

Up the garden path

In ancient Suzhou, a renowned scholar was trying to understand the patterns in the city's design. He noticed that the number of pathways in some of the classical gardens followed a specific pattern. In one particular garden, there were two types of pathways: stone and wooden. The scholar observed that for any given number of pathways, n , the total number of ways to choose a combination of stone and wooden pathways was given by the binomial coefficient nCk , where k is the number of wooden pathways.



- If a garden has 5 pathways in total, how many ways can the scholar choose 3 wooden pathways?
- In another garden with 6 pathways, what is the total number of ways to choose pathways if there are at least 2 wooden ones?
- Using the binomial theorem, expand the expression $(x + y)^4$ to represent the number of ways to choose pathways in a garden with 4 pathways, where x represents stone pathways and y represents wooden pathways.

Shanghai skyline

An architect in Shanghai is inspired by the city's skyline and decides to design a building with multiple sections. Each section of the building can be made of either glass or concrete. The architect uses the binomial theorem to determine the number of different designs possible for the building.



- If the building has 4 sections, in how many ways can the architect design the building with 2 sections made of glass?
- For a building with 5 sections, what is the total number of design combinations if at least 3 sections are made of glass?
- Using the binomial theorem, expand the expression $(g + c)^3$ to represent the number of design combinations for a building with 3 sections, where g represents sections made of glass and c represents sections made of concrete.