21. Calculate standard deviation and coefficient of variation from the following data.

Sales	40-50	50-60	60-70	70-80	80-90
No of shops	3	5	8	7	2

Group - 'D' [4× 5 = 20]

22. Optimize the objective function F= 4x + 9y under the constraints: x+ y ≤ 6, x − y ≥ - 2, x ≥ 0, y ≥ 0. Also find the difference between maximum and minimum values.

23. The three points (1, 2), (3, 4) and (5, 2) are located at the circumference of a wheel such that the distance from the fixed point to these points is always equal. Find the co-ordinates of the fixed point and then derive the equation representing the locus that contains all three points.

24. Prove by vector method that the mid-point of the hypotenuse of a right- angled triangle is equidistance from the vertices.

25. A triangle ABC with vertices A(2 , 4), B(2 , 2) and C(6 , 2) is reflected in the line y = -x and then the image so obtained is rotated through half turn in a clock direction about center at origin. Write the coordinates of the images of the vertices of the Δ ABC and present each of them in the same graph paper. Also, state the single transformation equivalent to the combination of two transformations.

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Students are encouraged to write answers in their own words as far as practicable. The figures in the margin indicate the full marks.
Attempt all the questions

Group - 'A' [10× 1 = 10]

1. a. Define trigonometric function.
   b. What is the nth term of an A.P. with common difference 'd', number of terms 'n' and first term 'a'.? Write it.

2. a. Write the mathematical notation $\lim_{x \to a^+} f(x)$ in a sentence.
   b. What is the determinant of identity matrix of order 2×2? Write it.

3. a. Write the condition of co-incident of the lines represented by $ax^2 + 2hxy + by^2 = 0$.
   b. If the intersection plane is parallel to the generator of a right circular cone, which conic does it form? Write it.

4. a. Express Sinθ + Sinα in the product form of Sine and Cosine.
   b. Express Cos3β in terms of Cosβ.

5. a. Write the formula to find the angle between two vectors $\vec{a} \ and \ \vec{b}$.
   b. If the radius of the adjoining inversion circle is 8 cm, what is the value of OP× O P' ? Write it.
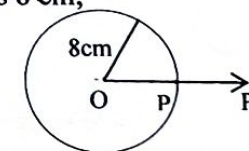
Group - 'B' [13× 2= 26]

6. a. If the function $f = \{x, 2x − 7\}$, find the $f^{-1}(x) \ and \ f^{-1}(−7)$.
   b. Find the value of k if (x − 2) is a factor of $x^3 + 6x^2 − kx − 30$.
   c. Find the quotient and remainder when $3x^3 + 7x^2 + 9$ is divided by x + 3 using synthetic division method.

7. a. For what value of x, does the inverse of the matrix A = $\begin{pmatrix} x-2 & -3 \\ x & x+2 \end{pmatrix}$ not exist?

b. Find the inverse of the matrix M = $\begin{bmatrix} 13 & 9 \\ 10 & 7 \end{bmatrix}$.

8. a. Find the value of k so that the lines represented by equation $k^2x^2 + 10xy + (k-2)y^2 = 0$ are perpendicular.

b. If two lines $ax - by + c = 0$ and $px + qy + r = 0$ are parallel to each other, show that $aq + bp = 0$.

9. a. If $\cos\theta = \dfrac{\sqrt{3}}{2}$, find the value of $\cos 2\theta$.

b. Prove that $\dfrac{\sin\frac{\theta}{2} - \sqrt{1+\sin\theta}}{\cos\frac{\theta}{2} - \sqrt{1+\sin\theta}} = \cot\frac{\theta}{2}$.
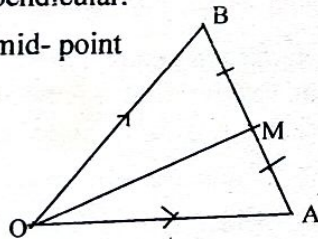
c. Solve: $2\sin\theta - \sqrt{3} = 0$ $(0 \le \theta \le 180°)$

10. a. Find the value of 'k' so that the vectors $\vec{a} = \begin{pmatrix} k+4 \\ -3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 3 \\ 2k \end{pmatrix}$ are mutually perpendicular.

b. In a triangle OAB, M is the mid-point of AB. If $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$,

Show that $\overrightarrow{OM} = \dfrac{\vec{a}+\vec{b}}{2}$.

c. If the lower and upper quartiles of a data are 40 and 60 respectively, calculate quartile deviation and its coefficient.

Group - 'C' [11× 4= 44]

11. If $f(x) = 3x - 7$, $5\,g(x) = x + 2$ and $g^{-1}of(x) = f(x)$, find the value of x.

12. If A.M. and G.M. between two positive and unequal numbers are 5 and 4 respectively, find the numbers.

13. Examine the continuity or discontinuity of the function
$g(x) = \begin{cases} 3x + 5 \text{ for } x > 2 \\ 7x - 3 \text{ for } x \le 2 \end{cases}$ at $x = 2$ by calculating left hand limit, right hand limit and functional value.

14. Solve by inverse matrix method: $\dfrac{3x+4y}{4} = \dfrac{-4y+6x}{5} = 4$

15. The opposite corners A and C of a rhombus have coordinates (-2 ,7) and (4, -3) respectively. Find the equation of the diagonal BD.

16. Prove: $CotA - Cot8A = Cosec2A + Cosec4A + Cosec8A$

17. If $A + B + C = 180°$, prove that
$Sin^2A + Sin^2B + Sin^2C = 2 + 2CosACosBCosC$

18. A 6m long ladder reaches a point 6m below the top of a vertical column. From the foot of the ladder, the elevation of the top of the column is 60°. Find the height of the column.

19. Find a 2×2 transformation matrix which transforms a unit square to the parallelogram with vertices (0,0), (3,1), (5,2) and (2,1).

20. Calculate the mean deviation from mean and its coefficient from the following data.

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
No. of students	2	3	6	5	4

20. Calculate the mean deviation from mean and its coefficient of the given table.

Marks	0 - 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	6	5	8	16	15

21. Find the standard deviation and coefficient of variation from the given table.

Income (Rs.)	40 - 50	50 – 60	60 – 70	70 – 80	80 – 90
No. of people	3	5	8	7	2

Group - 'D' [4× 5 = 20]

22. Optimize the objective function F= 10x + 9y under the constraints: x+ y ≤ 6, x − y ≥ - 2, x ≥ 0 , y ≥ 0. Also find the difference between maximum and minimum values.

23. The three points (3,1), (1,-1) and (3,-3) are located at the circumference of a wheel such that the points are equidistant from the fixed point. Find the co-ordinates of the fixed point and then derive the equation the locus that lies all these three points.

24. Prove by vector method that the inscribed angle at semi-circle is always a right angle.

25. A triangle ABC with vertices A (2,4), B (2,2) and C (6,2) is reflected in the line y = x and then the image so obtained is rotated through 180^0 in an anti- clock direction about center at origin. Write the coordinates of the images of the vertices of the Δ ABC and present each of them in the same graph paper. Also, state the single transformation equivalent to the combination of two transformations.

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Sainik Schools' Examination Board
### SEE Preparation Exam-2079

Subject: Opt- I Mathematics   **Time: 3 Hrs.**   **F.M: 100**

Students are encouraged to write answers in their own words as far as practicable. The figures in the margin indicate the full marks

Attempt all the questions

Group - 'A' [10× 1 = 10]

1. a. Define linear function.
   b. What is the nth term of a G.P. whose first term, common ratio and number of terms are a, r and n respectively? Write it.

2. a. Write the mathematical notation $\lim_{x \to a^-} f(x)$ in a sentence.
   b. What is the determinant of a unit matrix of order 2×2? Write it.

3. a. Write the formula to calculate the angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$.
   b. If the intersection plane is parallel to the base of a right circular cone, which conic does it form? Write it.

4. a. Express SinA - SinB in the product form Sine and Cosine.
   b. Express Sin3β in terms of Sinβ.

5. a. Write the formula to find the angle between two vectors $\vec{m}$ and $\vec{n}$.
   b. If the radius of the adjoining inversion circle is 6 cm, what is the value of OP×OP'? Write it.
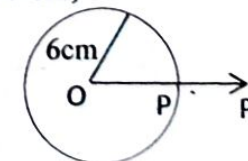
Group - 'B' [13× 2= 26]

6. a. If the function is defined as $f: x \rightarrow 4x + 5$, find the $f^{-1}(x)$ and $f^{-1}(5)$.

 b. Find the value of m if $(x +2)$ is a factor of $x^3 + 6x^2 - mx - 30$.

 c. Find the quotient and remainder when $3x^3 - 5x^2 - 3$ is divided by $x - 2$ using synthetic division method.

7. a. For what value of x, does the matrix $A = \begin{pmatrix} x - 2 & -3 \\ x & x + 2 \end{pmatrix}$ become a singular matrix?

 b. Find the inverse of the matrix $M = \begin{bmatrix} 13 & -9 \\ -10 & 7 \end{bmatrix}$.

8. a. Find the value of k so that the lines represented by equation $k^2 x^2 + 15xy + (k - 2)y^2 = 0$ are perpendicular.

 b. If two lines $ax + by + c = 0$ and $px - qy + r = 0$ are perpendicular to each other, show that $ap - bq = 0$.

9. a. If $Sin\theta = \frac{\sqrt{3}}{2}$, find the value of $Cos2\theta$.

 b. Prove that $\frac{cos\frac{\theta}{2} - \sqrt{1+sin\theta}}{sin\frac{\theta}{2} - \sqrt{1+sin\theta}} = tan\frac{\theta}{2}$.

 c. Solve: $2Cos\theta - \sqrt{3} = 0$ $(0 \leq \theta \leq 180^\circ)$

10. a. Find the value of 'm' so that the vectors $\vec{p} = \begin{pmatrix} m + 4 \\ -3 \end{pmatrix}$ and $\vec{q} = \begin{pmatrix} 3 \\ 2m \end{pmatrix}$ are mutually perpendicular.

 b. In a triangle OAB, C is the mid- point of AB. If $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OB} = \vec{b}$,
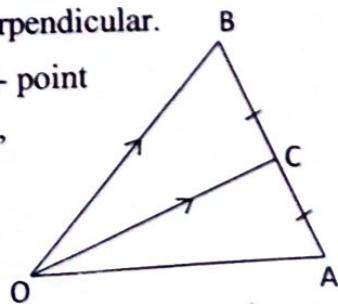
Show that $\overrightarrow{OC} = \frac{\vec{a}+\vec{b}}{2}$

 c. If the first and third quartiles of a data are 15 and 35 respectively, calculate semi- interquartile range and its coefficient.

Group - 'C' [11× 4= 44]

11. If $g(x) = 3x - 7$, $5f(x) = x + 2$ and $f^{-1}og(x) = g(x)$, find the value of x.

12. Find the two numbers whose arithmetic mean and geometric mean are 25 and 20 respectively.

13. Examine the continuity or discontinuity of the function $f(x) = \begin{cases} 5x - 2 & for\ x \geq 2 \\ 4x & for\ x < 2 \end{cases}$ at $x = 2$ by calculating left hand limit, right hand limit and functional value.

14. Solve by inverse matrix method: $\frac{3x-4y}{10} = \frac{3y+5x}{7} = 2$

15. The opposite corners P and R of a square have coordinates (2 ,4) and (8, 10) respectively. Find the equation of the diagonal QS.

16. Prove: $Cot\theta - Cot8\theta = Cosec2\theta + Cosec4\theta + Cosec8\theta$

17. If $A + B + C = 180^0$, prove that $Cos^2A + Cos^2B + Cos^2C = 1 - 2CosACosBCosC$

18. An 8m long ladder reaches a point 8m below the top of a vertical flagstaff. From the foot of the ladder, the elevation of the top of the flagstaff is 60^0. Find the height of the flagstaff.

19. Find a 2×2 transformation matrix which transforms a unit square to the parallelogram with vertices (0,0), (3,0), (4,1) and (1,1).