INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

MAA

EXERCISES [MAA 5.2]

DERIVATIVES – BASIC RULES

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O. Practice questions

1. [Maximum mark: 20] ***[without GDC]***

Differentiate the following functions:

$f(x)$	$f'(x)$
$f(x)=2x^5$	
$f(x)=\frac{2}{x^5}$	
$f(x)=x^3+\ln x$	
$f(x)=2\sin x+3\cos x+5e^x$	
$f(x)=5x^3+2x^2+3x+7$	
$f(x)=\frac{5}{x^3}+\frac{2}{x^2}+\frac{3}{x}+7$	
$f(x)=\sqrt{x}-x+1$	
$f(x)=6\sqrt[3]{x^5}$	
$f(x)=mx+c$	
$f(x)=ax^2+bx+c$	

2. [Maximum mark: 24] ***[without GDC]***

Differentiate the following functions **without using the product or the quotient rules**:

$f(x)$	Simplify $f(x)$	$f'(x)$
$f(x)=x^2(2x+3)$	$=2x^3+3x^2$	
$f(x)=(3x+2)(2x+3)$		
$f(x)=2x^3+\frac{5}{x^3}+1$		
$f(x)=1+\frac{2}{x}+\frac{3}{x^2}$		
$f(x)=x^2(1+\frac{2}{x}+\frac{3}{x^2})$		
$f(x)=\frac{1+x+x^2}{x^2}$		
$f(x)=\frac{3x^5}{2}+\frac{2}{3x^4}$		
$f(x)=\frac{2x^5+5x^2+1}{x^2}$		
$f(x)=\frac{2x^5+5x^2+1}{3x^2}$		
$f(x)=3x(\sqrt{x}+1)$		
$f(x)=\sqrt{x}(2x+3\sqrt{x})$		
$f(x)=\frac{2x+3\sqrt{x}}{\sqrt{x}}$		

3. [Maximum mark: 28] ***[without GDC]***

Differentiate the following functions:

$y = f(x)$	$\frac{\mathrm{d}y}{\mathrm{d}x}$
$y = \mathrm{e}^x \sin x$	
$y = \mathrm{e}^x \cos x$	
$y = x^3 \mathrm{e}^x$	
$y = x^2 \ln x$	
$y = \sqrt{x} \sin x$	
$y = (2x+3)\cos x$	
$y = \frac{\mathrm{e}^x}{\sin x}$	
$y = \frac{\sin x}{\mathrm{e}^x}$	
$y = \frac{\mathrm{e}^x + 1}{\cos x}$	
$y = \frac{2x-1}{3x+5}$	
$y = \frac{2x+3}{\cos x}$	
$y = \frac{\cos x}{2x+3}$	
$y = \frac{7x^3}{5} + \frac{7}{5x^3} + \frac{4x}{3} - \frac{4}{3x}$	
$y = x^2 + \ln x + x^2 \ln x$	

4. [Maximum mark: 6] ***[with / without GDC]***

Let $f(x) = 2x^3 + \ln x$

(a) Find (i) $f'(x)$ (ii) $f''(x)$. [4]

(b) Find (i) $f'(1)$ (ii) $f''(1)$. [2]

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5. [Maximum mark: 6] ***[with GDC]***

Let $f(x) = \dfrac{x^2}{\sin x}$

(a) Find $f'(x)$. [3]

(b) Find the gradient of the curve $y = f(x)$.

(i) at $x = \dfrac{\pi}{2}$ (ii) at $x = 1$ rad. [3]

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6. [Maximum mark: 12] ***[without GDC]***

Given the following values at $x=1$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	2	3	4	5

Calculate the derivatives of the following functions at $x=1$

(i) $y=2f(x)+3g(x)$ (ii) $y=f(x)g(x)$

(iii) $y=\dfrac{f(x)}{g(x)}$ (iv) $y=2x^3+1+5f(x)$

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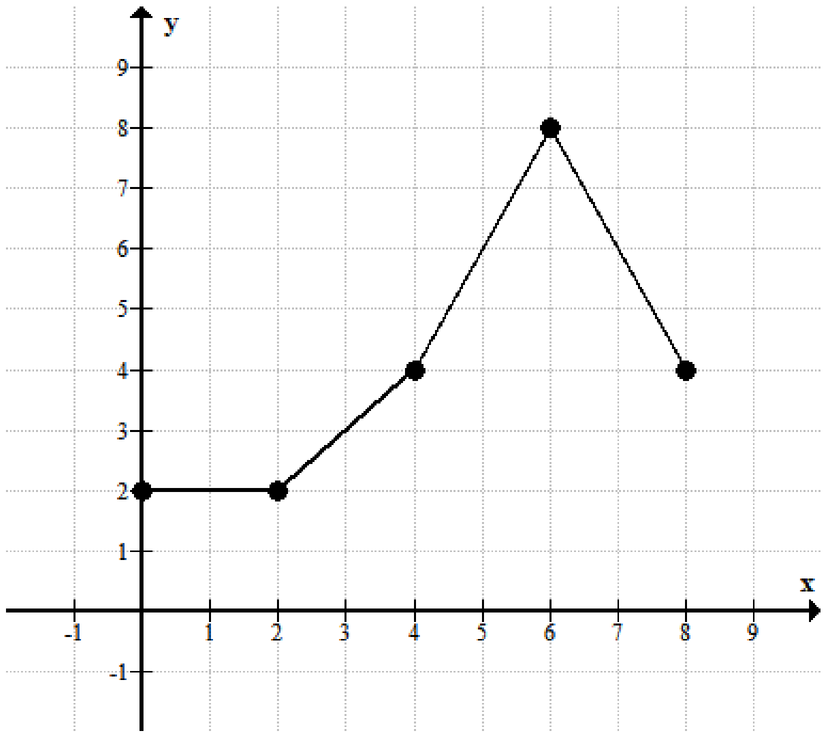
7. [Maximum mark: 10] ***[without GDC]***

The diagram shows the graph of a function $y = f(x)$, for $0 \le x \le 8$.



(a) Complete the table below.

x	1	3	5	7
$f(x)$				

[2]

(b) Complete the table below.

x	1	3	5	7
$f'(x)$				

[4]

(c) Complete the table below.

x	1.7	4.1	5.8	6.5
$f'(x)$				

[2]

(d) Solve the equation $f(x) = 6$. [2]

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8. [Maximum mark: 5] ***[without GDC]***

Let $f(x) = 5x^2 - 3x$. Find the coordinates of the point where the gradient is 7.

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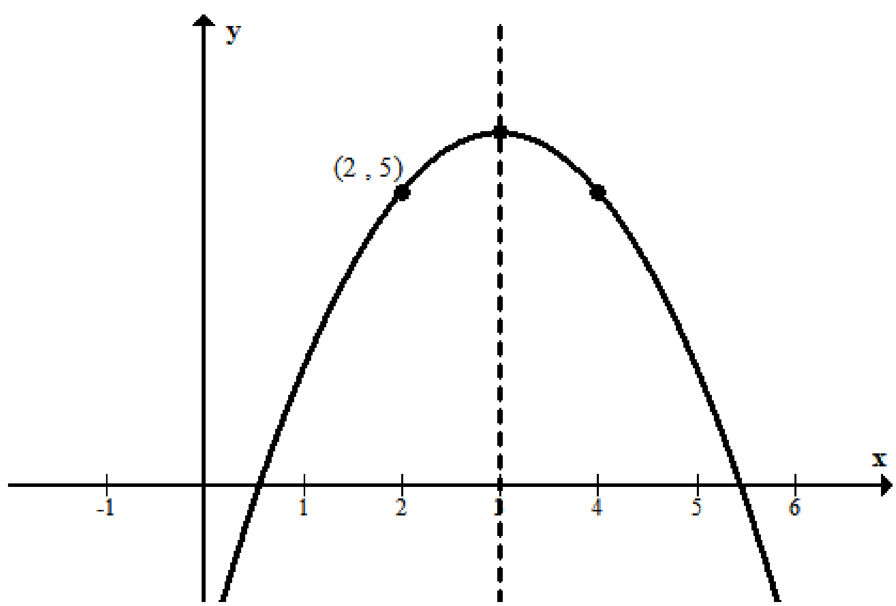
9. [Maximum mark: 4] ***[without GDC]***

The graph of the quadratic function below has a vertex at $x = 3$.



(a) Write down the value of $f'(3)$. [1]

(b) Given that $f(2) = 5$, write down the value of $f(4)$. [1]

(c) Given that $f'(2) = 2$, write down the value of $f'(4)$. [2]

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A. Exam style questions (SHORT)

10. [Maximum mark: 4] ***[without GDC]***

Let $f(x) = x^3 - 2x^2 - 1$.

(a) Find $f'(x)$ [2]

(b) Find the gradient of the curve of $f(x)$ at the point $(2, -1)$. [2]

11. [Maximum mark: 4] ***[without GDC]***

Let $f(x) = 6\sqrt[3]{x^2}$. Find $f'(x)$.

12. [Maximum mark: 6] ***[without GDC]***

Let $h(x) = \dfrac{6x}{\cos x}$. Find $h'(0)$

13. [Maximum mark: 6] ***[without GDC]***

Let $g(x) = 2x \sin x$.

(a) Find $g'(x)$. [3]

(b) Find the gradient of the graph of g at $x = \pi$. [3]

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14. [Maximum mark: 5] ***[without GDC]***

Let $f(x) = \dfrac{3x^2}{5x-1}$.

(a) Write down the **equation** of the vertical asymptote of $y = f(x)$. [1]

(b) Find $f'(x)$. Give your answer in the form $\dfrac{ax^2 + bx}{(5x-1)^2}$ where a and $b \in \mathbb{Z}$. [4]

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15. [Maximum mark: 6] ***[without GDC]***

Given the function $f(x)=x^2-3bx+(c+2)$, determine the values of b and c such that $f(1)=0$ and $f'(3)=0$.

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16. [Maximum mark: 7] ***[without GDC]***

Consider the function $f(x)=x\ln x-x$.

(a) Find $f'(x)$. [3]

(b) Find the coordinates of the point where the gradient of the curve is 1. [4]

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17. [Maximum mark: 5] ***[without GDC]***

Consider the curve of the function $f(x) = 2e^{x} - 4x + 1$.

Find the x-coordinate of the point where the gradient of the curve is 0.

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18. [Maximum mark: 8] ***[without GDC]***

Let $f(x) = \tan x$.

(a) Show that $f'(x) = \dfrac{1}{\cos^{2} x}$. [3]

(b) Show that the gradients of the curve of $f(x)$ at $x = 0$, $x = \dfrac{\pi}{4}$ and $x = \dfrac{\pi}{3}$ are in geometric sequence. [5]

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19. [Maximum mark: 5] ***[without GDC]***

Consider the function $f(x) = kx^3 - 30x + 1$, where k is a constant.

The gradient of the curve of $f(x)$ at $x = 2$ is 6. Find the value of k.

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20. [Maximum mark: 4] ***[without GDC]***

Consider the function $f(x) = k\sin x + 3x$, where k is a constant.

(a) Find $f'(x)$.

(b) When $x = \frac{\pi}{3}$, the gradient of the curve of $f(x)$ Is 8. Find the value of k.

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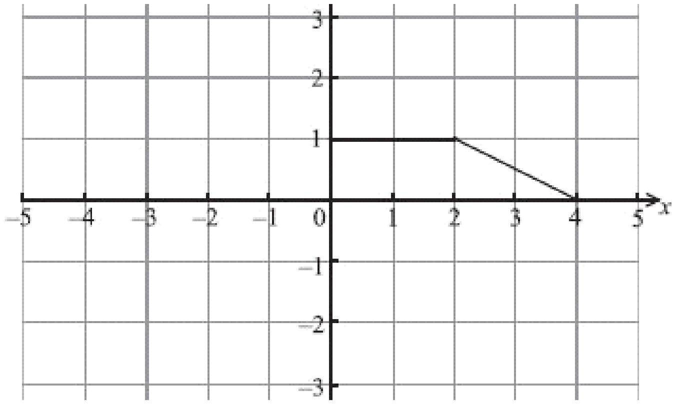
21. [Maximum mark: 4] ***[without GDC]***

The graph of the function $y = f(x)$, $0 \le x \le 4$, is shown below.



(a) Write down the value of (i) $f(1)$ (ii) $f(3)$

(b) Write down the value of (i) $f'(1)$ (ii) $f'(3)$

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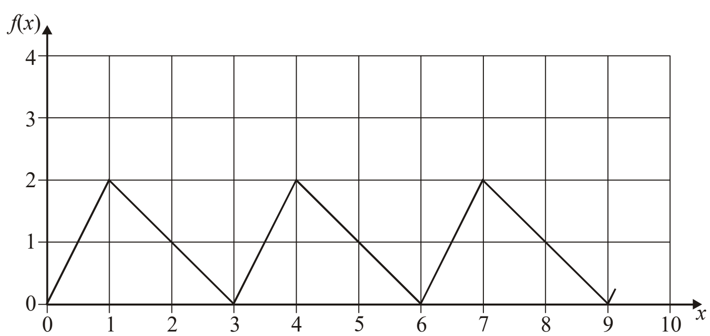
22. [Maximum mark: 6] ***[without GDC]***

Part of the graph of the periodic function f is shown below. The domain of f is $0 \le x \le 15$ and the period is 3.



(a) Find (i) $f(2)$ (ii) $f'(6.5)$ (iii) $f'(14)$

(b) How many solutions are there to the equation $f(x) = 1$ over the given domain?

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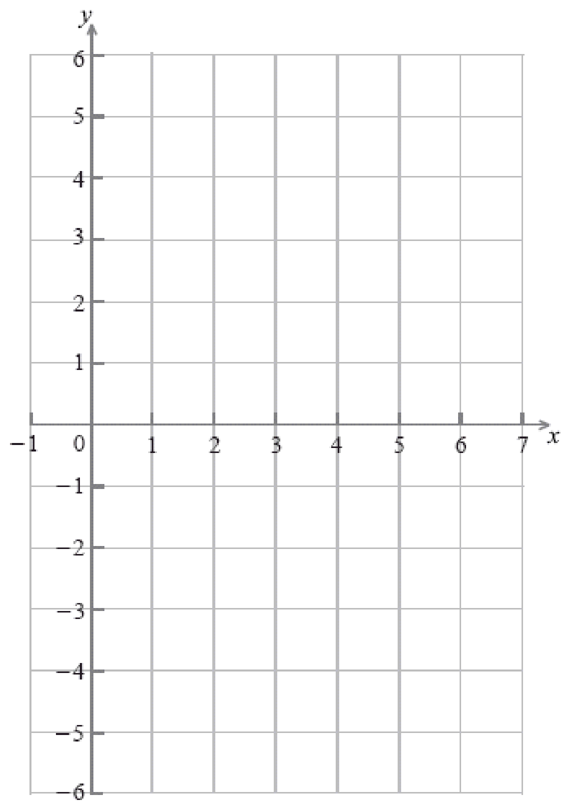
23. [Maximum mark: 8] ***[with GDC]***

Let $f(x) = x\cos x$, for $0 \le x \le 6$.

(a) Find $f'(x)$. [3]

(b) On the grid below, sketch the graph of $y = f'(x)$. [3]

(c) Write down the range of the function $y = f'(x)$, for $0 \le x \le 6$ [2]



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24. [Maximum mark: 6] ***[without GDC]***

Let $f(x) = ax^2 + bx + c$. Given that $f(0) = 2$, $f'(0) = -3$, $f''(0) = 6$, find the values of a, b and c

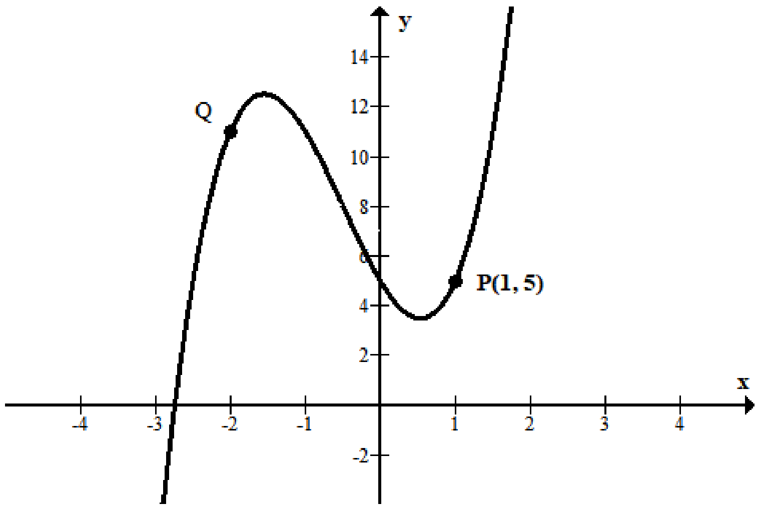
25*. [Maximum mark: 6] ***[without GDC]***

Let f be a cubic polynomial function. Given that $f(0) = 2$, $f'(0) = -3$, $f(1) = f'(1)$ and $f''(-1) = 6$, find $f(x)$.

B. Exam style questions (LONG)

26. [Maximum mark: 18] ***[without GDC]***

The following diagram shows part of the graph of the function $f(x)=2x^3+ax^2-5x+b$ and two points P(1,5) and Q which lie on this curve.



(a) Show that $a+b=8$. [2]

The gradient of the curve at point P is 7.

(b) Find an expression for $f'(x)$. [2]

(c) **Hence** show that $a=3$ and find the value of b. [3]

(d) Find the gradient of the curve at $x=0$. [1]

The gradient of the curve at point Q is equal to the gradient at point P.

(e) Find the coordinates of Q. [6]

(f) Find the equation of the line (PQ) in the form $y=mx+c$. [4]

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27*. [Maximum mark: 16] ***[without GDC]***

The n-th derivative of a function f is denoted by $f^{(n)}(x)$

(a) Let $f(x) = x^4$. Find the first four derivatives of f [i.e. up to $f^{(4)}(x)$]. [4]

(b) Let $g(x) = x^4 + ax^3 + bx^2 + cx + d$. Write down the value of $g^{(4)}(x)$. [2]

(c) Let $h(x) = x^m$. Find the value of $g^{(m)}(x)$ in terms of m. [2]

(d) Let $k(x) = \dfrac{1}{x}$

(i) Show that $k^{(4)}(x) = \dfrac{24}{x^5}$. [5]

(ii) Guess a formula for $k^{(n)}(x)$, the n-th derivative of $k(x)$. [3]

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