The Euler Method

Dr. Kevin Thompson

We are going to explore a method to approximate the solution to a differential equation called the Euler Method. https://www.geogebra.org/m/n46r2swb

1) Suppose we start with the differential equation: $\frac{dy}{dx} = \sin(x)$ and examine the slope field.

   Find the specific solution where y(½)= ½ .

   y(x)= ____________________

2) Determine the value for y(1)= _______

3) Suppose we couldn't find that antiderivative though but are only able to differentiate the given function. Using the initial point A (½ , ½) on the solution determine the equation of the tangent line for the solution that goes through this point. (NOTE THAT YOU DO NOT NEED THE ANTIDERIVATIVE TO GET THIS ANSWER)
   a. L(x)= ________________________________
      (This should match the function in GeoGebra as a check of your work which must be shown below)

   b. We know that if a function is ______________ _______________ that the tangent line does not stray very far from the function over short distances.

   c. Use this line to approximate the value y(1) using L(1). $y(1) \approx$ ____________

   d. We know have a second point (____, _____) that is *near* the solution to the differential equation.

4) This works well if I desire a single point and an approximation of a single value, but what if I am attempting to approximate the entire function? At this point we have one point that we know is on the function (point A) and a second point that is approximately on the function. (1, L(1)). Why would a segment connecting these points approximate your answer to number 1 over the interval [0,1]?

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   ____

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5) Click the button next to *L(x)* in GeoGebra. You can see that if we would continue on this path, the approximation becomes very poor using this line. Now, even though we know the answer to the last problem is not on the function, what if we assumed it was? So, what if we would construct a second line at our approximate point? What we assume is that our approximate point is very close to the curve. Use that point to find a second "tangent line" $L_2(x)$. Use this new line to get a second approximate point on the function you answer in #1. The approximate point is (1.5, ________)

6) So, we now have 3 points. {A; (1, L(1)) ; (1.5, $L_2(1)$)} Change n-slider to the value of 2 which should now show the graphing of the three points simultaneously with the solution to the differential equation.

7) Determine a 4^{th} point that is near the curve of the solution (by assuming it is on the solution).

(_________, __________)

8) Set the n-slider to 3 and examine the list **l5** in GeoGebra to confirm your four points.
Confirmed Y / N

If no, go back and fix your mistakes.

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9) Now we will explore by taking advantage of the computer. Increase/decrease the n-slider. Increase/decrease the d-slider. Describe the effect that each has in combination with the other. Address how the Euler method allows someone to find a very accurate approximation to the solution for a differential equation when you are not able to come up with the antiderivative. Use sentences.

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10) Now let us examine more careful the formula used in GeoGebra. Notice that A is the only point that we know on the solution to the differential equation and is a given. To find the next point, I wrote the following command: ***IterationList(A + (d, d h'(x(A))), A, {A}, n)***

Effectively, this is taking the coordinates of point A and adding the coordinates of the point ***(d, d*h'(x(A)))*** where h'(x) is the original differential equation given and x(A) is the x coordinate of point A.

The iterationList command repeats this process n times. Explain why **A + *(d, d*h'(x(A)))*** is the second point.

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11) Most of the time, textbooks show the following table to generate the new points.

X	Y	m=dy/dx	$\Delta y = m \cdot \Delta x$	$y_{new}=y+\Delta y$
x_1	y_1			
$x_1+\Delta x$	y_{new}			

JUSTIFY THIS TABLE:

____
____

Read this for fun: https://www.insidescience.org/news/exploring-math-hidden-figures

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