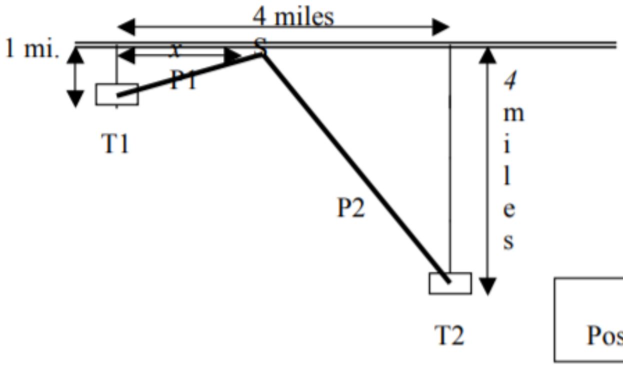
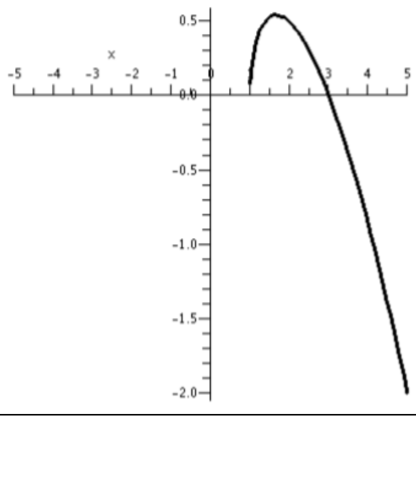
Virga Jessecollege **Taak WISKUNDE**
Hasselt **leerkracht: Karel Appeltans**
Datum: **schooljaar**
Klas: VI **Studierichting:**
Naam:... **Aantal uren wiskunde:**

<table>
<tr><td>1</td><td></td><td>Bepaal de vergelijkingen van de asymptoten van de grafiek van
$f(x) = \sqrt{x^2 - 6x + 3} + x - 1$</td></tr>
<tr><td>2</td><td></td><td>On the same side of a river with straight banks are two towns (T1 and T2) and a pumping station (S) that supplies water to both towns (see diagram below). The pumping station is at the river's edge with pipes extending straight to the distribution points in each town. Where should the pumping station be located (i.e., at what value of x in the diagram) to minimize the total length of the pipes P1 and P2?

Position of pumping station: x = ____________</td></tr>
<tr><td>3</td><td></td><td>Bepaal de vergelijkingen van de asymptoten van de grafiek van
$f(x) = \sqrt{x^2 - 3x + 4} + x$</td></tr>
<tr><td>4</td><td></td><td>Given the function $y = (x-1)^{1/2} - \frac{1}{2}(x-1)^{3/2}$, find all points at which the function has a horizontal tangent and all points at which the function has a vertical tangent if any such points exist. NOTE: You MUST show some appropriate calculations—approximating any answers from the graph below will gain you NO POINTS although you may use the graph to check the reasonability of your calculations…
Horizontal tangents at x = ____________
Vertical tangents at x = ____________
(if none, write NONE in the space above)
</td></tr>
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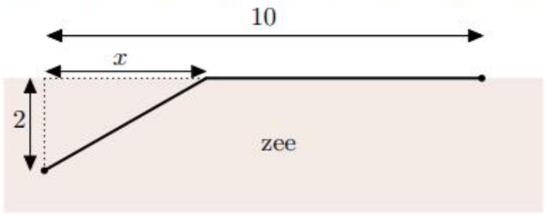
<tr><td>5</td><td></td><td>Een man bevindt zich in een roeiboot 2 km verwijderd van een rechte kustlijn. Hij wenst een plaats te bereiken die 10 km verder langs deze kustlijn is gelegen (zie bijgevoegde figuur). Deze man roeit aan een (constante) snelheid van 3 km per uur en wandelt aan een (constante) snelheid van 6 km per uur. Het totaal aantal uren (in functie van x) dat de man nodig heeft om de afstand af te leggen langs het in het zwart aangeduide traject is gelijk aan


(A) $\frac{\sqrt{x^2+4}}{3}+\frac{10-x}{6}$
(B) $\frac{x}{6}+\frac{7}{3}$
(C) $\frac{\sqrt{4-x^2}}{3}+\frac{x-10}{6}$
(D) $3\sqrt{x^2+4}+6(10-x)$
(E) $\frac{\sqrt{4+(x-10)^2}}{3}+\frac{x}{6}$</td></tr>
<tr><td>6</td><td></td><td>Gegeven is de functie $f : x \to x + \sqrt{p-2x}$ voor een of ander getal p.
Neem eerst $p = 10$, dus: $f(x) = x + \sqrt{10-2x}$.
a Bereken exact de coördinaten van het punt op de grafiek van f waar de raaklijn horizontaal is.
b Voor welk getal p heeft de grafiek van f een horizontale raaklijn in het punt met eerste coördinaat 2?
c Voor welke waarde van p is de lijn $y = 5$ een horizontale raaklijn aan de grafiek van f?</td></tr>
<tr><td>7</td><td></td><td>Bepaal de vergelijkingen van de asymptoten van de grafiek van $f(x) = \sqrt{x^2+4x-7}$</td></tr>
<tr><td>8</td><td></td><td>5. (10 points) An amphibious vehicle requires 5 minutes to travel each mile over land, but only 4 minutes to travel each mile in the water. The vehicle is currently located on water, in a canal that runs in the east-west direction. Its destination point is on land, 3 miles due east and 2 miles due north of its current position. What route will minimize the time required for the vehicle to reach its destination? (You should assume that the time required for the vehicle to transition from water to land is negligible, and that the canal's width is also negligible.) Justify your answer completely.</td></tr>
<tr><td>9</td><td></td><td>**Problem 4.** (6 points)
A. (3 points) Construction costs. An oil refinery is located on the north bank of a straight river that is 2 km wide. A pipeline is to be constructed from the refinery to storage tanks located on the south bank of the river 6 km east of the refinery. The cost of laying the pipe is $400,000/km over land to a point P on the north bank and $800,000/km under the river to the tanks. To minimize the cost of the pipeline, where should be P located?
Hint: $\sqrt{3} \approx 1.732...$</td></tr>
<tr><td>10</td><td></td><td>Bepaal de afleidbaarheid van $f(x) = x\sqrt{x+3}$ in de randpunten van het domein</td></tr>
<tr><td>11</td><td></td><td>Bepaal het verloop van $f(x) = \sqrt{x^2-4x}$</td></tr>
<tr><td>12</td><td></td><td>Bepaal de asymtoten van $f(x) = \sqrt{x^2+5} - x$</td></tr>
<tr><td>13</td><td></td><td>Bepaal domein en asymptoten van $f(x) = x - \sqrt{x^2+5x}$</td></tr>
</table>

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Gegeven de functie $f : [a, +\infty[\rightarrow \mathbb{R} : x \mapsto f(x) = x\sqrt{2x+3}$, met $a \in \mathbb{R}$ de kleinste waarde waarvoor $x\sqrt{2x+3}$ gedefinieerd is. A is het punt op de grafiek van f met x-coördinaat a. B is het punt op de grafiek van f met x-coördinaat 3. C is het snijpunt van de x-as met de raaklijn in B aan de grafiek van f. Bepaal de oppervlakte O van de driehoek ABC.

(A) $O = \frac{27}{8}$ (B) $O = \frac{81}{8}$ (C) $O = \frac{135}{8}$ (D) $O = \frac{243}{8}$

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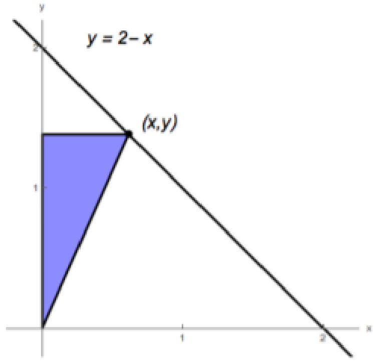
. (18 pts) A **right** triangle is constructed in the first quadrant.

Its hypotenuse runs from the origin to a point on the line $y = 2 - x$.
One of its sides lies on the $y-$axis, and the other side is parallel to the $x-$axis (see figure).
Find the values x and y which **minimize the perimeter** of the triangle.



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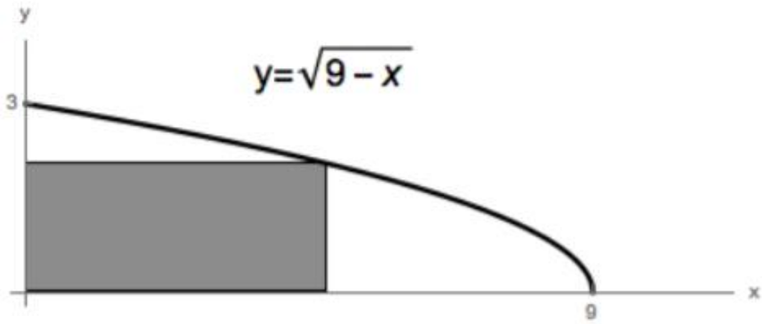
4. (18 pts) A **rectangle** is constructed in the **first quadrant** with one side on the x-axis, one side on the y-axis, and the vertex opposite the origin on the curve $y = \sqrt{9-x}$ (see figure).

Find the **area** of the **largest** such rectangle.



Solve the problem by following the steps indicated below.

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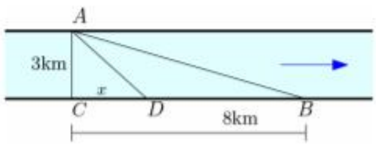
. An animal wants to get from point A on a bank of a straight river, 3km wide, to point B 8km down the stream on the opposite bank of the river as quickly as possible. The animal will swim straight to a point D on the other bank x km from C towards B with speed 6 km/h and then run straight along the bank to the point B with speed 8 km/h. Find the point D which minimizes the total time needed for the animal to reach B.



(a) Assuming time $= \frac{\text{distance}}{\text{rate}}$, write the formula for the swimming time, the running time, and the total time $T(x)$ in terms of x.

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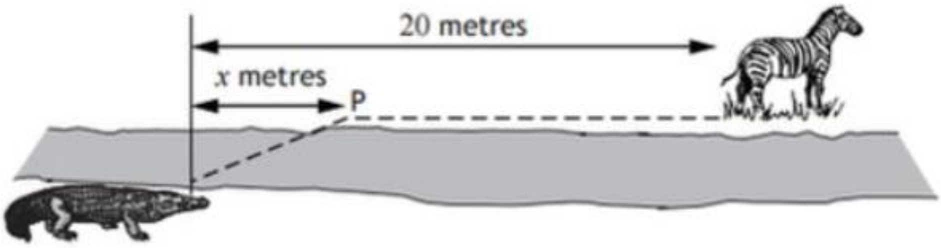
A crocodile is stalking prey located 20 metres further upstream on the opposite bank of a river.

Crocodiles travel at different speeds on land and in water.

The time taken for the crocodile to reach its prey can be minimised if it swims to a particular point, P, x metres upstream on the other side of the river as shown in the diagram.



The time taken, T, measured in tenths of a second, is given by

$$T(x)=5\sqrt{36+x^2}+4(20-x)$$

(a) (i) Calculate the time taken if the crocodile does not travel on land.

(ii) Calculate the time taken if the crocodile swims the shortest distance possible.

(b) Between these two extremes there is one value of x which minimises the time taken. Find this value of x and hence calculate the minimum possible time.

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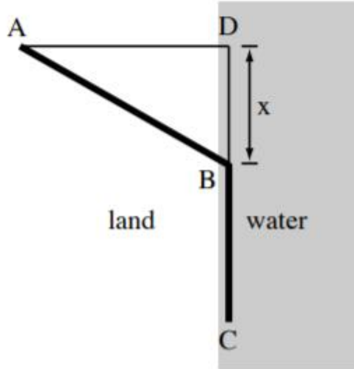
[6 pts.] Shown in the figure below is the view from above of the path taken by a penguin from point A to a feeding area on the shore at point C. The penguin must choose the point B toward which it starts walking. It takes twice as much energy per unit distance for a penguin to walk over land (AB) as to swim through water (BC). The distance AD is 300 m and the distance DC is 400 m. Calculate the value of the distance x (and hence the location of the point B - see figure) that minimizes the energy spent on the entire trip.



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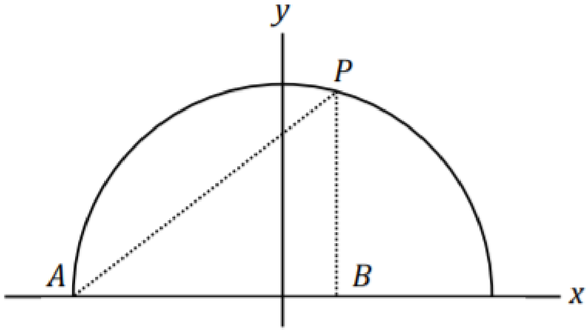
[marks] Find the coordinates of the point P on the semicircle $y = \sqrt{1-x^2}$ of radius 1 (pictured below) for which the right triangle ABP has maximal area.



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Consider the function $f : D_f \to \mathbb{R}$ defined by

$$f(x) = \frac{x}{1 - \sqrt{x^2+1}}.$$

a) Find the (maximal) domain D_f of f.

b) Is f an even function? Is f an odd function? [Explain your answers.]

c) On what interval(s) is f increasing? And on what interval(s) is it decreasing?

d) Calculate the following limits, or explain why the limit does not exist:

$$\lim_{x\to\infty} f(x), \quad \lim_{x\to-\infty} f(x) \quad \text{and} \quad \lim_{x\to 0} f(x).$$

e) Find the range R_f of f.

22

Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \frac{x}{\sqrt{x^2+4}}.$$

a) Prove that f is one-to-one on $\mathbb{R}$.

b) Prove that f has an inverse function f^{-1} with domain $(-1, 1)$.

c) Calculate $(f^{-1})'(0)$.

23

Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by

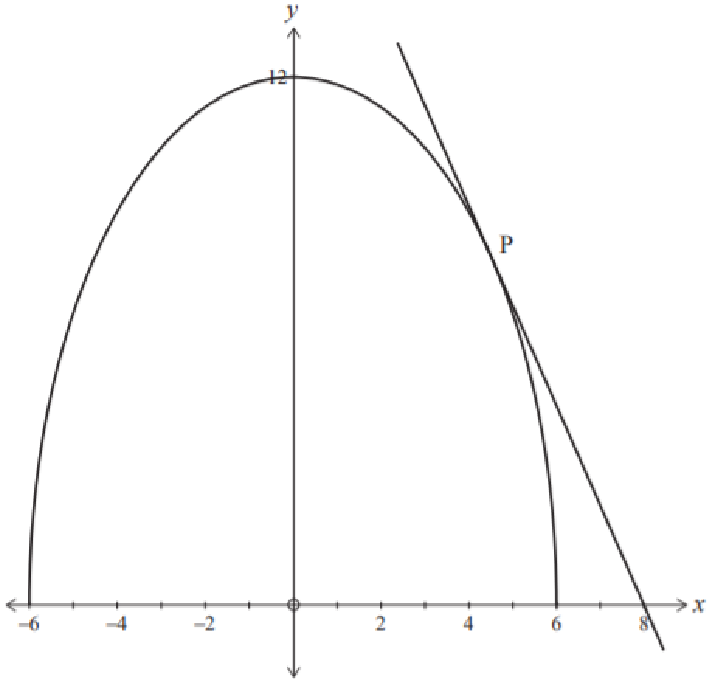
$$f(x) = x\sqrt{x^2+3}.$$

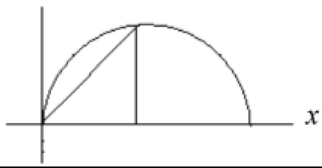
a) Prove that f is one-to-one on $\mathbb{R}$.

b) Prove that f has an inverse function f^{-1} with domain $\mathbb{R}$.

c) Calculate $(f^{-1})'(0)$.

24		1. (5 = 2+3 punten) De functie $f : D_f \to \mathbb{R}$ wordt gegeven door $$f(x) = x\sqrt{4x - x^2}.$$ a) Wat is het domein van f? (Dus: bepaal alle x waarvoor de gegeven formule betekenis heeft.) b) Bepaal de extreme waarden van f op het in a) bedoelde domein.
25		1. De functie $f : D_f \to \mathbb{R}$ wordt gegeven door $$f(x) = x\sqrt{6 - x - x^2}.$$ a) Wat is het domein van f? [Dus: bepaal alle x waarvoor de gegeven formule betekenis heeft.] b) Bepaal de vergelijking van de raaklijn aan de grafiek van f in het punt $(1, 2)$.
26		1. Gegeven is de functie $$f(x) = \sqrt{4\sqrt{x} - x}.$$ a) Wat is het domein van f? [Dus: bepaal alle x waarvoor de gegeven formule betekenis heeft.] b) Bepaal de plaats en grootte van de extreme waarden van f op het interval $[1, 9]$. c) Bepaal de vergelijking van de raaklijn aan de grafiek van f in het punt $(1, \sqrt{3})$.

<table>
<tr><td>27</td><td></td><td>The graph below shows the function $y = 2\sqrt{36 - x^2}$, and the tangent to that function at point P.
The tangent intersects the x-axis at the point (8,0).


Find the x-coordinate of point P.</td></tr>
<tr><td>28</td><td></td><td>38. The hypotenuse of the right triangle shown at the right is the line segment from the origin to a point on the graph of $y = \sqrt{4 - (x-2)^2}$. Find the coordinates on the graph that will maximize the area of the right triangle.

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