NEMATYC 2012 Conference
Math: Conquered in Concord
March 30-31, 2012

Title: GeoGebra for Everyone

Abstract: Geogebra is a free dynamic mathematics software package combining elements of dynamic geometry software (Sketchpad, Cabri, Cinderella) with elements of computer algebra systems (Maple, Mathematica, Maxima). Geogebra has interactive graphics, algebra and spreadsheet capabilities the can be used to improve the teaching of mathematics from elementary school through the university level. This software package can be used to create demonstrations, facilitate student experimentation and as an authoring tool for creating images and dynamic applets. In this presentation, I will demonstrate the range of Geogebra via applets suitable for algebra, geometry, trigonometry, statistics and calculus. Geogebra is for everyone!

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Saint Joseph College
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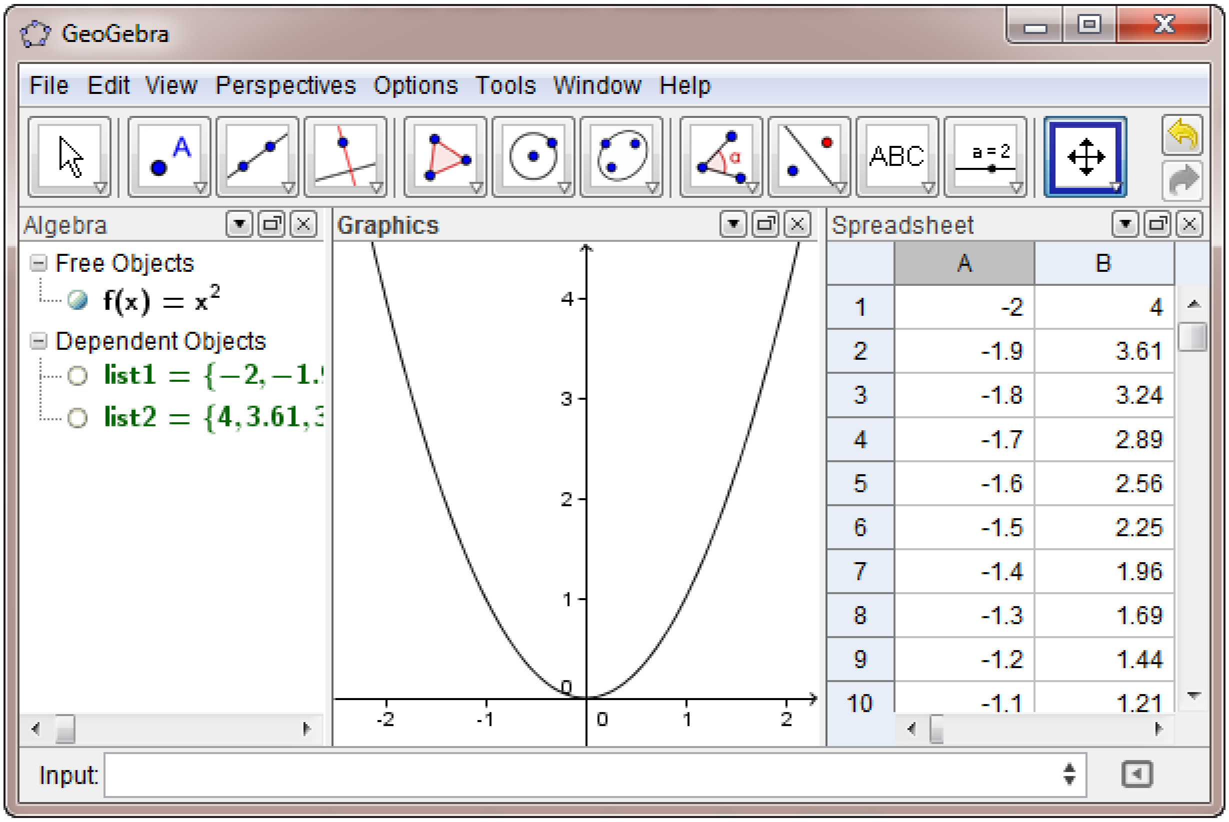
Introduction

In his book, The Third Industrial Revolution (2011), Jeremy Rifkin observes that fossil fuels (coal, oil, natural gas) are by their very nature located only in certain selected locations (Ghawar oil field in Saudi Arabia) and thus give rise to large centralized energy companies (ExxonMobil Corporation). In the future, he argues that sources of renewable energy (solar, wind, hydro, geothermal, biomass, ocean waves and tides) are far more disperse and will require a different economic model that he refers to as distributed capitalism. One consequence of this is a push towards a more lateral as opposed to hierarchical economic structures. For example, consider the changes in the computer software industry which was once completely dominated by the traditional top-down approach by companies such as Microsoft. This business model is being seriously challenged by open source software networks such as Linux, in which thousands of computer programmers collaborate to produce software used by millions. I have personally noticed that the quality of open source software is increasing and there are many open source programs which are very competitive with their commercial counterparts. Geogebra is only one small part of these larger trends.

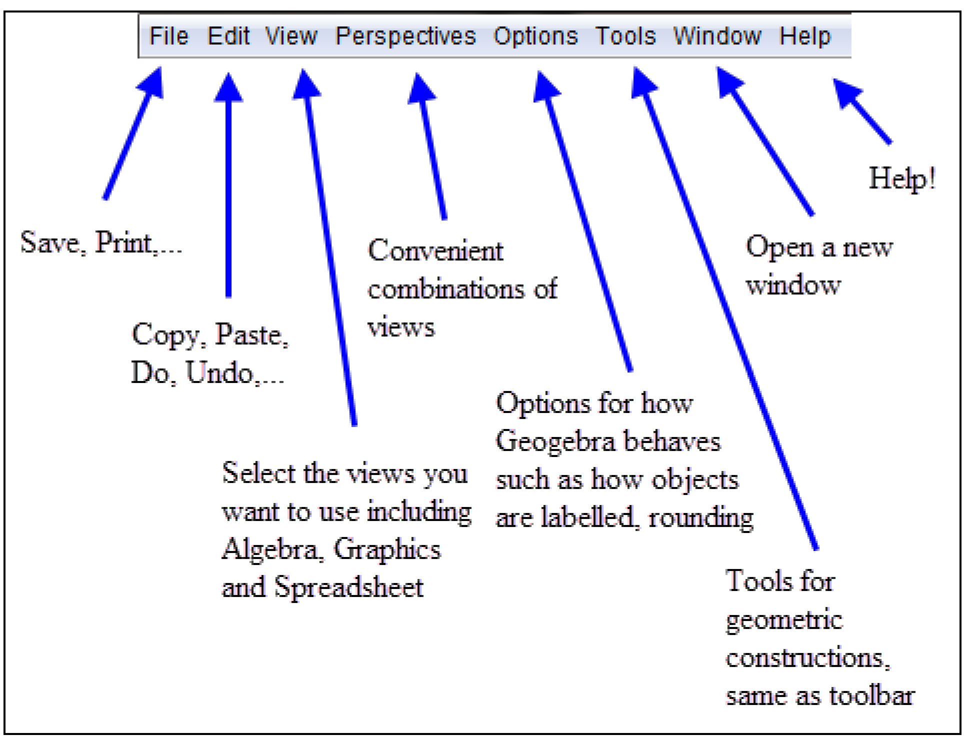
Geogebra was created by Markus Hohenwarter who began working on the project part of his Master's thesis in Computer Science and Mathematics Education at the University of Salzburg (Austria) in 2002. He designed the software to combine features of dynamic geometry software with features of computer algebra systems. Markus continued his project at Florida Atlantic University (2006–2008), Florida State University (2008–2009), and now at the University of Linz together with the help of open-source developers and translators all over the world.

Geogebra 4.0 fundamentals

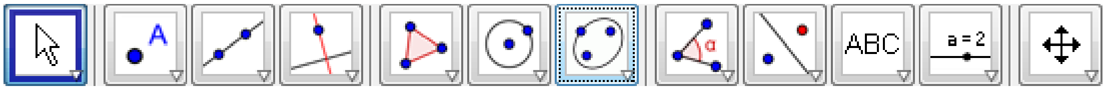
Figure: A parabola shown in in Algebra View, Graphics View and Spreadsheet View.



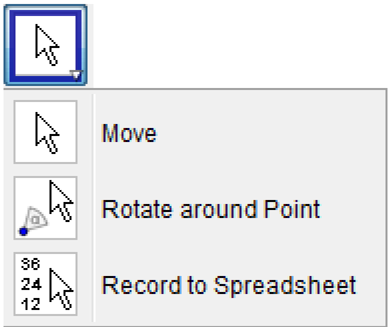
Menu



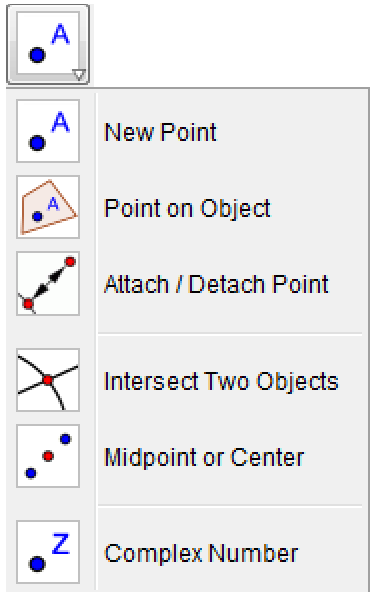
Toolbar



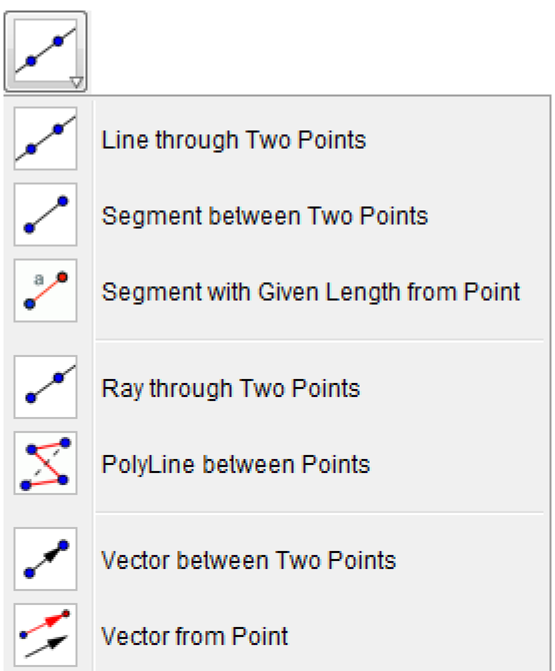
Move



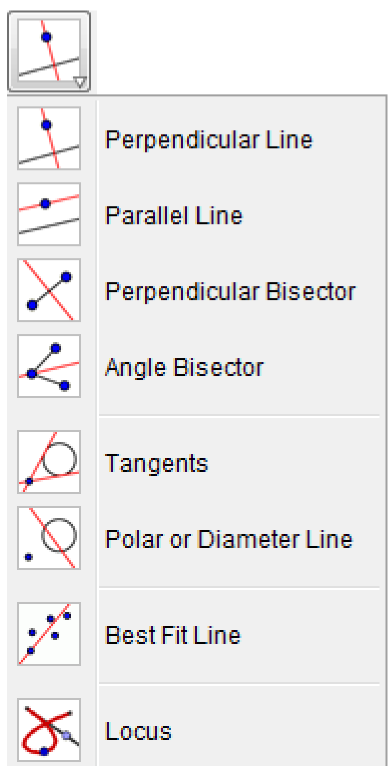
New Point



Line throught two points

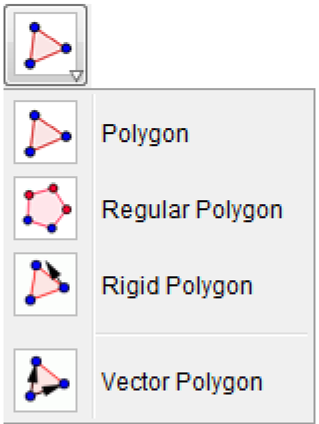


Perpendicular line

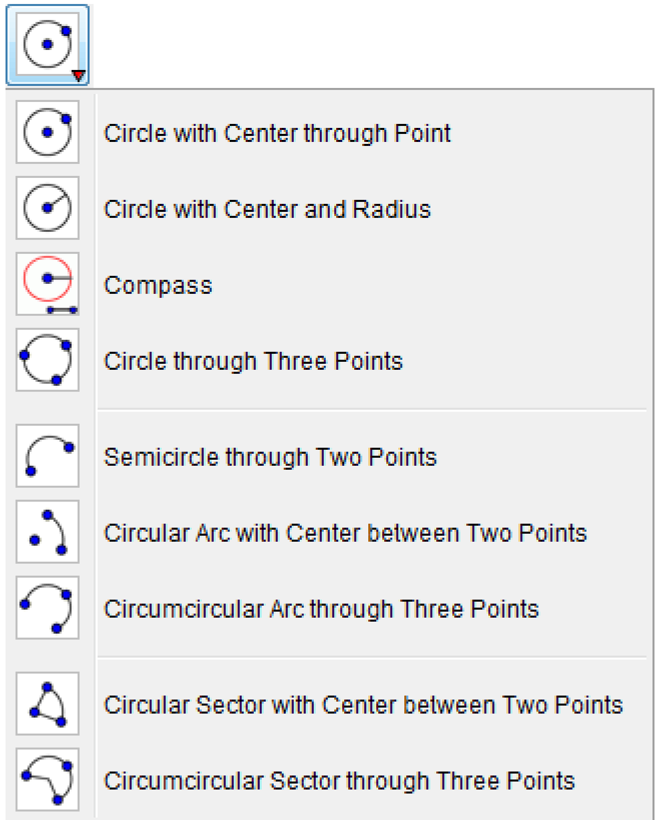


Comment: The Move tool is the most used tool and is used to interact with all of the objects in Algebra and Graphics View.

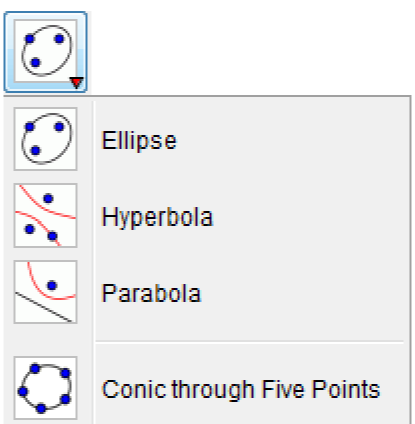
Polygon



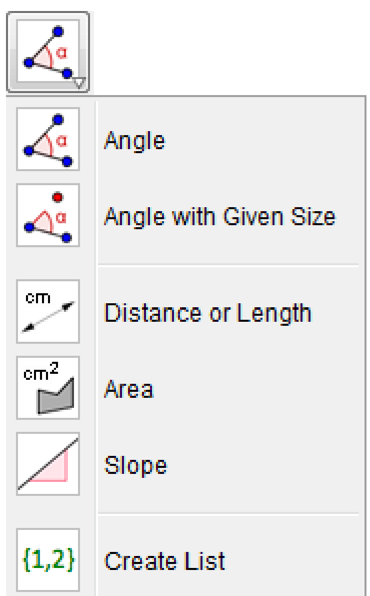
Circle with Center through Point



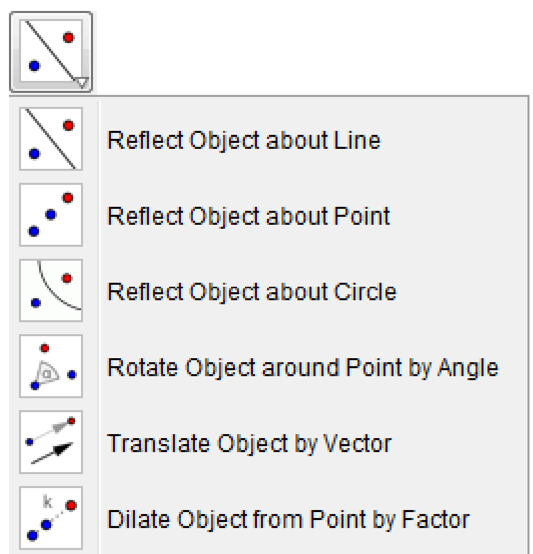
Ellipse



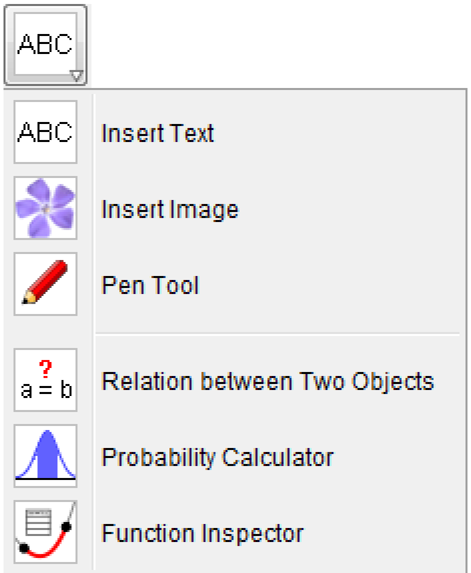
Angle



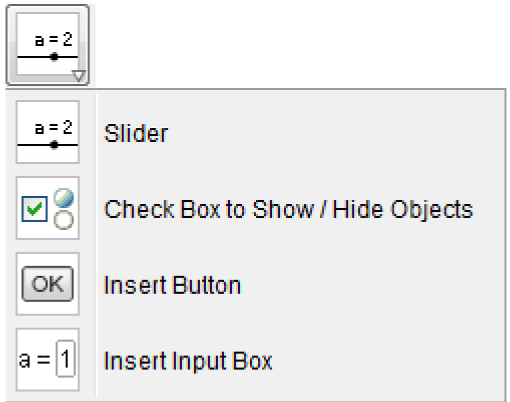
Reflect Object about Line



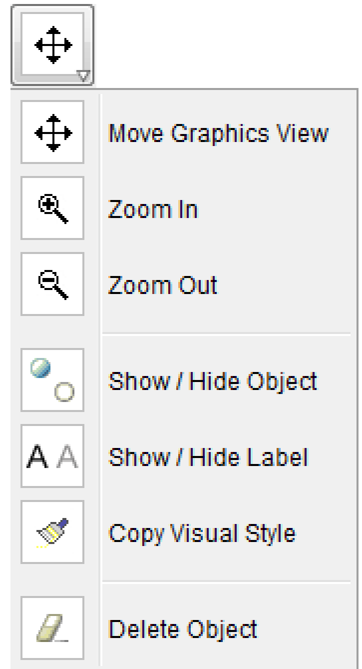
Insert text



Slider



Move Graphics View

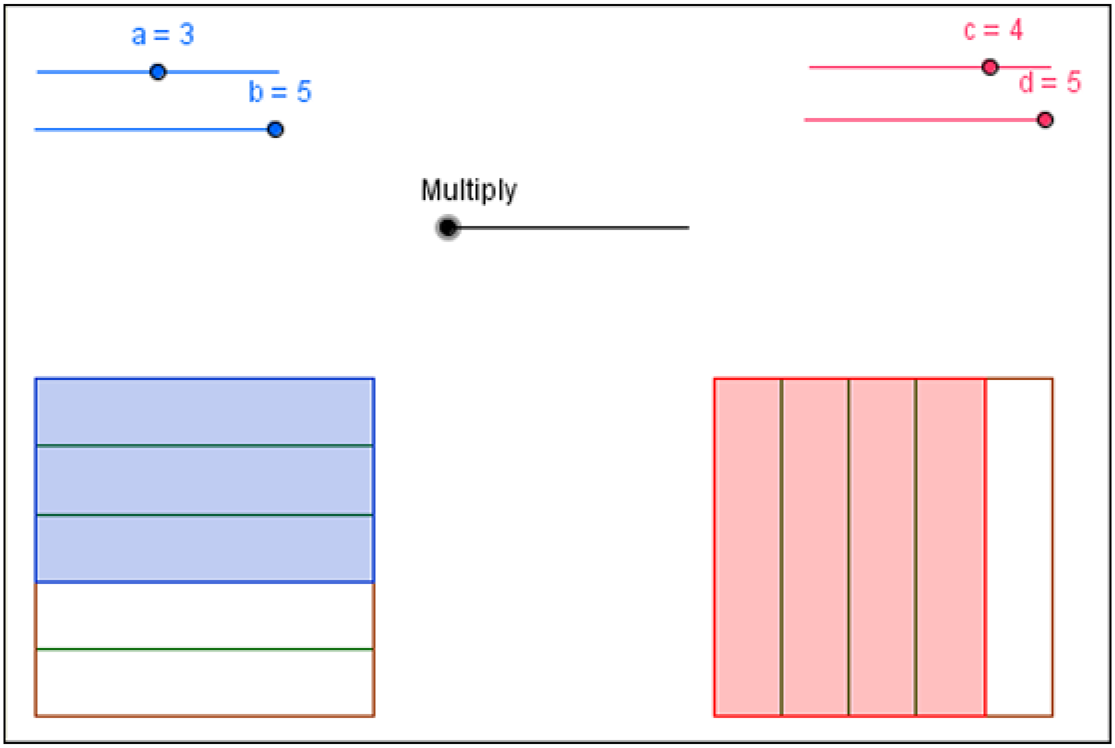


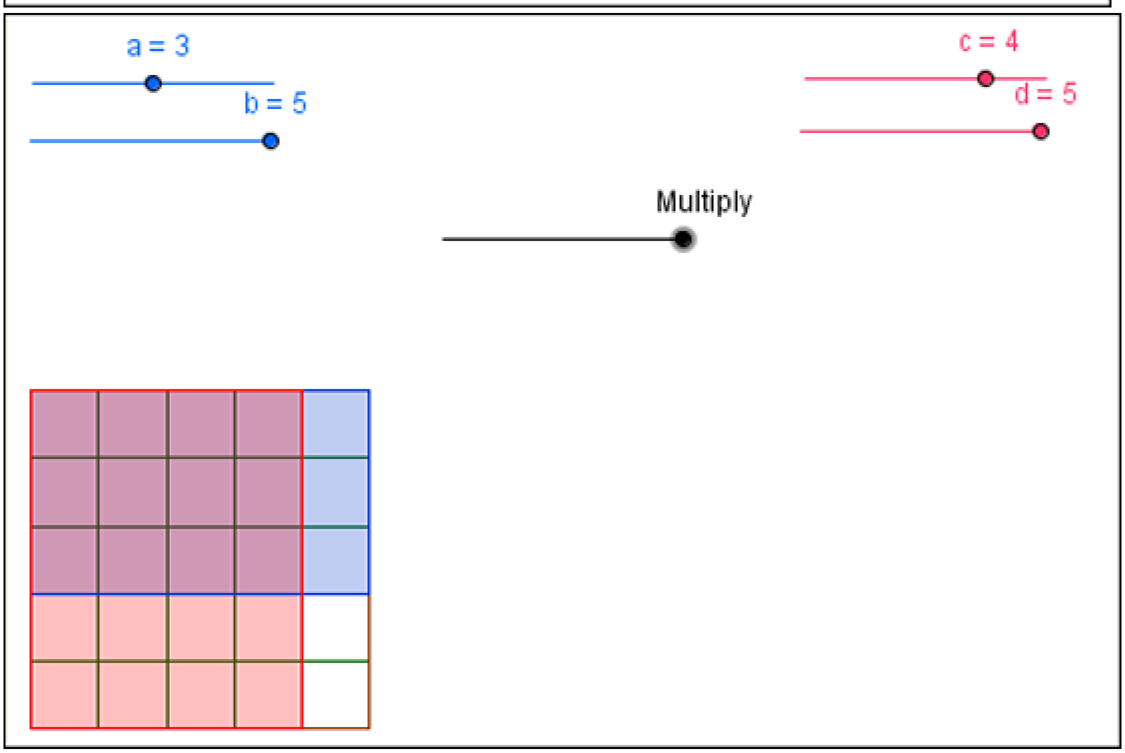
Input bar

Commands can be entered using the toolbars or by using the input bar. The toolbar is easier to use, but not as powerful as some commands can only be entered via the input bar.

Example 1: Multiplying fractions

Here is an applet for demonstrating how to multiply fractions. The first view shows the fractions before they are multiplied and the second view shows them after they are multiplied.





Directions

To create the multiplication applet, use the following steps. The bold commands should be typed into the Input bar.

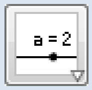
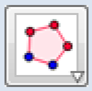
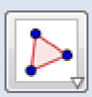
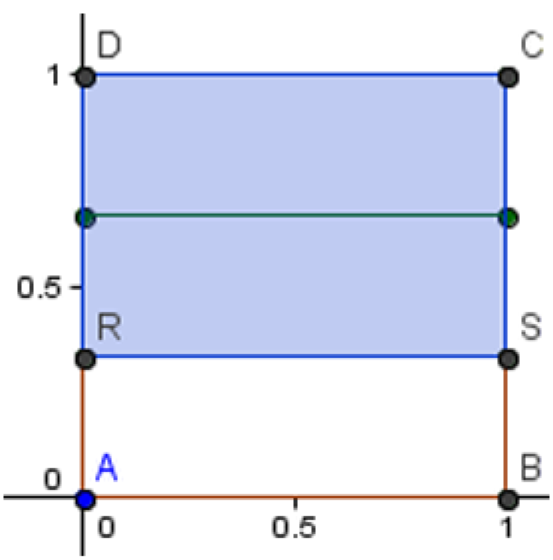
Step 1		**Create the sliders.** Sliders *a*, *b*, *c* and *d* vary from 1 to 5 in steps of 1. Slider *Multiply* will vary from 0 to 1 in steps of 0.1. See the dialog box below.
Step 2	**A = (0,0)** **B = A + (0,1)** 	**Create the first square.** Define the first square by creating points A and B. Use the regular polygon tool, select points A and B and enter 4 to create a regular polygon with 4 sides.
Step 3	**L1 = Sequence[A + k (D – A), k, 0, 1, 1/b]** **L2 = Sequence[B + k (C – B), k, 0, 1, 1/b]** **L3 = Sequence[Segment[Element[L1, k], Element[L2, k]], k, 2, b]**	**Divide the first square into horizontal strips.** Create a sequence of equally spaced points along the left side of the first square to split this side into segments of length 1/b. Repeat along the right side of the square. Then connect points in the first list with the points in the second list to create a sequence of equally spaced horizontal line segments.
Step 4	**R = D – (0,a/b)** **S = C – (0,a/b)** 	**Shade the horizontal strips corresponding to the first fraction.** Define points R and S. Then the polygon DCSR represents the fraction a/b. Use the polygon tool to create a polygon with vertices DCSR. Shade this polygon using a color of your choice.

Figure: First square



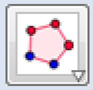
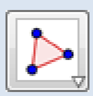
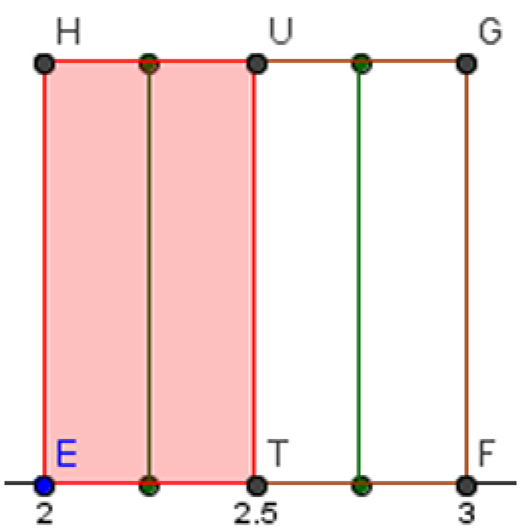
Step 5	**E = A + (2 – 2****Multiply***, 0)** **F = E + (1, 0)** 	**Create the second square.** Define the second square by creating points E and F. We define point E in terms of point A and the slider Multiply to allow us to move the second square so as to coincide with the first square. Use the regular polygon tool, select points A and B and enter 4 to create a regular polygon with 4 sides.
Step 6	**L4 = Sequence[H + k (G – H), k, 0, 1, 1/d]** **L5 = Sequence[E + k (F – E), k, 0, 1, 1/d]** **L6 = Sequence[Segment[Element[L4, k], Element[L5, k]], k, 2, d]**	**Divide the second square into vertical strips.** Create a sequence of equally spaced points along the top of the second square to split this side into segments of length 1/d. Repeat along the bottom of the square. Then connect points in the first list with the points in the second list to create a sequence of equally spaced vertical line segments.
Step 7	**T = E + (c/d, 0)** **U = H + (c/d, 0)** 	**Shade the vertical strips corresponding to the second fraction.** Define points T and U. Then the polygon HUTE represents the fraction a/b. Use the polygon tool to create a polygon with vertices HUTE. Shade this polygon using a color of your choice.

Figure: Second square

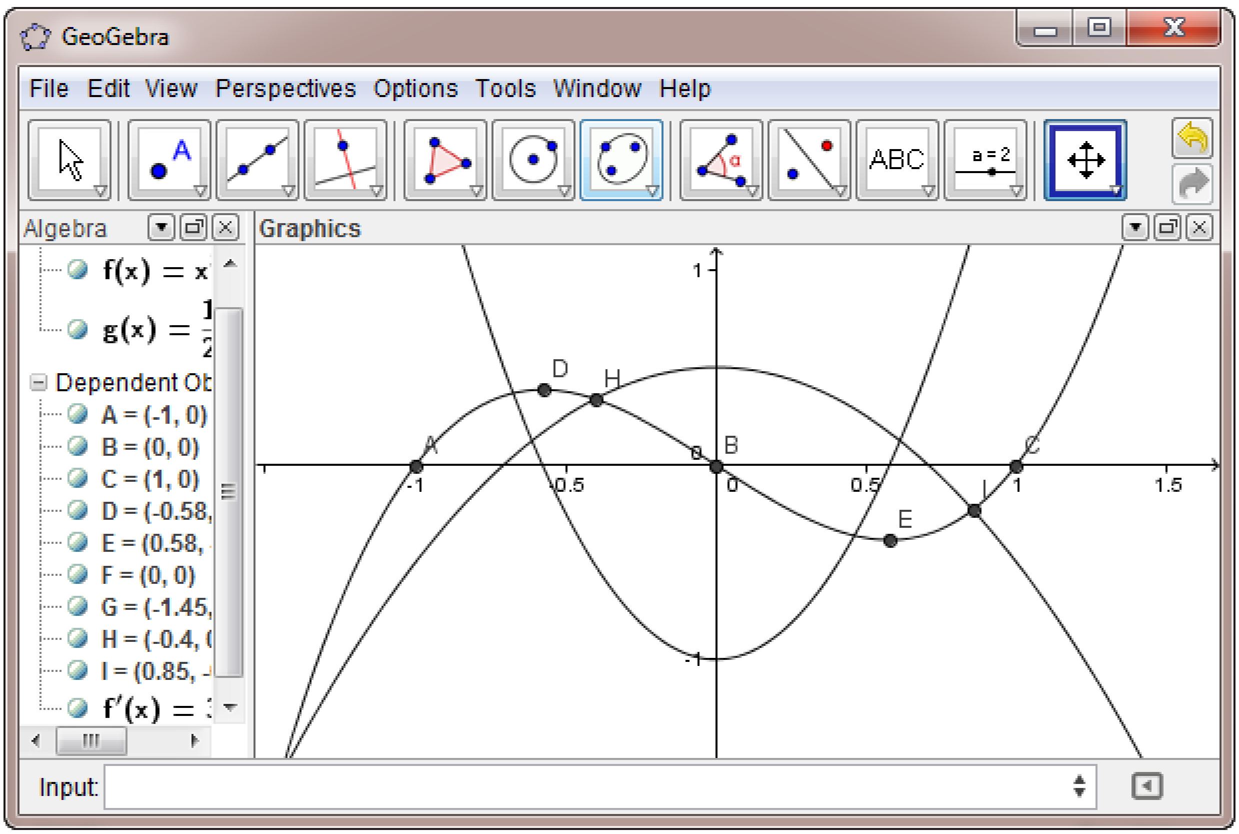


Example 2: Graphing

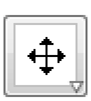
Functions

Graph functions by entering them into the Input bar. We can also study functions in a manner similar to that of a graphing calculator by using additional commands.

f(x) = x^3 - x
Root[f]
Extremum[f]
Factors[f]
InflectionPoint[f]
Derivative[f] OR f'(x)
g(x) = 1/2 - x^2
Intersect[f, g]



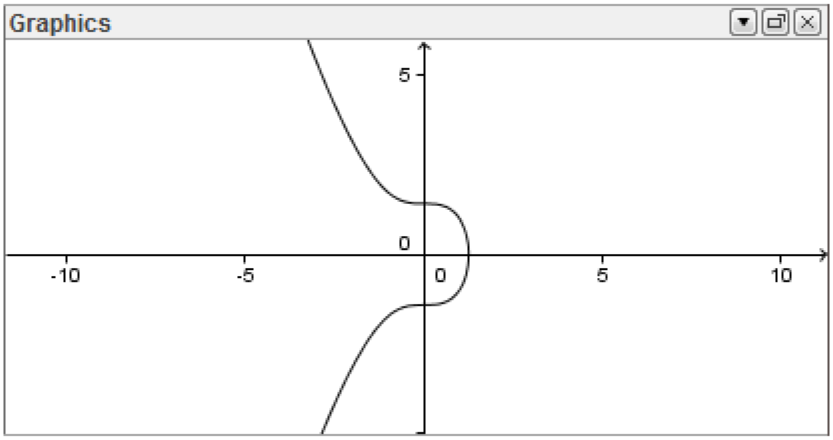
To restrict the domain of the graph of a function, use a command of the form
f(x) = Function[x^3 – x, -1, 1]. Use the Move Graphics View tool to move the graph and/or scale the *x*-axis or *y*-axis.



Implicit functions

Graph implicit functions by entering them into the Input bar. In the current version of Geogebra, only polynomials in *x* and *y* can be graphed.

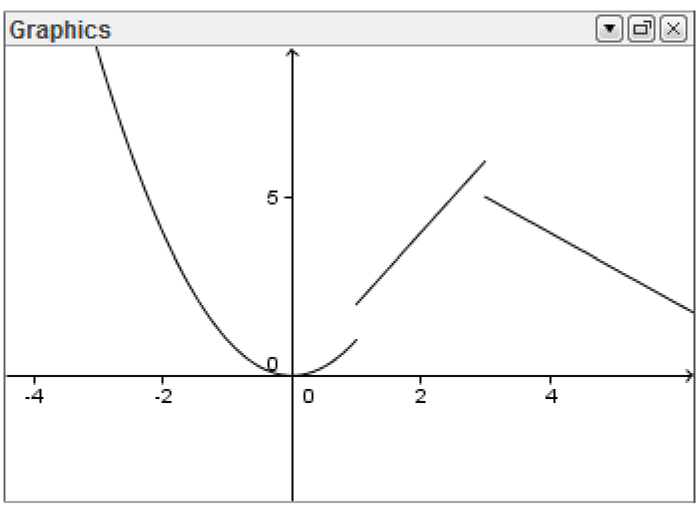
x^3 + y^2 = 2



Piecewise defined functions

The best way to graph piecewise defined functions is to use if statements. Enter the following command in the Input bar. If the piecewise function has more than two parts, use nested if statements,

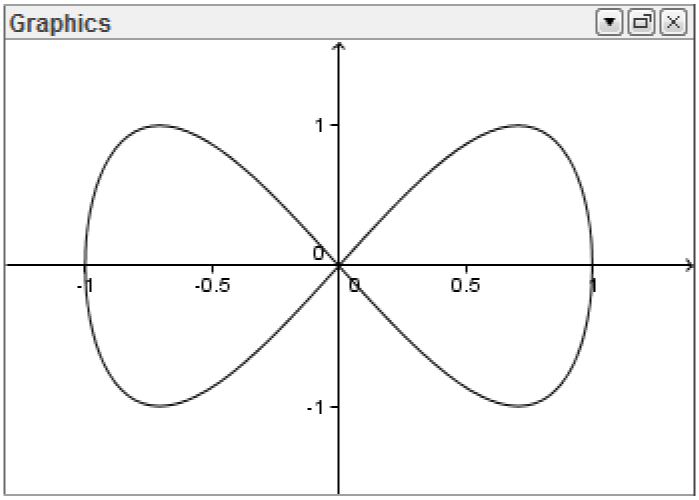
f(x) = If[x < 1, x^2, 2x]
f(x) = If[x < 1, x^2, If[x < 3, 2*x, 8-x]



Parametric equations

Graph parametric equations using the Curve command.

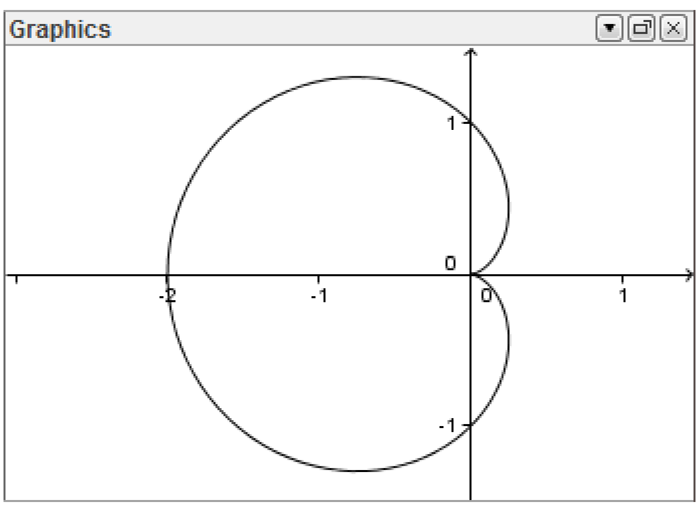
Curve[cos(t), sin(2*t), t, 0, 2*Pi]



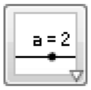
Polar coordinates

Polar curves can also be graphed by using the Curve command.

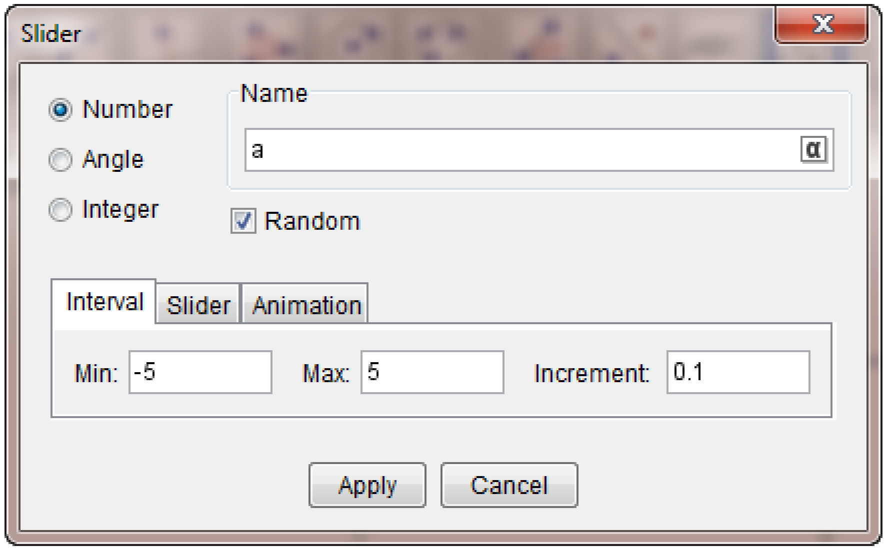
r(x) = 1 – cos(x)
Curve[r(t)*cos(t), r(t)*sin(t), t, 0, 2*Pi]



Sliders

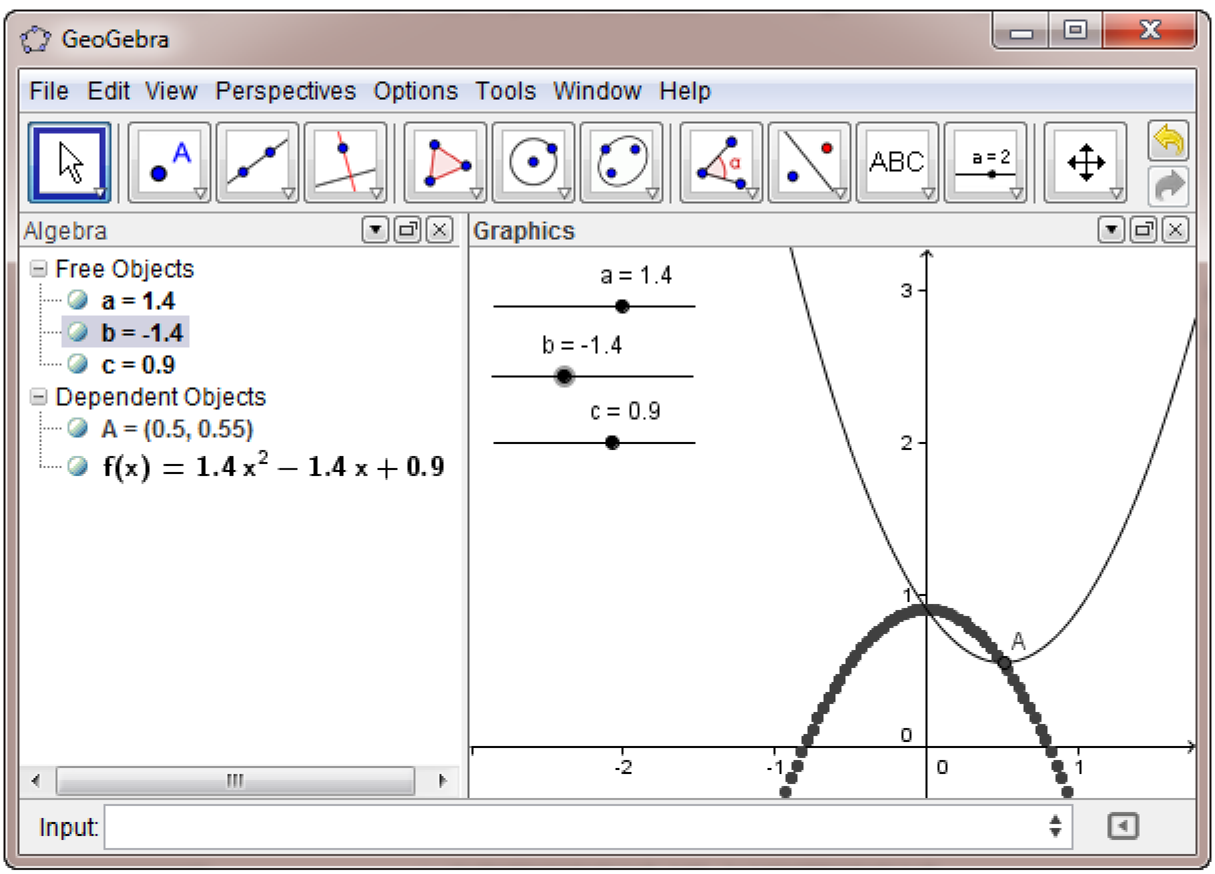


One of the more powerful features of Geogebra is the use of sliders. A slider is a visual representation of a number and can be used to perform parameter studies. To create slider, select the slider tool and click on the location where you wish to place the slider. When the Slider dialog box opens up, enter the name, minimum value, maximum value and the increment for the slider. Create sliders for parameters *a*, *b* and *c*.



Enter the formula for a quadratic function which utilizes these parameters. Then create a point for the vertex. If you wish to view the trace of this point, right-click on the point and select Trace On.

f(x) = a*x^2 + b*x + c
A = (-b/(2*a), f(-b/(2*a)))

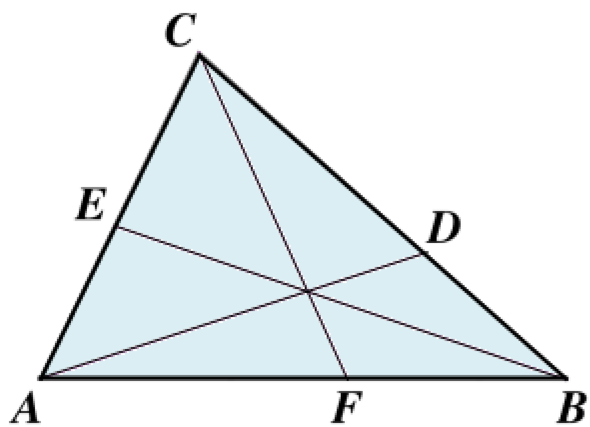


Example 3: Ceva's Theorem

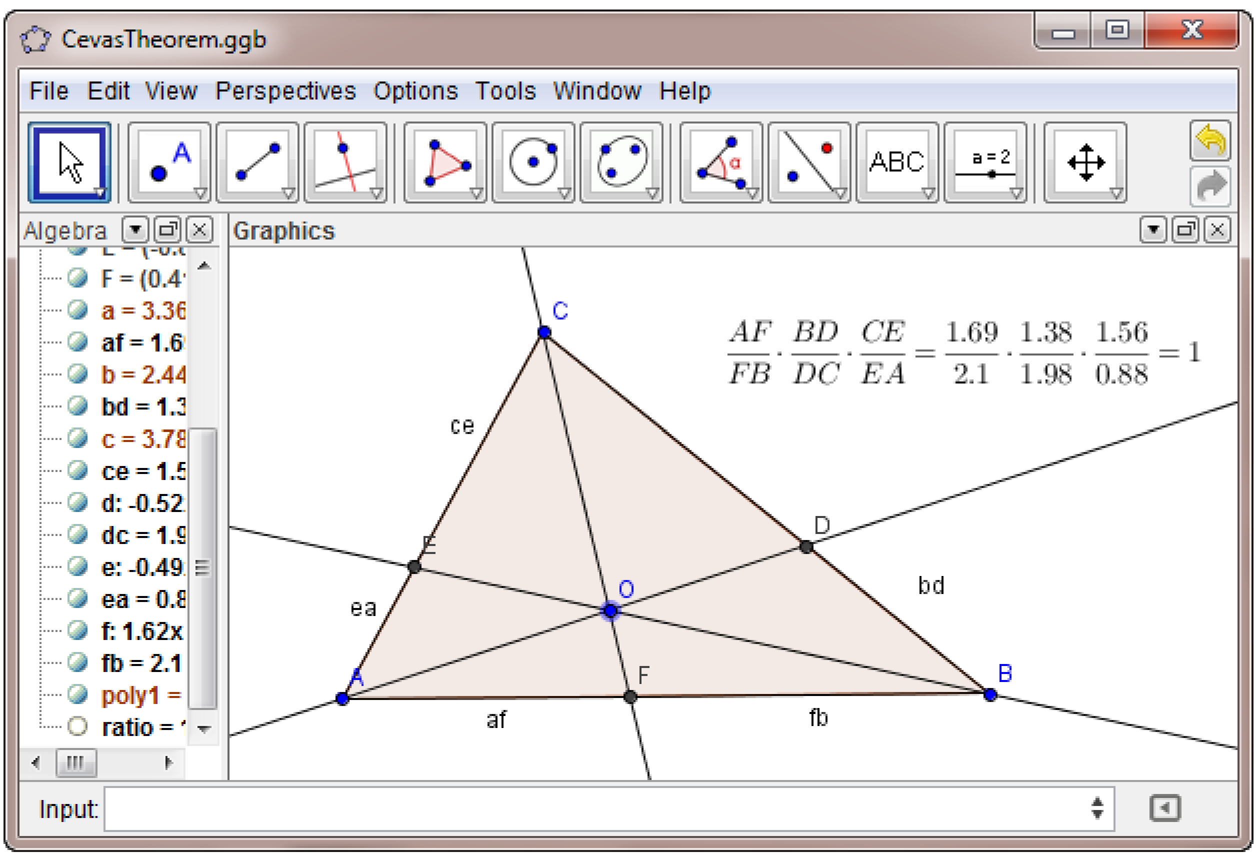
Geogebra is similar to Geometer's Sketchpad with regards to constructing geometric figures. However, Geometer's Sketchpad stays closer to the straightedge and compass constructions of classical Greek geometry.

Ceva's theorem: If the points D, E and F are on the sides BC, AC and AB of triangle ABC, then then the lines AD, BE and CF are concurrent if and only if

$$\frac{AF}{FB}\cdot\frac{BD}{DC}\cdot\frac{CE}{EA}=1$$



Move points A, B and C to change the triangle. Move the point O anywhere within the triangle.   What do you observe about the ratio?



Directions

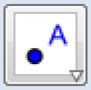
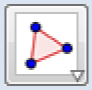
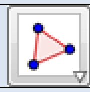
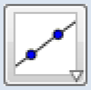
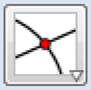
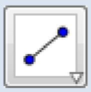
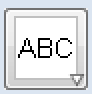
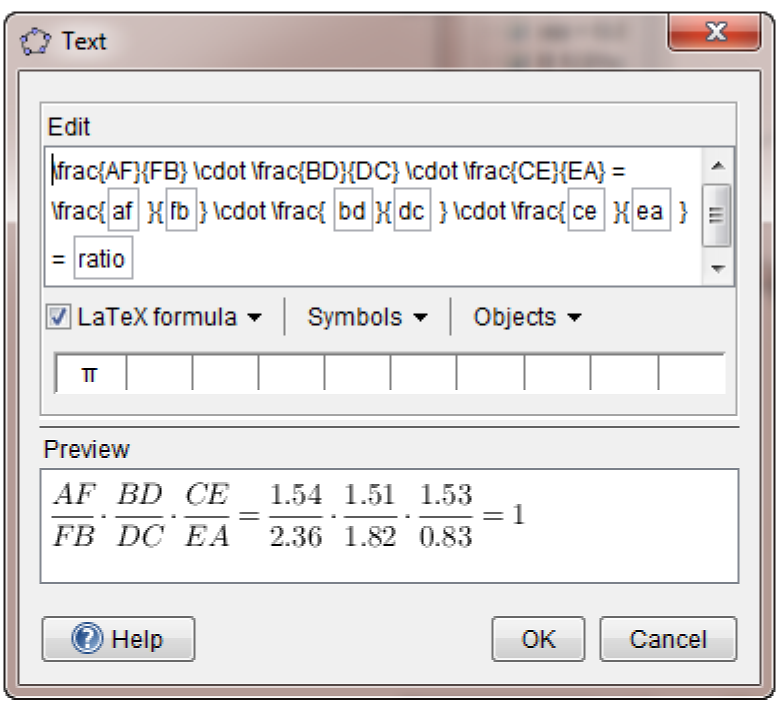
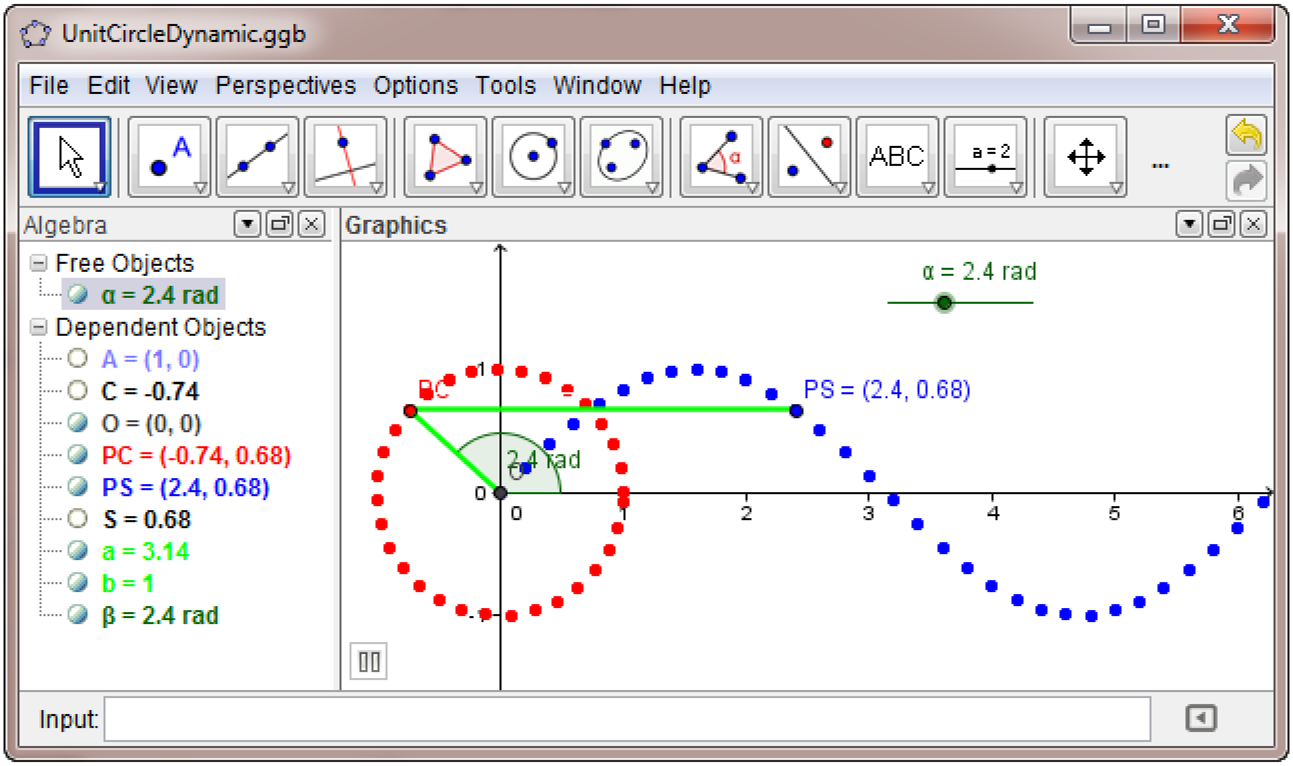
Step 1	 	**Create triangle ABC.** Use the New Points tool to create points A, B and C. Then use the polygon tool to connect the points.
Step 2		**Create point O.** Use the New Points tool to create a point in the interior of the triangle.
Step 3		**Create the Cevians.** Use the Line Through Two Points tool to create lines passing through the vertices of the triangle through point O.
Step 4		**Label points D, E and F.** Use the Intersect Two Objects tool (located under the New Point tool) to find the intersection of the cevians and the sides of the triangle.
Step 5		**Create line segments af, fb. bd, dc, ce and ae.** Use the Segment between Two Points tool (located under the Line through Two Points tool).
Step 6	**ratio = (af/fb)(bd/dc)(ce/ea)**	**Compute Ceva's ratio.** Enter the formula for Ceva's ratio in the Input bar.
Step 7		**Add the dynamic text.** Add the text to show the value of the ratio and the values of each number within the ratio. The fractions are created using LaTeX. The lengths of the line segments in Ceva's ratio are entered by clicking on them in either Algebra or Graphics View.

Figure: Text dialog box showing the LaTeX code. Compare the input in the Edit section with the output in the Preview section to better understand this code.



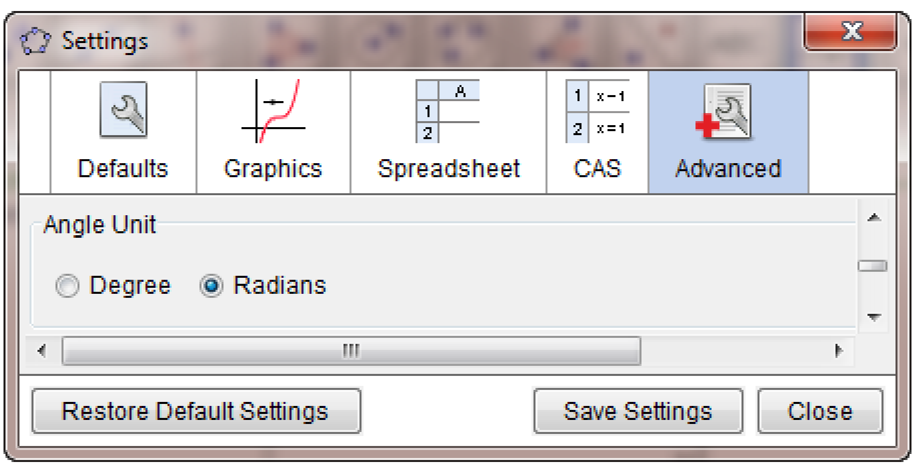
Example 4: Unit circle

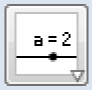
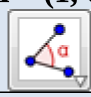
We can use Geogebra to animate the unit circle definition of the trigonometric functions. Here is an animation of the sine function.

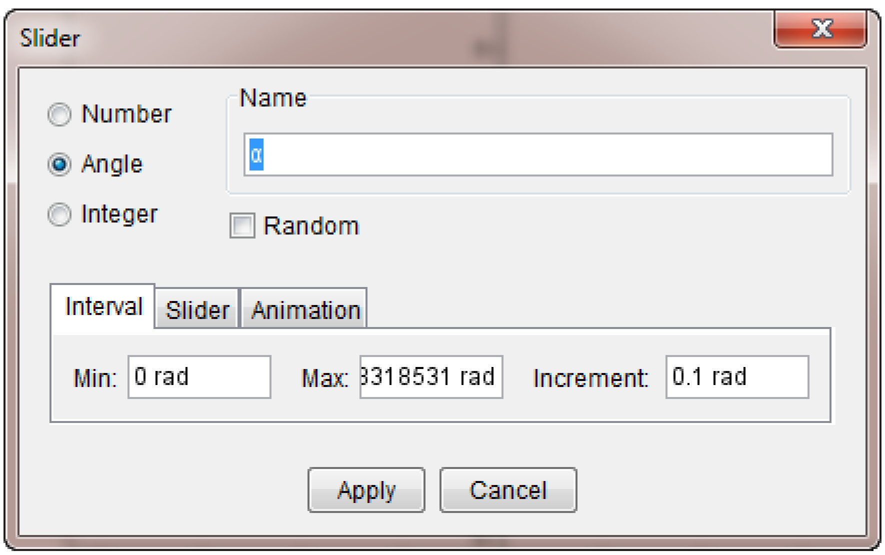


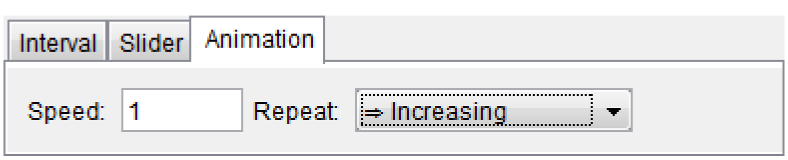
Directions

If you wish to measure the angle in radians, use Options/Settings, select the Advanced tab and then set the angle unit to radians.



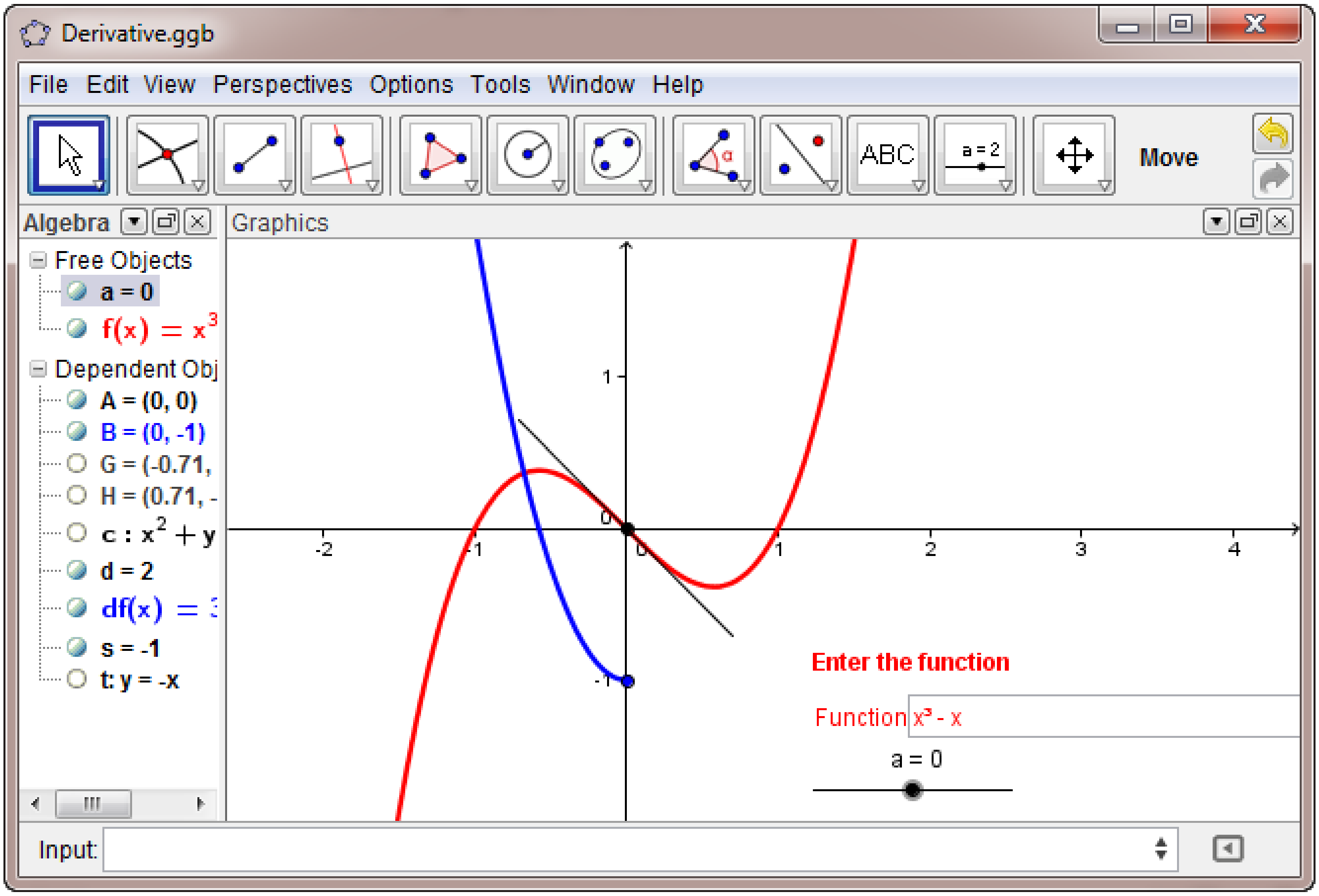
Step 1		**Create slider for an angle α.** Use the Slider tool to create slider for the angle α. Set the increment to 0.2. Select the Animation tab and set Repeat to increasing. See the diagrams below.
Step 2	**C = cos(α)**	**Create a number for cosine.**
Step 3	**S = sin(α)**	**Create a number for sine.**
Step 4	**PC = (C, S)**	**Create point PC on the unit circle.** Right-click on the point and use object properties to adjust the size and the color. Also, right click to turn on the tracing feature.
Step 5	**PS = (α, S)**	**Create a point PS on the sine function.** Right-click on the point and use object properties to adjust the size and the color. Also, right click to turn on the tracing feature.
Step 6	**O = (0, 0)** **A = (1, 0)** 	**Measure the angle.** Create points O and A. The point O is at the origin and A is the point where the unit circle intersects the positive *x*-axis. Then use the Angle tool to measure the angle defined by points A, O and PC.
Step 7	**Segment[O, PC]** **Segment[O, A]**	**Add some optional line segments.** To help guide the eyes while viewing the animation, use the Segment between Two Points tool (under the Line through Two Points) tool to create a line segment joining points O and PC and a line segment joining points PC and PS. Right-click on the line segments and use object properties to adjust their thickness and color.
Step 8		**Start the animation.** Right-click on the slider and select Animation On to start the animation.



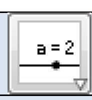
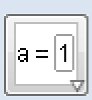


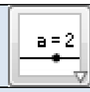
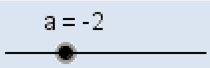
Example 5: Sketching derivatives

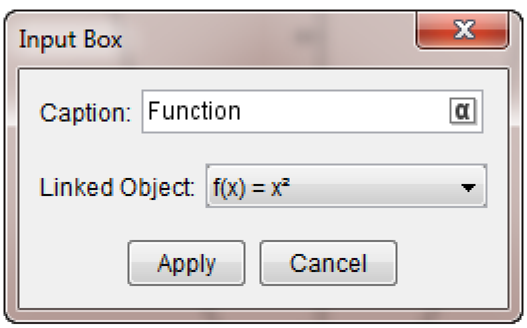
We can use Geogebra to animate the process of sketching the derivative of a function. In this case, we add an Input box to allow the user to easily enter a function of their choice.



Directions

Step 1		**Insert text.** Use the Insert Text tool to add the text “Enter the function”.
Step 2	**f(x) = x^2**	**Create a function.** Enter a function of your choice.
Step 3		**Create an input box.** Use the Insert Input Box tool (under the Slider tool) to create an input box so as to allow the user to enter a function of their choice. Provide a caption of your choice and use the drop down menu to select the function you have created as the Linked Object. When the user enters a function in the input box, it will overwrite the original function. See the diagram below these directions.

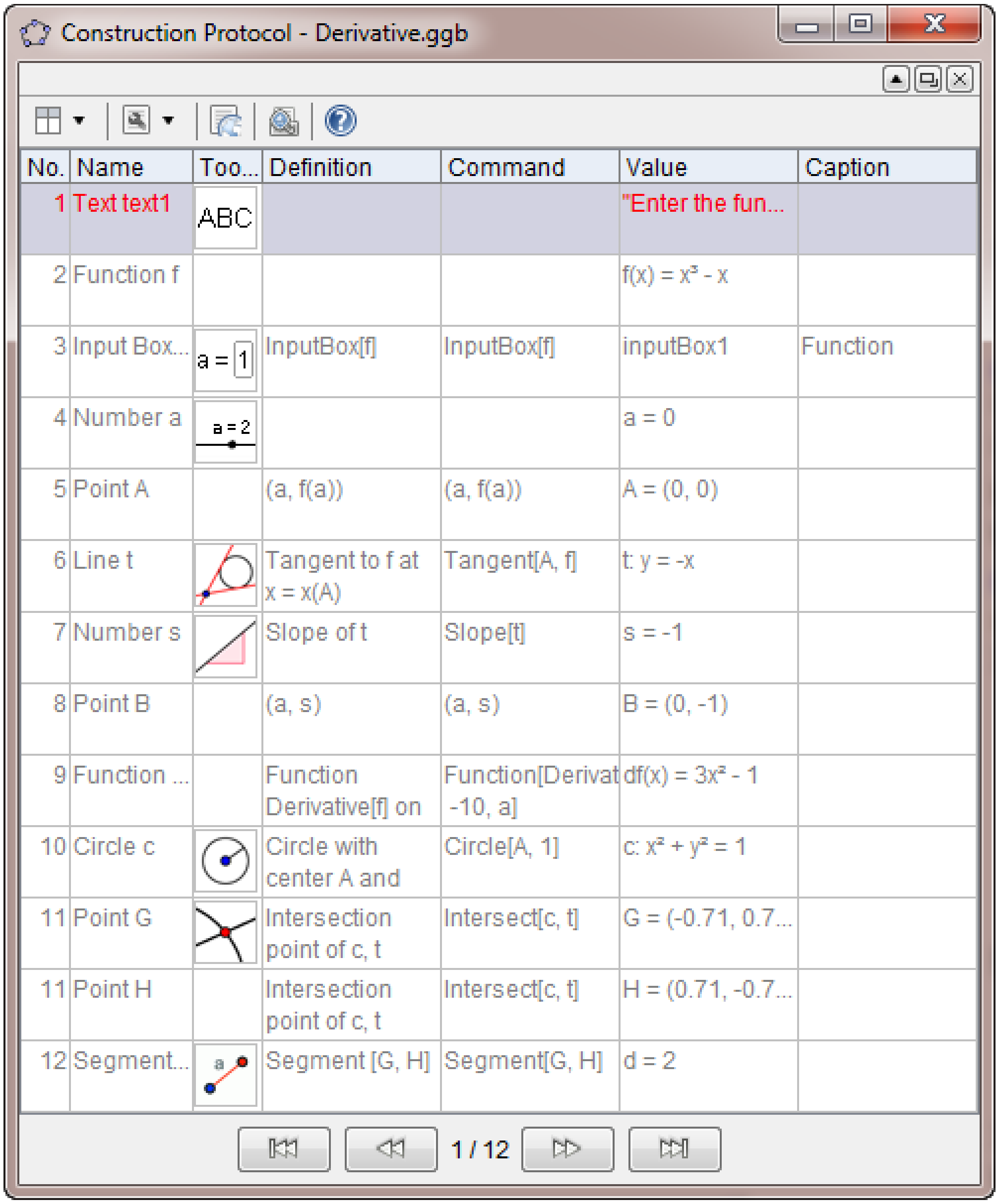
Step 4		**Create slider a.** This slider will control the location of a point on the graph of the function.
Step 5	**A= (a, f(a))**	**Create a point on the graph of the function.** This point can be moved by using the slider.
Step 6	**t = Tangent[A, f]** **s = Slope[t]**	**Construct a tangent line at point A.** Also, compute the slope of the tangent line.
Step 7	**df(x) = Function[Derivative[f],-10,a]**	**Construct the derivative.** Use the Function command to restrict the domain over which the derivative is visible. The left endpoint of -10 may need to be adjusted depending on the view you wish to achieve. The right endpoint is controlled by the slider a.
Step 9		**Make the applet beautiful.** Do not neglect this important step! Right-click on the objects you wish to modify to change their colors and sizes. In the applet shown above, I have hidden the tangent line and added a line segment. To hide an object, right-click on the object and select Show Object. If you repeat this step, the object will be made visible again.
Step 10		**Start the animation.** Right-click on the slider and select Animation On to start the animation.



Comments and resources

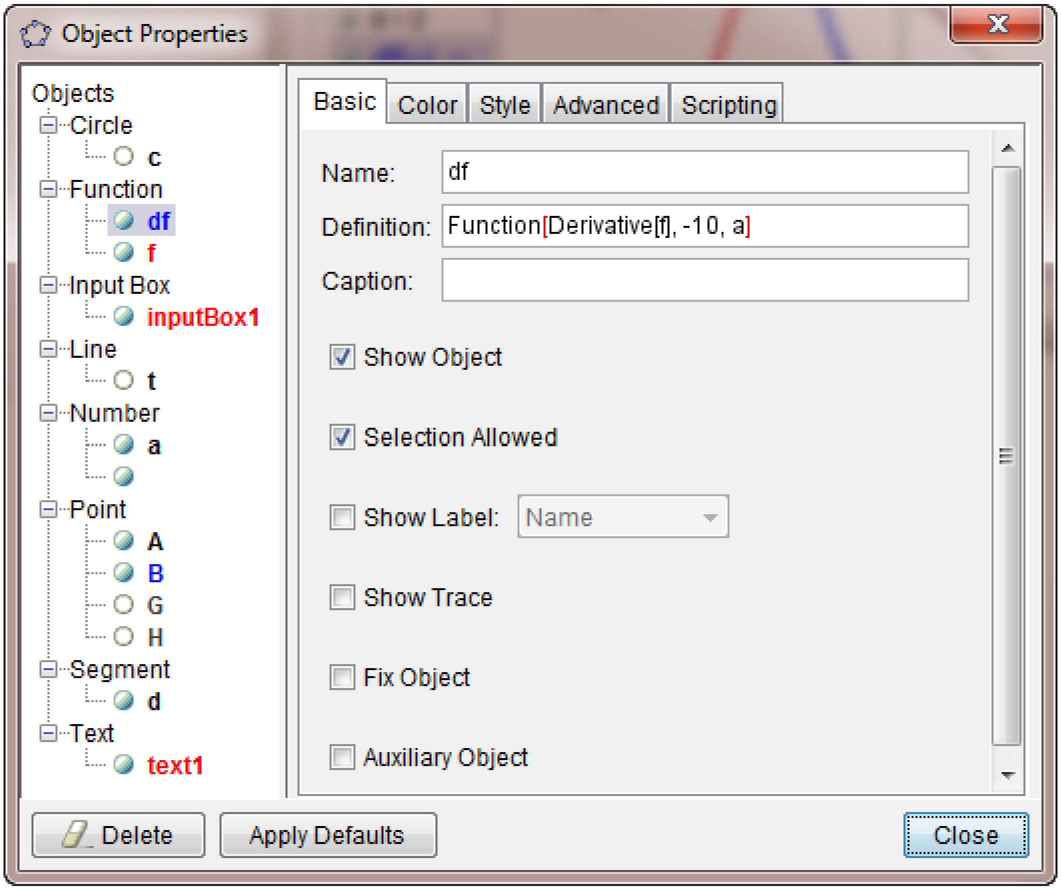
1. To download Geogebra and view the official documentation, go to http://www.geogebra.org/cms/.
2. For access to a wide range of Geogebra applets, visit http://www.geogebratube.org/.
3. For a series of outstanding tutorials on Geogebra, visit http://mathandmultimedia.com/ and select the Geogebra tab.
4. Another nice collection of applets can be found at http://media.lanecc.edu/users/gettyst/GeoGebraDemonstrations.html.
5. In future versions of Geogebra, there will be a Computer Algebra System (CAS) View and a 3D Graphics View.
6. Once you have obtained a Geogebra applet, you can deconstruct it to learn how it was made. To do this, use View/Construction Protocol. For example, the construction protocol for the

sketching derivatives applet above is shown below. By using the forward and reverse buttons at the bottom of this dialog box, you can watch the applet being constructed step by step.



No.	Name	Too...	Definition	Command	Value	Caption
1	Text text1	ABC			"Enter the fun...	
2	Function f				$f(x) = x^3 - x$	
3	Input Box...	a = 1	InputBox[f]	InputBox[f]	inputBox1	Function
4	Number a	a = 2			a = 0	
5	Point A		(a, f(a))	(a, f(a))	A = (0, 0)	
6	Line t		Tangent to f at x = x(A)	Tangent[A, f]	t: y = -x	
7	Number s		Slope of t	Slope[t]	s = -1	
8	Point B		(a, s)	(a, s)	B = (0, -1)	
9	Function ...		Function Derivative[f] on	Function[Derivat -10, a]	$df(x) = 3x^2 - 1$	
10	Circle c		Circle with center A and	Circle[A, 1]	$c: x^2 + y^2 = 1$	
11	Point G		Intersection point of c, t	Intersect[c, t]	G = (-0.71, 0.7...	
11	Point H		Intersection point of c, t	Intersect[c, t]	H = (0.71, -0.7...	
12	Segment...	a	Segment [G, H]	Segment[G, H]	d = 2	

7. If you wish to see how objects are defined and/or modify any of its properties, right-click on the object and select Object Properties.



8. The complete list of the available commands in Geogebra is on the website http://wiki.geogebra.org/s/en/index.php?title=Category:Commands&until=Maximize+Command This list allows one to quickly assess the full scope of the program. If a command seems interesting, one can look up how to use it.

9. There are many other open source software packages useful in mathematics. Here is a brief list.
   a. Computer algebra systems (Axiom, Maxima, Sage)
   b. Dynamic Geometry (Cinderella, GEONET)
   c. Statistics (R)
   d. Vector Graphics (Inkscape)
   e. Images (GIMP)