average math score decreased by 20%. Find the new average score.

b) By how many points must the average math score increase to get back to last year's average?

c) By what percent do scores need to increase next year to bring the average score back to 500?

34. During 2008, an auto plant produced 25% fewer cars than it had in 2007. Production fell by another 20% during 2009. By what percent will production have to increase in 2010 to make the same number of cars as the plant produced in 2007?

35. Tina invested $30,000 in a stock. In the first year, the stock increased in value by 10%. In the second year, the stock decreased in value by 20%. What percentage gain is required in the third year for Tina's stock to return to its original value? (Round to the nearest tenth of a percent.)

36. Khalid invested $100,000 in a mutual fund. In the first year, the mutual fund decreased in value by 25%. In the second year, the mutual fund increased in value by 20%. What percentage gain is required in the third year for Khalid's mutual fund to return to its original value? (Round to the nearest tenth of a percent.)

37. In 2007, Donald's stock portfolio decreased in value by 25%. In 2008, his stock portfolio decreased in value by another 40%. What percentage gain is required in 2009 for Donald's stock portfolio to return to the value it had at the beginning of 2007? (Round to the nearest tenth of a percent.)

38. In 2007, Giada's retirement fund decreased in value by 8%. In 2008, her retirement fund decreased in value by another 10%. What percentage gain is required in 2009 for Giada's retirement fund to return to the value it had at the beginning of 2007? (Round to the nearest tenth of a percent.)

39. Janet invested her savings in two accounts. One of the accounts paid 2% interest, and the other paid 5% interest. Janet put twice as much money in the account that paid 5% as she put in the account that paid 2%. If she earned a total of $84 in interest from the two accounts in one year, find the amount invested in each account.

40. One of Jaleel's mutual funds made a 10% profit last year, while the other mutual fund made an 8% profit. Jaleel had invested $1400 more in the fund that made an 8% profit than he did in the fund that made a 10% profit. If he made a $715 profit last year, how much was invested in each fund?

41. Kamiran deposits a total of $6500 in savings accounts at two different banks. The first bank pays 4% interest, while the second bank pays 5% interest. If he earns a total of $300 in interest in one year, how much was deposited at each bank?

42. Dianne was given $35,000 when she retired. She invested some at 7% interest and the rest at 9% interest. If she earned $2910 in interest in one year, how much was invested in each account?

43. Aurora invested $8000 in two stocks. One stock decreased in value by 8%; the other, by 20%. If she lost a total of $1000 on these two stocks, how much was invested in the stock that decreased in value by 8%?

44. Lebron invested $20,000 in two stocks. The first stock decreased in value by 3%, and the other decreased in value by 32%. If he lost a total of $5675 on these two stocks, how much was invested in the stock that decreased in value by 32%?

45. Marquis invested $4800 in two mutual funds. Last year one of the funds went up by 6% and the other fund went down by 5%. If his portfolio increased in value by $90 last year, how much did Marquis invest in the fund that earned a 6% profit?

46. Veronica received a $50,000 insurance settlement. She invested some of the money in a bank CD that paid 3.5% annual interest and invested the rest in a stock. In one year, the stock decreased in value by 30%. If Veronica lost $1600 from her $50,000 investment, how much did she put in the bank CD?

47. A mechanic has two antifreeze solutions. One is 70% antifreeze, and the other is 40% antifreeze. How much of each solution should be mixed to make 60 gallons of a solution that is 52% antifreeze?

48. A chemist has two saline solutions. One is 2% salt, and the other is 6% salt. How much of each solution should be mixed to make 2 liters of a saline solution that is 3% salt?

49. A dairyfarmer has some milk that is 5% butterfat as well as some lowfat milk that is 2% butterfat. How much of each needs to be mixed together to make 1000 gallons of milk that is 3.2% butterfat?

50. Patti has two solutions. One is 27% acid, and the other is 39% acid. How much of each solution should be mixed together to make 9 liters of a solution that is 29% acid?

51. A chemist has two solutions. One is 27% salt, and the other is 43% salt. How much of each solution should be mixed together to make 44 milliliters of a solution that is 39% salt?

52. A chemist has two solutions. One is 6% acid and the other is 9% acid. How much of each solution should be mixed together to make 72 milliliters of a solution that is 8.5% acid?

53. A chemist has two solutions. One is 40% alcohol, and the other is pure alcohol. How much of each solution should be mixed together to make 1.6 liters of a solution that is 52% alcohol?

54. A chemist has 60 milliliters of a solution that is 65% alcohol. She plans to add pure alcohol until the solution is 79% alcohol. How much pure alcohol should she add to this solution?

55. Paula has 400 milliliters of a solution that is 80% alcohol. She plans to dilute it so that it is only 50% alcohol. How much water must she add to do this?

56. A bartender has rum that is 40% alcohol. How much rum and how much cola need to be mixed together to make 5 liters of rum and cola that is 16% alcohol?

Solve the proportion.

57. $\frac{2}{3} = \frac{n}{48}$

58. $\frac{5}{8} = \frac{n}{32}$

59. $\frac{20}{n} = \frac{45}{81}$

60. $\frac{22}{30} = \frac{55}{n}$

61. $\frac{3}{4} = \frac{n}{109}$

62. $\frac{7}{10} = \frac{n}{86}$

63. $\frac{13.8}{n} = \frac{2}{9}$

64. $\frac{7.2}{n} = \frac{5}{8}$

65. $\frac{3}{4} = \frac{n + 12}{28}$

66. $\frac{5}{9} = \frac{3n - 2}{18}$

67. $\frac{2n + 15}{18} = \frac{5n + 13}{24}$

68. $\frac{n + 10}{40} = \frac{1 - n}{48}$

Set up a proportion and solve it for the following problems.

69. At a certain community college, 3 out of every 5 students are female. If the college has 9200 students, how many are female?

70. A day care center has a policy that there will be at least 4 staff members present for every 18 children. How many staff members are necessary to accommodate 63 children?

71. The directions for a powdered plant food state that 3 tablespoons of the food need to be mixed with 2 gallons of water. How many tablespoons need to be mixed with 15 gallons of water?

72. Eight out of every 9 people are right-handed. If a factory has 352 right-handed workers, how many left-handed workers are there at the factory?

73. If 5 out of every 6 teachers in a city meet the minimum qualifications to be teaching and the city has 864 teachers, how many do not meet the minimum qualifications?

74. A survey showed that 7 out of every 10 registered voters in a county are in favor of a bond measure to raise money for a new college campus. If there are 140,000 registered voters in the county, how many are not in favor of the bond measure?

Convert the given quantity to the desired unit. Round to the nearest tenth if necessary.

75. 60 cm to in.

76. 150 mi to km

77. 15.2 L to gal

78. 6 gal to L

79. 80 kg to lb

80. 35 oz to g

Writing in Mathematics

Answer in complete sentences.

81. Write a word problem for the following table and equation. Explain how you created the problem.

	Principal (P)	Rate (r)	Time (t)	Interest $I = P \cdot r \cdot t$
	x	0.03	1	$0.03x$
	$x + 10{,}000$	0.05	1	$0.05(x + 10{,}000)$
Total				2100

Equation: $0.03x + 0.05(x + 10{,}000) = 2100$

82. Write a word problem for the following table and equation. Explain how you created the problem.

Solution	Amount of Solution (ml)	% Alcohol	Amount of Alcohol (ml)
Solution 1	x	0.38	$0.38x$
Solution 2	$60 - x$	0.5	$0.5(60 - x)$
Mixture	60	0.4	24

Equation: $0.38x + 0.5(60 - x) = 24$

Quick Review Exercises

Section 2.4

Solve.

1. $4x + 17 = 41$
2. $5x - 19 = 3x - 51$
3. $2x + 2(x + 8) = 68$
4. $x + 3(x + 2) = 2(x + 4) + 30$

2.5 Linear Inequalities

OBJECTIVES

1. **Present the solutions of an inequality on a number line.**
2. **Present the solutions of an inequality using interval notation.**
3. **Solve linear inequalities.**
4. **Solve compound inequalities involving the union of two linear inequalities.**
5. **Solve compound inequalities involving the intersection of two linear inequalities.**
6. **Solve applied problems using linear inequalities.**

Suppose you went to a doctor and she told you that your temperature was normal. This would mean that your temperature was 98.6° F. However, if the doctor told you that you had a fever, could you tell what your temperature was? No, all you would know is that it was above 98.6° F. If you let the variable t represent your temperature, this relationship could be written as $t > 98.6$. The expression $t > 98.6$ is an example of an inequality. An inequality is a mathematical statement comparing two quantities using the symbols $<$ (*less than*) or $>$ (*greater than*). It states that one quantity is smaller than (or larger than) the other quantity.

Two other symbols that may be used in an inequality are $\leq$ and $\geq$. The symbol $\leq$ is read as *less than or equal to,* and the symbol $\geq$ is read as *greater than or equal to.* The inequality $x \leq 7$ is used to represent all real numbers that are less than 7 or are equal to 7. Inequalities involving the symbols $\leq$ or $\geq$ are often called **weak inequalities**, while inequalities involving the symbols $<$ or $>$ are called **strict inequalities** because they involve numbers that are strictly less than (or greater than) a given value.

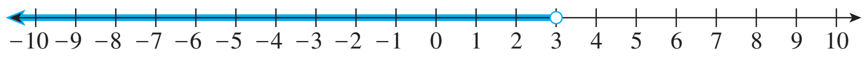
Presenting Solutions of Inequalities Using a Number Line

Objective 1 Present the solutions of an inequality on a number line.

Linear Inequality

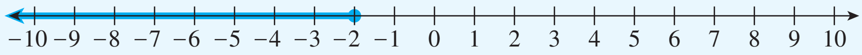
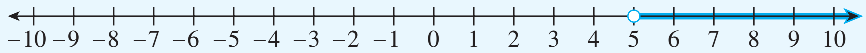
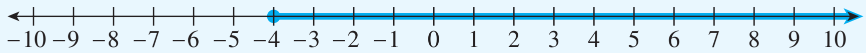
A **linear inequality** is an inequality containing linear expressions.

A linear inequality often has infinitely many solutions, and it is not possible to list every solution. Consider the inequality $x < 3$. There are infinitely many solutions of this inequality, but the one thing the solutions share is that they are all to the left of 3 on the number line. Values located to the right of 3 on the number line are greater than 3. All shaded numbers to the left are solutions of the inequality and can be represented on a number line as follows:



An **endpoint** of an inequality is a point on a number line that separates values that are solutions from values that are not. The open circle at $x = 3$ tells us that the endpoint is not included in the solution because 3 is not less than 3. If the inequality were $x \leq 3$, we would fill in the circle at $x = 3$.

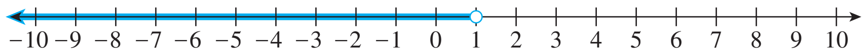
Here are three more inequalities with their solutions graphed on a number line:

Inequality	Solution
$x \leq -2$	
$x > 5$	
$x \geq -4$	

Saying that a is less than b is equivalent to saying that b is greater than a. In other words, the inequality $a < b$ is equivalent to the inequality $b > a$. This is an important piece of information, as we will want to rewrite an inequality so that the variable is on the left side of the inequality before we graph it on a number line.

EXAMPLE 1 Graph the solutions of the inequality $1 > x$ on a number line.

SOLUTION To avoid confusion about which direction should be shaded on the number line, before graphing the solutions, rewrite the inequality so that the variable is on the left side. Saying that 1 is greater than x is the same as saying that x is less than 1, or $x < 1$. The values that are less than 1 are to the left of 1 on the number line.



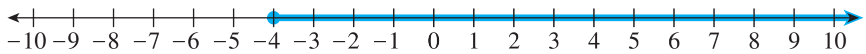
Quick Check 1

Graph the solutions of the inequality $8 < x$ on a number line.

Interval Notation

Objective 2 Present the solutions of an inequality using interval notation. Another way to present the solutions of an inequality is by using **interval notation**. A range of values on a number line, such as the solutions of the inequality $x \geq 4$, is called an **interval**. Inequalities typically have one or more intervals as their solutions. Interval notation presents an interval that is a solution by listing its left and right endpoints. We use parentheses around the endpoints if the endpoints are not included as solutions, and we use brackets if the endpoints are included as solutions.

When an interval continues on indefinitely to the right on a number line, we will use the symbol ∞ (infinity) in place of the right endpoint and follow it with a parenthesis. Let's look again at the number line associated with the solutions of the inequality $x \geq -4$.



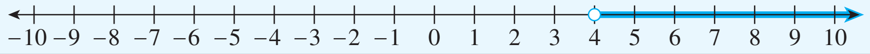
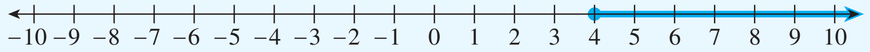
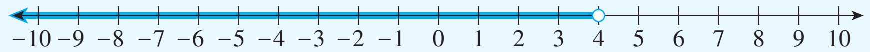
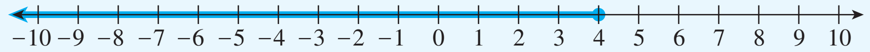
The solutions begin at -4 and include any number that is greater than -4. The interval is bounded by -4 on the left side and extends without bound on the right side, which can be written in interval notation as $[-4, \infty)$. The endpoint -4

is included in the interval, so we write a square bracket in front of it because it is a solution. We always write a parenthesis after ∞, because it is not an actual number that ends the interval at that point.

If the inequality had been $x > -4$ instead of $x \geq -4$, we would have written a parenthesis rather than a square bracket before -4. In other words, the solutions of the inequality $x > -4$ can be expressed in interval notation as $(-4, \infty)$.

For intervals that continue indefinitely to the left on a number line, we use $-\infty$ in place of the left endpoint.

Here is a summary of different inequalities with the solutions presented on a number line and in interval notation.

Inequality	Number Line	Interval Notation
$x > 4$		$(4, \infty)$
$x \geq 4$		$[4, \infty)$
$x < 4$		$(-\infty, 4)$
$x \leq 4$		$(-\infty, 4]$

Solving Linear Inequalities

Objective 3 Solve linear inequalities. Solving a linear inequality such as $7x - 11 \leq -32$ is similar to solving a linear equation. In fact, there is only one difference between the two procedures:

> Whenever you multiply both sides of an inequality by a negative number or divide both sides by a negative number, the direction of the inequality changes.

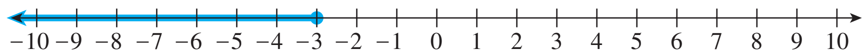
Why is this? Consider the inequality $3 < 5$, which is a true statement. If we multiply each side by -2, is the left side $(-2 \cdot 3 = -6)$ still less than the right side $(-2 \cdot 5 = -10)$? No, $-6 > -10$ because -6 is located to the right of -10 on the number line. Changing the direction of the inequality after multiplying (or dividing) by a negative number produces an inequality that is still a true statement.

EXAMPLE 2 Solve the inequality $7x - 11 \leq -32$. Present your solutions on a number line and in interval notation.

SOLUTION Think about the steps you would take to solve the equation $7x - 11 = -32$. You follow the same steps when solving this inequality.

$$7x - 11 \leq -32$$
$$7x \leq -21 \quad \text{Add 11 to both sides.}$$
$$x \leq -3 \quad \text{Divide both sides by 7.}$$

Now present the solution on a number line.



This can be expressed in interval notation as $(-\infty, -3]$.

Quick Check 2

Solve the inequality $4x + 3 < 31$. Present your solutions on a number line and in interval notation.

EXAMPLE 3 Solve the inequality $-2x - 5 > 9$. Present your solutions on a number line and in interval notation.

SOLUTION Begin by solving the inequality.

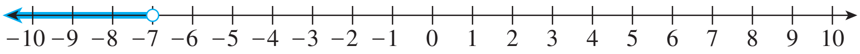
$$-2x - 5 > 9$$

$$-2x > 14 \quad \text{Add 5 to both sides.}$$

$$\frac{-2x}{-2} < \frac{14}{-2} \quad \text{Divide both sides by } -2 \text{ to isolate } x\text{. Notice that the direction of the inequality must change because you are dividing by a negative number.}$$

$$x < -7 \quad \text{Simplify.}$$

Now present the solutions.



In interval notation, this is $(-\infty, -7)$.

Quick Check 3

Solve the inequality $7 - 4x \le -13$. Present your solutions on a number line and in interval notation.

EXAMPLE 4 Solve the inequality $5x - 12 > 8 - 3x$. Present your solutions on a number line and in interval notation.

SOLUTION Collect all variable terms on one side of the inequality and all constant terms on the other side.

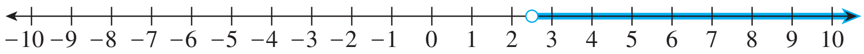
$$5x - 12 > 8 - 3x$$

$$8x - 12 > 8 \quad \text{Add } 3x \text{ to both sides.}$$

$$8x > 20 \quad \text{Add 12 to both sides.}$$

$$x > \frac{5}{2} \quad \text{Divide both sides by 8 and simplify.}$$

Here is the solution on a number line.



In interval notation, this is written as $\left(\frac{5}{2}, \infty\right)$.

Quick Check 4

Solve the inequality $3x + 8 < x + 2$. Present your solutions on a number line and in interval notation.

EXAMPLE 5 Solve the inequality $3(7 - 2x) + 6x \le 5x - 4$. Present your solutions on a number line and in interval notation.

SOLUTION As with equations, begin by simplifying each side completely.

$$3(7 - 2x) + 6x \le 5x - 4$$

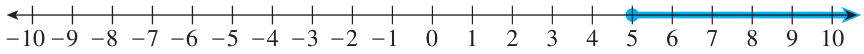
$$21 - 6x + 6x \le 5x - 4 \quad \text{Distribute.}$$

$$21 \le 5x - 4 \quad \text{Combine like terms.}$$

$$25 \le 5x \quad \text{Add 4 to both sides.}$$

$$5 \le x \quad \text{Divide both sides by 5.}$$

We can rewrite this solution as $x \geq 5$, which will help when you display the solutions on a number line.



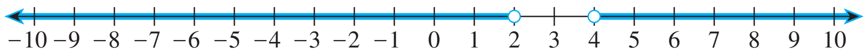
This can be expressed in interval notation as $[5, \infty)$.

Quick Check 5

Solve the inequality $(11x - 3) - (2x - 13) \geq 4(2x + 1)$. Present your solutions on a number line and in interval notation.

Compound Inequalities

Objective 4 Solve compound inequalities involving the union of two linear inequalities. A **compound inequality** is made up of two or more individual inequalities. One type of compound inequality involves the word *or*. In this type of inequality, we are looking for real numbers that are solutions of one inequality or the other. For example, the solutions of the compound inequality $x < 2$ or $x > 4$ are real numbers that are less than 2 or greater than 4. The solutions of this compound inequality can be displayed on a number line as follows:



To express the solutions using interval notation, we write both intervals with the symbol for **union** $(\cup)$ between them. The interval notation for $x < 2$ or $x > 4$ is $(-\infty, 2)\cup(4, \infty)$.

To solve a compound inequality involving *or*, we simply solve each inequality separately.

EXAMPLE 6 Solve $9x + 20 < -34$ or $5 + 4x \geq 19$.

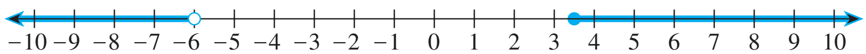
SOLUTION Solve each inequality separately. Begin with $9x + 20 < -34$.

$$9x + 20 < -34$$
$$9x < -54 \quad \text{Subtract 20 from both sides.}$$
$$x < -6 \quad \text{Divide both sides by 9.}$$

Now solve the second inequality, $5 + 4x \geq 19$.

$$5 + 4x \geq 19$$
$$4x \geq 14 \quad \text{Subtract 5 from both sides.}$$
$$x \geq \frac{7}{2} \quad \text{Divide both sides by 4 and simplify.}$$

Combine the solutions on a single number line.



The solutions can be expressed in interval notation as $(-\infty, -6)\cup\left[\frac{7}{2}, \infty\right)$.

Quick Check 6

Solve $3x - 2 \leq 10$ or $4x - 13 \geq 15$.

Objective 5 Solve compound inequalities involving the intersection of two linear inequalities. Another type of compound inequality involves the word *and*, and the solutions of this compound inequality must be solutions of each inequality. For example, the solutions of the inequality $x > -5$ and $x < 1$ are real numbers that are, at the same time, greater than -5 and less than 1. This type of inequality is generally written in the condensed form $-5 < x < 1$. You can think of the solutions as being between -5 and 1.

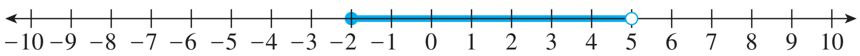
To solve a compound inequality of this type, we need to isolate the variable in the middle part of the inequality between two real numbers. A key aspect of the approach is to work on all three parts of the inequality at once.

EXAMPLE 7 Solve $-5 \le 2x - 1 < 9$.

SOLUTION We are trying to isolate x in the middle of this inequality. The first step is to add 1 to all three parts of this inequality, after which we will divide by 2.

$$-5 \le 2x - 1 < 9$$
$$-4 \le 2x < 10 \quad \text{Add 1 to each part of the inequality.}$$
$$-2 \le x < 5 \quad \text{Divide each part of the inequality by 2.}$$

The solutions are presented on the following number line:



These solutions can be expressed in interval notation as $[-2, 5)$.

Quick Check 7

Solve $-11 < 4x + 1 < 7$.

Applications

Objective 6 Solve applied problems using linear inequalities. Many applied problems involve inequalities rather than equations. Here are some key phrases and their translations into inequalities.

Key Phrases for Inequalities

x is greater than a x is more than a x is higher than a x is above a	$x > a$
x is at least a x is a or higher	$x \ge a$
x is less than a x is lower than a x is below a	$x < a$
x is at most a x is a or lower	$x \le a$
x is between a and b, exclusive x is more than a but less than b	$a < x < b$
x is between a and b, inclusive x is at least a but no more than b	$a \le x \le b$

EXAMPLE 8 A sign next to an amusement park ride says that you must be at least 48″ tall to get on the ride. Set up an inequality that shows the heights of people who can get on the ride.

SOLUTION Let h represent the height of a person. If a person must be at least 48″ tall, his or her height must be 48″ or above. In other words, the person's height must be greater than or equal to 48″. The inequality is $h \geq 48$.

A WORD OF CAUTION The phrase *at least* means "greater than or equal to" ($\geq$) and does not mean "less than."

EXAMPLE 9 An instructor tells her students that on the final exam, they need a score of 70 or higher out of a possible 100 to pass. Set up an inequality that shows the scores that are not passing.

SOLUTION Let s represent a student's score. Because a score of 70 or higher will pass, any score lower than 70 will not pass. This inequality can be written as $s < 70$. If we happen to know that the lowest possible score is 0, we also could write the inequality as $0 \leq s < 70$.

Quick Check 8

The Brainiac Club has a bylaw stating that a person must have an IQ of at least 130 to be admitted to the club. Set up an inequality that shows the IQs of people who are unable to join the club.

Now we will set up and solve applied problems involving inequalities.

EXAMPLE 10 If a student averages 90 or higher on the five tests given in a math class, the student will earn a grade of A. Sean's scores on the first four tests are 82, 87, 93, and 92. What score on the fifth test will give Sean an A?

SOLUTION To find the average of five test scores, add the five scores and divide by 5. Let x represent the score of the fifth test. The average can then be expressed as $\frac{82 + 87 + 93 + 92 + x}{5}$. We are interested in which scores on the fifth test give an average that is 90 or higher.

$$\frac{82 + 87 + 93 + 92 + x}{5} \geq 90$$

$$\frac{354 + x}{5} \geq 90 \quad \text{Simplify the numerator.}$$

$$5 \cdot \frac{354 + x}{5} \geq 5 \cdot 90 \quad \text{Multiply both sides by 5 to clear the fraction.}$$

$$354 + x \geq 450 \quad \text{Simplify.}$$

$$x \geq 96 \quad \text{Subtract 354 from both sides.}$$

Sean must score at least 96 on the fifth test to earn an A.

Quick Check 9

If a student averages lower than 70 on the four tests given in a math class, the student will fail the class. Bobby's scores on the first three tests are 74, 78, and 80. What scores on the fourth test will result in Bobby failing the class?

BUILDING YOUR STUDY STRATEGY

Using Your Textbook, 5 **A Word of Caution/Using Your Calculator** Two features of this textbook you may find helpful are labeled "A Word of Caution" and "Using Your Calculator." Each Word of Caution box warns you about common errors for certain problems and tells you how to avoid repeating the same mistake. Look through the section for this feature before attempting the homework exercises.

Using Your Calculator shows how you can use the Texas Instruments TI-84 calculator to help with selected examples in the text. If you own one of these calculators, look for this feature before beginning your homework.

Exercises 2.5

PRACTICE WATCH DOWNLOAD READ REVIEW

Vocabulary

1. A(n) ______________ is an inequality containing linear expressions.
2. When graphing an inequality, use a(n) __________ circle to indicate an endpoint that is not included as a solution.
3. You can express the solutions of an inequality using a number line or __________ notation.
4. When solving a linear inequality, you change the direction of the inequality whenever you multiply or divide both sides of the inequality by a(n) __________ number.
5. An inequality that is composed of two or more individual inequalities is called a(n) ___________ inequality.
6. Which inequality can be associated with the statement *The person's height is at least 80 inches*?

   a) $x \le 80$ **b)** $x < 80$ **c)** $x \ge 80$ **d)** $x > 80$

Graph each inequality on a number line and present it in interval notation.

7. $x < 3$

8. $x > -6$

9. $x \ge -1$

10. $x \le 13$

11. $-2 < x < 8$

12. $3 < x \le 12$

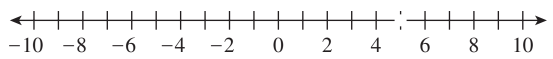
13. $x > \dfrac{9}{2}$

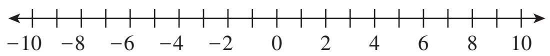
14. $x > 5.5$

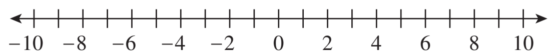
15. $x > 8$ or $x \le 2$

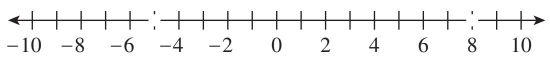
16. $x < -5$ or $x > 0$

Write an inequality associated with the given graph or interval notation.

17. 

18. 

19. 

20. 

21. $(-\infty, 2) \cup (9, \infty)$

22. $(-3, -2)$

23. $(-\infty, -4)$

24. $[-8, \infty)$

Solve. Present your solution on a number line and in interval notation.

25. $x + 9 < 5$

26. $x - 7 > 3$

27. $2x > 12$

28. $4x \geq -20$

29. $-5x > 15$

30. $-8x < -24$

31. $3x \geq -10.5$

32. $2x \leq 9.4$

33. $3x + 2 < 14$

34. $2x - 7 > 11$

35. $-2x + 9 \leq 29$

36. $7 - 4x > 19$

37. $8 < 3x - 13$

38. $1 \geq 5x + 16$

39. $\frac{2}{3}x > \frac{10}{9}$

40. $\frac{5}{9}x - \frac{4}{3} < 7$

41. $5x + 13 > 2x - 11$

42. $-2x + 51 < 9 - 8x$

43. $5(x + 3) + 2 > 3(x + 4) - 6$

44. $5(2x - 3) - 3 < 2(2x + 9) - 4$

45. $2x < -8$ or $x + 7 > 5$

46. $x - 3 \leq -5$ or $6x > -6$

47. $2x - 7 \leq -3$ or $2x - 7 \geq 3$

48. $3x - 6 < -9$ or $3x - 6 > 9$

49. $-2x + 17 < 5$ or $10 - x > 7$

50. $-4x + 13 \leq 9$ or $6 - 5x \geq 21$

51. $3x + 5 < 17$ or $2x - 9 < 5$

52. $7x - 1 \geq 13$ or $4x + 5 \geq -7$

53. $\frac{1}{4}x + \frac{7}{24} \leq \frac{2}{3}$ or $\frac{1}{4}x + \frac{9}{20} \geq \frac{6}{5}$

54. $\frac{1}{8}x - \frac{1}{5} < \frac{1}{12}x - \frac{3}{40}$ or $\frac{1}{2}x - \frac{3}{20} > \frac{1}{4}x + \frac{11}{10}$

55. $-4 < x - 3 < 1$

56. $2 \leq x + 7 \leq 5$

57. $-2 < 5x - 7 < 28$

58. $18 \leq 4x - 10 \leq 50$

59. $6 < 3x + 15 \leq 33$

60. $-12 \leq 4x - 18 < 7$

61. $\frac{1}{5} \leq \frac{1}{2}x - \frac{1}{3} \leq \frac{7}{4}$

62. $-\frac{3}{10} < \frac{1}{5}x - \frac{1}{2} < \frac{3}{5}$

63. In your own words, explain why the inequality $8 < 9x + 7 < 3$ has no solutions.

64. In your own words, explain why solving the compound inequality $x + 7 < 9$ or $x + 7 < 13$ is the same as solving the inequality $x + 7 < 13$ only.

65. Adrian needs to score at least 2 goals in the final game of the season to set a new scoring record. Write an inequality that shows the number of goals that will set a new record.

66. If Eddie sells fewer than 7 cars this week, he will be fired. Write an inequality that shows the number of cars that will result in Eddie being unemployed.

67. A certain type of shrub is tolerant to 30° F, which means that it can survive at temperatures down to 30° F. Write an inequality that shows the temperatures at which the shrub cannot survive.

68. To be eligible for growth funding from the state, a community college must have 11,200 or more students this semester. Write an inequality that shows the number of students that makes the college eligible for growth funding.

For 69–76, write an inequality for the given situation and solve it.

69. John is paid to get signatures on petitions. He gets paid 10 cents for each signature. How many signatures does he need to gather today to earn at least $30?

70. Carlo attends a charity wine-tasting festival. There is a $10 admission fee. In addition, there is a $4 charge for each variety tasted. If Carlo brings $40 with him, how many varieties can he taste?

71. Angelica is going out of town on business. Her company will reimburse her up to $85 for a rental car. She rents a car for $19.95 plus $0.07 per mile. How many miles can Angelica drive without going over the amount her company will reimburse?

72. An elementary school's booster club is holding a fundraiser by selling cookie dough. Each tub of cookie dough sells for $11, and the booster club gets to keep half of the proceeds. If the booster club wants to raise at least $12,000, how many tubs of cookie dough does it need to sell?

73. Charles told his son Harry that he should give at least 15% of his earnings to charity. If Harry earned $37,500 last year, how much should he have given to charity?

74. Karina is looking for a formal dress to wear to a charity event, and she has $400 to spend. If the store charges 7% sales tax on each dress, what price range should Karina be considering?

75. Students in a real estate class must have an average score of at least 80 on their exams in order to pass. If Jacqui has scored 92, 93, 85, and 96 on the first four exams, what scores on the fifth exam would allow her to pass the course?

76. Students with an average test score below 70 after the third test will be sent an Early Alert warning. Robert scored 62 and 59 on the first two tests. What scores on the third exam will save Robert from receiving an Early Alert warning?

Writing in Mathematics

Answer in complete sentences.

77. In your own words, explain why you must change the direction of an inequality when dividing both sides of that inequality by a negative number.

78. ***Solutions Manual*** Write a solutions manual page for the following problem:

Solve. $-10 < 3x + 2 \leq 29$

CHAPTER 2 SUMMARY

Section 2.1 Introduction to Linear Equations

Linear Equations, pp. 63–64 An equation is a mathematical statement of equality between two expressions. A linear equation has a single variable, and that variable does not have an exponent that is greater than 1.	$3x = 18$
Solution of an Equation, p. 64 A value that when substituted for the variable in the equation produces a true statement	Is $x = -3$ a solution of $4x - 8 = 3x - 11$? $4(-3) - 8 = 3(-3) - 11$ $-12 - 8 = -9 - 11$ $-20 = -20$ $x = -3$ is a solution.
Multiplication Property of Equality, pp. 64–67 For any algebraic expressions A and B and any nonzero number n, if $A = B$, then $n \cdot A = n \cdot B$.	Solve $-5x = 40$. $-5x = 40$ $\frac{-5x}{-5} = \frac{40}{-5}$ $x = -8$ $\{-8\}$
Addition Property of Equality, pp. 67–68 For any algebraic expressions A and B and any number n, if $A = B$, then $A + n = B + n$ and $A - n = B - n$.	Solve $x - 6 = -13$. $x - 6 = -13$ $x - 6 + 6 = -13 + 6$ $x = -7$ $\{-7\}$

Section 2.2 Solving Linear Equations: A General Strategy

Solving Linear Equations, pp. 71–75 **1.** Simplify each side of the equation completely. **2.** Collect all variable terms on one side of the equation. **3.** Collect all constant terms on the other side of the equation. **4.** Divide both sides of the equation by the coefficient of the variable term. **5.** Check your solution.	Solve $2(4x + 5) - 5x = x + 6$. $2(4x + 5) - 5x = x + 6$ $8x + 10 - 5x = x + 6$ $3x + 10 = x + 6$ $2x + 10 = 6$ $2x = -4$ $x = -2$ $\{-2\}$
Contradictions, p. 75 A contradiction is an equation that has no solution, so its solution set is the empty set { }. The empty set also is known as the null set and is denoted by the symbol ∅.	Solve $2(x + 3) + 7 = 2x + 1$. $2(x + 3) + 7 = 2x + 1$ $2x + 6 + 7 = 2x + 1$ $2x + 13 = 2x + 1$ $13 = 1$ $\varnothing$
Identities, pp. 75–76 An identity is an equation that is always true. The solution set for an identity is the set of all real numbers, denoted ℝ. An identity has infinitely many solutions.	Solve $3x + 8 - 9 = 2x + 7 + x - 8$. $3x + 8 - 9 = 2x + 7 + x - 8$ $3x - 1 = 3x - 1$ $-1 = -1$ $\mathbb{R}$
Literal Equations, pp. 76–77 A literal equation is an equation that contains two or more variables. To solve a literal equation for a particular variable, write it in terms of the other variable(s).	Solve $3x + 2y = 9$ for y. $3x + 2y = 9$ $2y = -3x + 9$ $y = \frac{-3x + 9}{2}$

Section 2.3 Problem Solving; Applications of Linear Equations

Solving Applied Problems, pp. 80–87

1. Read the problem.
2. List all of the important information.
3. Assign a variable to the unknown quantity.
4. Find an equation relating the known values to the variable expressions representing the unknown quantities.
5. Solve the equation.
6. Present the solution.

Seven less than 3 times a number is 59. Find the number.

Unknown Number: x

$$\begin{aligned} 3x - 7 &= 59 \\ 3x &= 66 \\ x &= 22 \end{aligned}$$

The number is 22.

Geometric Formulas and Definitions, pp. 81–83

- Perimeter of a triangle with sides s_1, s_2, and s_3: $P = s_1 + s_2 + s_3$
- Perimeter of a square with side s: $P = 4s$
- Perimeter of a rectangle with length L and width W: $P = 2L + 2W$
- Circumference of a circle with radius r: $C = 2\pi r$

The length of a rectangle is 5 inches more than twice its width. If the perimeter is 46 inches, find the dimensions of the rectangle.

Unknowns Length: $2x + 5$; width: x

$$\begin{aligned} 2(2x + 5) + 2x &= 46 \\ 4x + 10 + 2x &= 46 \\ 6x + 10 &= 46 \\ 6x &= 36 \\ x &= 6 \end{aligned}$$

Length: $2x + 5 = 2(6) + 5 = 12 + 5 = 17$

Width: $x = 6$

The length is 17 inches, and the width is 6 inches.

Distance, pp. 84–85

When an object such as a car moves at a constant rate of speed (r) for an amount of time (t) the distance traveled (d) is given by the formula $d = r \cdot t$.

If a person drives at a constant speed of 65 mph, how long will it take to drive 338 miles?

Unknown Time driving: t

$$\begin{aligned} 338 &= 65 \cdot t \\ \frac{338}{65} &= t \\ 5.2 &= t \end{aligned}$$

It will take 5.2 hours.

Section 2.4 Applications Involving Percents; Ratio and Proportion

Basic Percent Equation, pp. 90–92

The amount is a percentage of the base.

$$\text{Amount} = \text{Percent} \cdot \text{Base}$$

The percent must be expressed as a decimal or a fraction rather than as a percent.

What percent of 68 is 17?

$$\underline{17} \text{ is } \underline{n}\ \% \text{ of } \underline{68}.$$

$$\begin{aligned} 17 &= n \cdot 68 \\ \frac{17}{68} &= n \\ 0.25 &= n \end{aligned}$$

17 is 25% of 68.

Percent Increase/Decrease, pp. 92–93

Percent increase is a measure of how much a quantity has increased from its original value. Percent decrease is a similar measure of how much a quantity has decreased from its original value. The amount is the amount of increase/decrease, and the base is the original value.

A painting was purchased for \$540 and sold a year later for \$648. Find the percent increase in the value of the painting.

Amount of increase: \$108; base: \$540

$$\underline{108} \text{ is } \underline{n}\ \% \text{ of } \underline{540}.$$

$$\begin{aligned} 108 &= n \cdot 540 \\ \frac{108}{540} &= n \\ 0.2 &= n \end{aligned}$$

The painting's value increased by 20%.

Interest, pp. 93–94

If r is the interest rate, the simple interest (I) owed on a principal (P) is given by the formula $I = P \cdot r \cdot t$, where t is time in years.

Mark invested a total of \$10,000 in two accounts. One account paid 4% annual interest, and the other paid 5% annual interest. In the first year, Mark earned \$460 in interest. How much did he invest in each account?

Acct.	P	r	t	$I = P \cdot r \cdot t$
First	x	0.04	1	$0.04x$
Second	$10{,}000 - x$	0.05	1	$0.05(10{,}000 - x)$
Total	\$10,000			\$460

$$\begin{aligned} 0.04x + 0.05(10{,}000 - x) &= 460 \\ 0.04x + 500 - 0.05x &= 460 \\ -0.01x + 500 &= 460 \\ -0.01x &= -40 \\ x &= 4000 \end{aligned}$$

Mark invested \$4000 at 4% and \$6000 at 5%.

Ratio, p. 95

A ratio is a comparison of two quantities using division and is usually written as a fraction.

$$\frac{63 \text{ miles}}{2 \text{ hours}}$$

Proportions, p. 95

A proportion is a statement of equality between two ratios.

$$\frac{n}{40} = \frac{17}{24}$$

Cross Products of a Proportion, pp. 96–97

If $\frac{a}{b} = \frac{c}{d}$, then $a \cdot d = b \cdot c$.

Solve $\frac{n}{12} = \frac{48}{64}$.

$$\begin{aligned} 64n &= 12 \cdot 48 \\ 64n &= 576 \\ n &= \frac{576}{64} \\ n &= 9 \end{aligned}$$

Section 2.5 Linear Inequalities

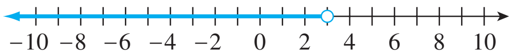
Linear Inequalities, p. 101

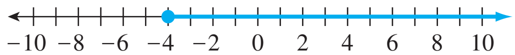
A linear inequality is an inequality containing linear expressions.

$$2x - 3 \geq 9$$

Solutions: Number Line, pp. 101–102

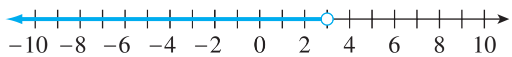
The solutions to a linear inequality can be represented on a number line.
If the endpoint is a solution, you use a closed circle on the number line.
If the endpoint is not a solution, you use an open circle.

$x < 3$:


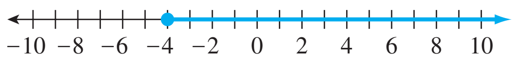
$x \geq -4$:


Solutions: Interval Notation, pp. 102–103

Interval notation presents an interval by listing its left and right endpoints. Parentheses are used around endpoints not included in the interval, and square brackets are used when the endpoints are included in the interval.



; $(-\infty, 3)$



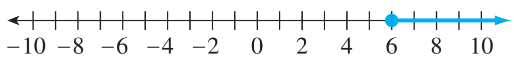
; $[-4, \infty)$

Solving Linear Inequalities, pp. 103–105

Solving a linear inequality is similar to solving a linear equation except that multiplying or dividing both sides of an inequality by a negative number changes the direction of the inequality.

Solve $-3x + 2 \leq -16$.

$$\begin{aligned} -3x + 2 &\leq -16 \\ -3x &\leq -18 \\ \frac{-3x}{-3} &\geq \frac{-18}{-3} \\ x &\geq 6 \end{aligned}$$



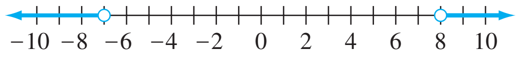
, $[6, \infty)$

Compound Inequalities, pp. 105–106

A compound inequality is made up of two or more individual inequalities.

Solve $x + 3 < -4$ or $x - 2 > 6$.

$$x + 3 < -4 \qquad x - 2 > 6$$
$$x < -7 \qquad x > 8$$

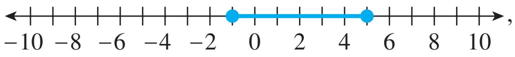


;

$(-\infty, -7) \cup (8, \infty)$

Solve $-5 \le 2x - 3 \le 7$.

$$-5 \le 2x - 3 \le 7$$
$$-2 \le 2x \le 10$$
$$-1 \le x \le 5$$



, $[-1, 5]$

SUMMARY OF CHAPTER 2 STUDY STRATEGIES

Using your textbook as more than a source of the homework exercises and answers to the odd problems will help you learn mathematics.

- Read the section the night before your instructor covers it to familiarize yourself with the material.
- Reread the section after it is covered in class. After you read through an example, attempt the Quick Check exercise that follows it.
- Use the textbook to create a series of note cards for each section.
- In each section, look for the feature labeled “A Word of Caution.” This feature points out common mistakes and offers advice to help you avoid making the same mistakes.
- Finally, the feature labeled “Using Your Calculator” will show you how to use the TI-84 calculator to help you solve the problems in that section.

CHAPTER 2 REVIEW

Determine whether the given value is a solution of the equation. **[2.1]**

1. $x = 7, 4x + 9 = 19$

2. $a = -9, (4a - 15) - (2a + 7) = -2(11 - a)$

Solve using the multiplication property of equality. **[2.1]**

3. $2x = 32$

4. $-9x = 54$

Solve using the addition property of equality. **[2.1]**

5. $x - 3 = -9$

6. $a + 14 = 8$

For each of the following problems, set up a linear equation and solve it. **[2.1]**

7. A local club charged $8 per person to see a "Battle of the Bands." After paying the winning band a prize of $250, the club made a profit of $694. How many people came to see the "Battle of the Bands?"

8. Thanks to proper diet and exercise, Randy's cholesterol dropped 23 points in the last two months. If his new cholesterol level is 189 points, what was his cholesterol level two months ago?

Solve. **[2.2]**

9. $5x - 8 = 27$

10. $-11 = 3a + 16$

11. $-4x + 15 = 7$

12. $51 - 2x = -21$

13. $\frac{x}{6} - \frac{5}{12} = \frac{5}{4}$

14. $\frac{1}{5}x - \frac{1}{2} = \frac{5}{4}$

15. $4x - 19 = 5x + 14$

16. $2m + 23 = 17 - 8m$

17. $2n - 11 + 3n + 4 = 7n - 39$

18. $3(2x - 7) - (x - 9) = 2x - 33$

Solve the following literal equations for the specified variable. **[2.2]**

19. $7x + y = 8$ for y

20. $15x + 7y = 30$ for y

21. $C = \pi \cdot d$ for d

22. $P = 3a + b + 2c$ for a

23. One number is 26 more than another number. If the sum of the two numbers is 98, find the two numbers. [2.3]

24. The width of a rectangle is 7 feet less than its length, and the perimeter is 50 feet. Find the length and the width of the rectangle. [2.3]

25. The sum of three consecutive even integers is 204. Find the three even integers. [2.3]

26. If Jason drives at a speed of 72 miles per hour, how long will it take him to drive 342 miles? [2.3]

27. Alycia's piggy bank has dimes and quarters in it. The number of dimes is 4 less than 3 times the number of quarters. If the total value of the coins is $7.85, how many dimes are in the piggy bank? [2.3]

28. Twenty percent of 95 is what number? [2.4]

29. What percent of 136 is 51? [2.4]

30. Thirty-two percent of what number is 1360? [2.4]

31. Fifty-five percent of the students at a college are female. If the college has 10,520 students, how many are female? [2.4]

32. In a group of 320 college graduates, 204 decided to pursue a master's degree. What percent of these students decided to pursue a master's degree? [2.4]

33. Jerry's bank account earned $553.50 interest last year. This account pays 4.5% interest. How much money did Jerry have in the account at the start of the year? [2.4]

34. A store is discounting all of its prices by 20%. If a sweater's original price was $59.95, what is the price after the discount? [2.4]

35. Karl deposits a total of $2000 in savings accounts at two banks. The first bank pays 6% interest, while the second bank pays 5% interest. If he earns a total of $117 interest in the first year, how much did he deposit at each bank? [2.4]

36. Merle has two solutions. One is 17% alcohol, and the other is 42% alcohol. How much of each solution should be mixed to make 80 milliliters of a solution that is 32% alcohol? [2.4]

Solve the proportion. **[2.4]**

37. $\frac{n}{6} = \frac{18}{27}$

38. $\frac{4}{9} = \frac{n}{72}$

39. $\frac{32}{13} = \frac{192}{n}$

40. $\frac{38}{n} = \frac{57}{75}$

Set up a proportion and solve it to solve the following problems. **[2.4]**

41. At a community college, 3 out of every 7 students plan to transfer to a four-year university. If the college has 14,952 students, how many plan to transfer to a four-year university?

Worked-out solutions to Review Exercises marked with ● can be found on page AN–4.

42. Six tablespoons of a concentrated weed killer must be mixed with 1 gallon of water before spraying. How many tablespoons of the weed killer need to be mixed with $2\frac{1}{2}$ gallons of water? [2.4]

***Solve. Present your solution on a number line and in interval notation.* [2.5]**

43. $x + 5 < 2$

44. $x - 3 > -8$

45. $2x - 7 \geq 3$

46. $3x + 4 \leq -8$

47. $3x < -18$ or $x + 3 > 10$

48. $x + 5 \leq -3$ or $4x \geq -12$

49. $-21 \leq 3x - 6 \leq 24$

50. $-11 < 2x - 9 < 11$

51. Erin needs to make at least 7 sales this week to qualify for a bonus. Write an inequality that shows the number of sales that will qualify Erin to receive a bonus. [2.5]

52. A company is trying to raise capital and offers 2 million shares of stock for sale. If the company needs to raise at least \$69.5 million, what price does it need to receive for the stock? [2.5]

53. If Steve's test average is at least 70 but lower than 80, his final grade will be a C. His scores on the first four tests were 58, 72, 66, and 75. What scores on the fifth exam will give him a C for the class? (Assume that the highest possible score is 100.) [2.5]

CHAPTER 2 TEST

For Extra Help
CHAPTER Test Prep VIDEOS

Step-by-step test solutions are found on the Chapter Test Prep Videos available via the Video Resources on DVD, in MyMathLab, and on YouTube (search "WoodburyElemIntAlg" and click on "Channels").

1. Solve $-8x = 56$ using the multiplication property of equality.

2. Solve $x + 17 = -22$ using the addition property of equality.

Solve.

3. $7x - 20 = 36$

4. $72 = 30 - 8a$

5. $3x + 34 = 5x - 21$

6. $3(5x - 16) - 9x = x - 63$

7. $\frac{1}{8}x - \frac{2}{3} = \frac{5}{6}$

8. $\frac{n}{100} = \frac{22}{40}$

9. Solve the literal equation $4x + 3y = 28$ for y.

10. What number is 16% of 175?

11. What percent of 660 is 99?

Solve. Present your solution on a number line and in interval notation.

12. $6x + 8 \leq -7$

13. $x + 4 \leq -5$ or $2x + 3 \geq 0$

14. $-23 < 3x + 7 < 23$

15. The length of a rectangle is 3 feet less than twice its width, and the perimeter is 72 feet. Find the length and the width of the rectangle.

16. Barry's wallet contains \$5 and \$10 bills. He has 7 more \$5 bills than \$10 bills. If Barry has \$185 in his wallet, how many \$5 bills are in the wallet?

17. Ernesto invested a total of \$10,000 in two mutual funds. In the first year, one account earned a 4% profit and the other earned an 11% profit. If Ernesto's total profit was \$862, how much did he invest in the mutual fund that earned an 11% profit?

18. A chemist has two solutions. One is 13% salt, and the other is 21% salt. How much of each solution should be mixed to make 100 milliliters of a solution that is 18% salt?

19. At a certain university, 2 out of every 9 professors earned their degree outside the United States. If the university has 396 professors, how many of them earned their degree outside the United States?

20. Wayne earns a commission of \$500 for every car he sells. How many cars does he need to sell to earn at least \$17,000 in commissions this month?

Mathematicians in History

George Pólya was a Hungarian mathematician who spent much of his career on problem solving and wrote the landmark text *How to Solve It*. Alan Schoenfeld has said of this book, "For mathematics education and the world of problem solving it marked a line of demarcation between two eras, problem solving before and after Pólya."

Write a one-page summary (*or* make a poster) of the life of George Pólya and his accomplishments. Also look up Pólya's most famous quotes and list your favorite.

Interesting issues:

- Where and when was George Pólya born?
- What circumstances led to Pólya leaving Göttingen after Christmas in 1913?
- The political situation in Europe in 1940 forced Pólya to leave Zürich for the United States. At which university did Pólya work after arriving in the United States?
- When was Pólya's monumental book *How to Solve It* published?
- Summarize Pólya's strategy for solving problems.
- At which American university did Pólya spend most of his career?
- What was Pólya's mnemonic for the first fourteen digits of π?
- How old was Pólya when he taught his last class? What was the subject?