[MAA 2.1] LINES

SOLUTIONS

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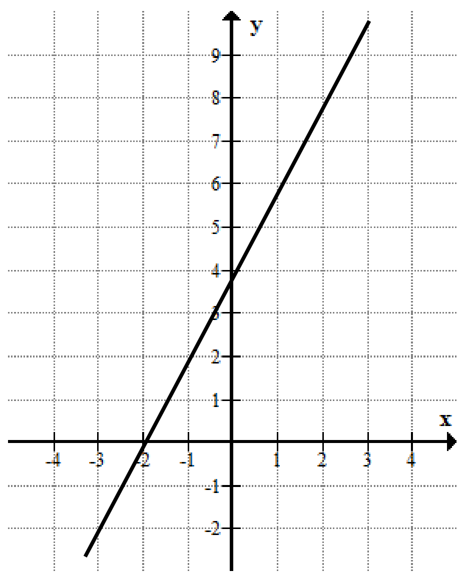
O. Practice questions

1. (a) $m_{AB} = \frac{4}{3}$ (b) $m_{AB} = -\frac{3}{4}$ (c) M($\frac{7}{2}$,9) (d) $d = 5$ (e) (8,15)

2. (a) $a = 1$ (b) $a = 11$ (c) $a = 5$ (d) $a = 0$ or $a = 6$

3. (a) (i) $m = 2$ (ii) $y = 4$ (iii) $x = -2$

(b)



(c) A does not lie on the line since 2×7+4 =18 ≠ 19 while B lies on the line since 2×8+4 =20

4. (a) $m_{AB} = \frac{3}{2}$, $y - 4 = \frac{3}{2}(x - 3)$ (b) $y = \frac{3}{2}x - \frac{1}{2}$ (c) $3x - 2y = 1$

5. (i) $3x + 2y = 18$, For x=0 $2y = 18 \Rightarrow y = 9$ therefore A (0,9)

(ii) For $y = 0$, $3x = 18 \Rightarrow x = 6$ therefore B (6,0)

(iii) midpoint between (0, 9) and (6, 0): $\left(\frac{0+6}{2}, \frac{9+0}{2}\right)$ = (3, 4.5)

6. (a) $x = 2$ (b) $y = 5$ (c) P(2,5)

7. (a) $y = 3$ (b) $x = 2$ (c) $y = \frac{3}{2}x$

8. (a) (i) $m = -2$ (ii) $y = 5$ (iii) $x = 2.5$ (= $\frac{5}{2}$)

(b) $y = -2x + 5$

(c) $y = 3$, $x = 2$ (d) $a = 5$, $b = 15$

A. Exam style questions (SHORT)

9. (a) $y=-2x+3$ gradient of line $L_1=-2$

(b) **METHOD 1**

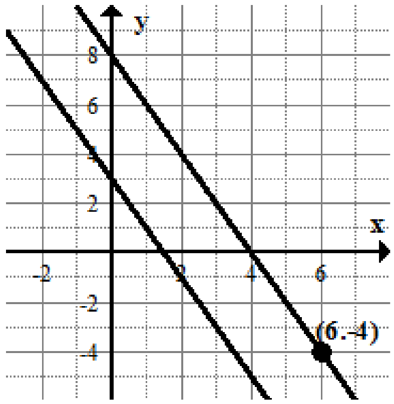
$(y-y_1)=m(x-x_1)\Rightarrow\left(y-(-4)\right)=-2(x-6)$

$y+4=-2x+12 \;\Rightarrow\; y=-2x+8$

METHOD 2

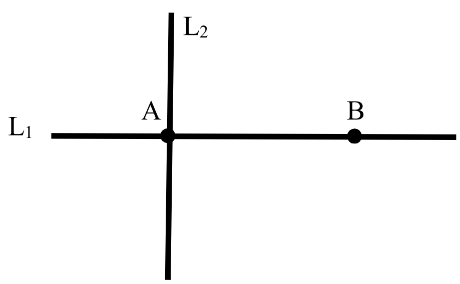
Substituting the point $(6,-4)$ in $y=mx+c \Rightarrow c=8$ so $y=-2x+8$

(c) when line L_1 cuts the x-axis, $y=0\Rightarrow y=-2x+8\Rightarrow x=4$



10. (a) (i) $m_{AB}=3$ (ii) $y=3x-1$ (b) $y=-\frac{1}{3}x+\frac{17}{3}$ (c) $3x-y=1$ and $x+3y=17$

(d) A(2,5). The solution is in fact the point of intersection which is A as expected.



11. (a) $m_{AB}=\frac{2}{3}$ (b) M(1,7) (c) $y=-\frac{3}{2}x+\frac{17}{2}$ (d) $d=\sqrt{13}$

12. (a) $m_{AB}=\frac{2}{4}=\frac{1}{2}$, $m_{\perp}=-2$, Midpoint (4,2), $y-2=-2(x-4)$ or $y=-2x+10$

(b) $y=10$

13. $m_{AB}=\frac{28}{12}=\frac{7}{3}$, $m_{\perp}=-\frac{3}{7}$,

Midpoint (14,27),

$y-27=-\frac{3}{7}(x-14)$

$\Leftrightarrow 7y-189=-3x+42$

$\Leftrightarrow 3x+7y=231$

14. Let P(x,x+1) be on L_1.

$\sqrt{x^2+(x+1)^2}=5 \Leftrightarrow x^2+(x+1)^2=25 \Leftrightarrow x=-4 \text{ or } x=3$,

Therefore, the point is P(-4,-3) or P(3,4)

15. Let A(a,a+1) and B(b,2b+1) be the points on L_1 and L_2 respectively. Then

$$\frac{a+b}{2}=5 \text{ and } \frac{a+1+2b+1}{2}=8,$$

that is

$$a+b=10$$
$$a+2b=14$$

hence $a=6, b=4$

and finally A(6,7), B(4,9)

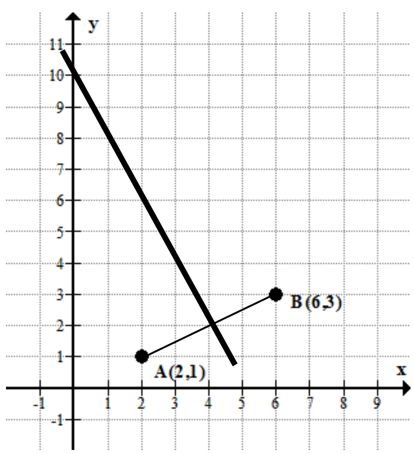
16. Let R be $(x_1, 5-x_1)$ and Q be $\left(x_2, \frac{1}{2}x_2-2\right)$

Since P is the mid-point of [QR]

$$\frac{x_1+x_2}{2}=1 \text{ and } \frac{5-x_1+\frac{1}{2}x_2-2}{2}=1$$

$\Rightarrow x_1+x_2=2$ and $-2x_1+x_2=-2$

$\Rightarrow x_1=\frac{4}{3}$ and $x_2=\frac{2}{3}$ $\Rightarrow R$ is $\left(\frac{4}{3},\frac{11}{3}\right)$ and Q is $\left(\frac{2}{3},\frac{-5}{3}\right)$



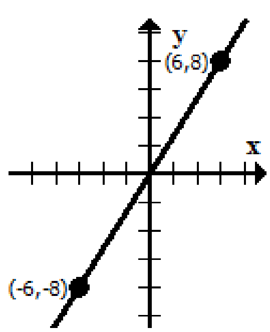
B. Exam style questions (LONG)

17. (a) For $x=3k$, we obtain $y=4k$

(b) $\sqrt{(3k)^2+(4k)^2}=10 \Leftrightarrow 25k^2=100 \Leftrightarrow k^2=4 \Leftrightarrow k=\pm 2$ (or by GDC)

(c) (6,8) and (-6,-8)

(d)

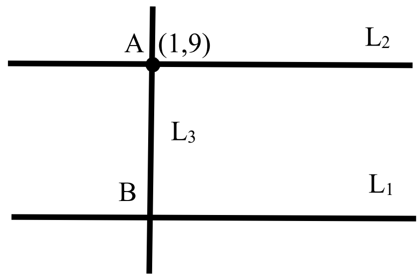


18. (a) L_2 : $y = 2x + 7$ $\quad L_2 : y = -\frac{1}{2}x + \frac{19}{2}$

(b) B(5,7)

(c) $d = \sqrt{20} = 2\sqrt{5}$

(d) In fact the distance from A to the line L_1 is $2\sqrt{5}$



19. (a) $x = 5$ (b) $y = 5$ (c) $P(5,5)$ $\quad \frac{3+7}{2} = 5 \quad \frac{2+8}{2} = 5$

(d) (i) 12 (ii) 6 (iii) 6

(e) $m_{BC} = -\frac{3}{2}$, $m_{\perp} = \frac{2}{3}$, Midpoint (5,5), $y - 5 = \frac{2}{3}(x - 5) \Rightarrow 2x - 3y = -5$

(f) For $x = 3$, $y = \frac{11}{3} \neq 2$

20. (a) A(6,-1) (b) (i) B(-2,7) (ii) C(-2,-5) (c) Area = $\frac{12 \times 8}{2} = 48$