Evoluta de la elipse.

Para la elipse

$$f(t) = (a \cos t, b \text{ sen } t)$$

Se tiene

$$f'(t) = (-a \, sen\, t, b \cos t); \quad \|f'(t)\| = \sqrt{a^2 sen^2 t + b^2 \cos^2 t}$$

$$\perp f'(t) = (-b \cos t , -a \text{ sen } t)$$

$$N(t) = \frac{1}{\sqrt{a^2 sen^2 t + b^2 \cos^2 t}} (-b \cos t, -a \, sen\, t)$$

$$f''(t) = (-a \cos t , -b \text{ sen } t)$$

De donde

$$\kappa(t) = \frac{1}{\|f'(t)\|^3} f''(t) \cdot \perp f'(t) = \frac{1}{(a^2 sen^2 t + b^2 \cos^2 t)^{3/2}} (ab \cos^2 t + ab \, sen^2 t)$$

Simplificando

$$\boxed{\kappa(t) = \frac{ab}{(a^2 sen^2 t + b^2 \cos^2 t)^{3/2}}}$$

Para la ecuación de la evoluta queda

$$c(t) = (a \cos t, b \, sen\, t) + \frac{a^2 sen^2 t + b^2 \cos^2 t}{ab} (-b \cos t, -a \, sen\, t)$$

$$\boxed{c(t) = \left(a \cos t - \frac{(a^2 sen^2 t + b^2 \cos^2 t) \cos t}{a}; \quad b \, sen\, t - \frac{(a^2 sen^2 t + b^2 \cos^2 t) sen\, t}{b} \right)}$$

A esta curva se le denomina *astroide.*