PRAVAC

· implicitni oblik: $Ax + By + C = 0$

- eksplicitni oblik: $y = kx + l$ → odsječak na osi ordinata

↓ nagib (koef. smjera)

$k = \dfrac{y_B - y_A}{x_B - x_A}$

$k = tg\,\varphi$

- okomiti pravci: $k_2 = -\dfrac{1}{k_1}$

· segmentni oblik: $\dfrac{x}{m} + \dfrac{y}{n} = 1$

Zad 1. Odredi jednadžbu pravca koji prolazi kroz točku $T(-4, 2)$ i okomit je na pravac p_1 koji prolazi točkama $A(-3,2)$ i $B(2,4)$.

$A(-3,2)$

$B(2,4)$

$T(-4,-2)$

$y = kx + l$

$2 = -3k + l \quad /\cdot(-1)$

$4 = 2k + l$

$-2 = 3k - l$

$4 = 2k + l$

$2 = 5k$

$k = \dfrac{2}{5} = 0.4$

$-2 = 3 \cdot \dfrac{2}{5} - l$

$l = \dfrac{6}{5} + 2$

$l = 3.2$

$p_1 \ldots \quad y = 0.4x + 3.2$

$p_2 \ldots \quad y = kx + l$

$k_2 = -\dfrac{1}{k_1}$

$k_2 = -\dfrac{1}{\frac{2}{5}} = -\dfrac{5}{2} = -2.5$

$-2 = -4 \cdot \left(-\dfrac{5}{2}\right) + l$

$l = -12$

$\boxed{y = -2.5x - 12}$

· kut između pravaca: $\operatorname{tg}\varphi = \left|\dfrac{k_2 - k_1}{1 + k_1 k_2}\right|$

ZAD 2. Za koju vrijednost broja m pravci $mx + y + 3 = 0$ i $x - 2y - 1 = 0$ zatvaraju kut od $45°$?

$$mx + y + 3 = 0 \Rightarrow y = -mx - 3$$

$$x - 2y - 1 = 0 \Rightarrow y = \frac{1}{2}x - \frac{1}{2}$$

$$\varphi = 45°$$

$$m = ?$$

$$\operatorname{tg}\varphi = \left|\frac{k_1 - k_2}{1 + k_1 k_2}\right|$$

$$\operatorname{tg}45° = \left|\frac{-m - \frac{1}{2}}{1 - \frac{1}{2}m}\right|$$

$$1 = \left|\frac{-m - \frac{1}{2}}{1 - \frac{1}{2}m}\right| \longrightarrow \frac{-m - \frac{1}{2}}{1 - \frac{1}{2}m} = -1 \quad / \cdot \left(1 - \frac{1}{2}m\right) \neq 0$$

$$m \neq 2$$

$$-m - \frac{1}{2} = -\left(1 - \frac{1}{2}m\right)$$

$$\frac{3}{2}m = \frac{1}{2} \quad / \cdot \frac{2}{3}$$

$$m = \frac{1}{3} \ \checkmark$$

$$\frac{-m - \frac{1}{2}}{1 - \frac{1}{2}m} = 1 \quad / \cdot \left(1 - \frac{1}{2}m\right) \neq 0$$

$$\frac{1}{2}m \neq 1$$

$$m \neq 2$$

$$-m - \frac{1}{2} = 1 - \frac{1}{2}m$$

$$\frac{1}{2}m = \frac{-3}{2} \quad / \cdot 2$$

$$m = -3 \ \checkmark$$

ZAD 3. Odredi jednadžbu simetrale dužine $\overline{AB}$. $A(1,4)$, $B(5,2)$

$A(1,4)$
$B(5,2)$

polovište AB: $x = \frac{x_A + x_B}{2} = \frac{1+5}{2} = 3$

$y = \frac{y_A + y_B}{2} = \frac{4+2}{2} = 3$

$P(3,3)$

$k_{AB} = \frac{y_A - y_B}{x_A - x_B} = \frac{4-2}{1-5} = -\frac{2}{4} = -\frac{1}{2}$

$k_s = -\frac{1}{k_{AB}} = 2$

j. simetrale: $y = kx + l$

$3 = 2 \cdot 3 + l$

$l = -3$

$\boxed{y = 2x - 3}$

ZAD 4. Odredi točku simetričnu točki $T(2,4)$ s obzirom na pravac $y = \frac{3}{4}x - 5$

$T(2,4)$
$p_1 \ldots y = \frac{3}{4}x - 5$

$k_1 = \frac{3}{4}$

$k_2 = -\frac{1}{k_1} = -\frac{4}{3}$

$p_2 \ldots$ $y = kx + l$

$4 = 2 \cdot \left(-\frac{4}{3}\right) + l$

$l = \frac{20}{3}$

$y = -\frac{4}{3}x + \frac{20}{3}$

$p_1 \cap p_2 = P$ $\quad \frac{3}{4}x - 5 = -\frac{4}{3}x + \frac{20}{3} \quad / \cdot 12$

$9x - 60 = -16x + 80$

$25x = 140 \quad / : 25$

$x_P = \frac{28}{5}$

$y_P = -\frac{4}{5}$

$\boxed{T(9.2, -5.6)}$

$x_P = \frac{x_T + x_{T'}}{2} \Rightarrow x_{T'} = 2x_P - x_T = 2 \cdot \frac{28}{5} - 2 = 9.2$

$y_P = \frac{y_T - y_{T'}}{2} \Rightarrow y_{T'} = 2y_P - y_T = 2 \cdot \left(-\frac{4}{5}\right) - 4 = -5.6$

ZAD 5. Stranice trokuta ABC pripadaju pravama $x+3y-2=0$, $x-y+2=0$, $3x+y-14=0$. Odredi vrhove trokuta i ortocentar.

$a \ldots\ x+3y-2=0 \Rightarrow y=-\frac{1}{3}x+\frac{2}{3} \qquad k=-\frac{1}{3}$

$b \ldots\ x-y+2=0 \Rightarrow y=x+2 \qquad k=1$

$c \ldots\ 3x+y-14=0 \Rightarrow y=-3x+14 \qquad k=-3$

$a \cap b \ldots\ -\frac{1}{3}x+\frac{2}{3}=x+2$

$-x+2=3x+6$

$-4x=4$

$x=-1$

$y=-1+2$

$y=1$

$\Rightarrow \boxed{C(-1,1)}$

$a \cap c \ldots\ -\frac{1}{3}x+\frac{2}{3}=-3x+14 \quad /\cdot 3$

$-x+2=-9x+42$

$8x=40$

$x=5$

$y=-3\cdot 5+14$

$y=-1$

$\Rightarrow \boxed{B(5,-1)}$

$b \cap c \ldots\ x+2=-3x+14$

$4x=12$

$x=3$

$y=3+2$

$y=5$

$\Rightarrow \boxed{A(3,5)}$

$\{H\}=v_a \cap v_c$

$3x-4=\frac{1}{3}x+\frac{4}{3} \quad /\cdot 3$

$9x-12=x+4$

$8x=16$

$x=2 \qquad y=2$

$\boxed{H(2,2)}$

$v_c \ldots\ k_{v_c}=-\frac{1}{k_c}=\frac{1}{3}$

$y=kx+l$

$1=-\frac{1}{3}+l$

$l=\frac{4}{3}$

$y=\frac{1}{3}x+\frac{4}{3}$

$v_a \ldots\ k_{v_a}=-\frac{1}{k_a}=3$

$y=kx+l$

$5=9+l$

$l=-4$

$y=3x-4$

· udaljenost točke od pravca: $d(T,p)=\dfrac{|Ax+By+C|}{\sqrt{A^2+B^2}}$

ZAD 6. Točka T (2,3) jedan je vrh kvadrata, a jedna stranica kvadrata leži na pravcu $3x-2y+13=0$. Kolika je površina kvadrata?

$T(2,3)$

$3x-2y+13=0$

$P=?$

$a=d(T,p)$

$$a=\frac{|Ax+By+C|}{\sqrt{A^2+B^2}}=\frac{|3\cdot2+(-2)\cdot3+13|}{\sqrt{9+4}}=\sqrt{13}$$

$\boxed{P=a^2=\sqrt{13}^2=13}$

ZAD 7. Dvije stranice pravokutnika leže na pravcima $x+2y-3=0$ i $2x-y+3=0$. Ako je jedan vrh pravokutnika točka (8,5), kolika je površina pravokutnika?

$p_1 \dots\ x+2y-3=0$

$p_2 \dots\ 2x-y+3=0$

$T(8,5)$

$P=?$

$a=d(T,p_1)$

$b=d(T,p_2)$

$$a=\frac{|Ax+By+C|}{\sqrt{A^2+B^2}}=\frac{|8+10-3|}{\sqrt{1+4}}=3\sqrt{5}$$

$$b=\frac{|16-5+3|}{\sqrt{4+1}}=\frac{14\sqrt{5}}{5}$$

$$\boxed{\begin{aligned}P&=a\cdot b\\&=3\sqrt{5}\cdot\frac{14\sqrt{5}}{5}\\&=42\end{aligned}}$$