## Task A: Angle at Centre and Angle at Circumference

## Step 1

Create a circle.

Rename the centre as O and the point on the circumference as Z .

Hide point Z .


## Step 2

Create points A, B and C on the circumference.

Create their opposite points by using "Reflect Object about Point" $\bullet^{\circ}$.

Rename the points $\mathrm{A}^{\prime}, \mathrm{B}^{\prime}$ and $\mathrm{C}^{\prime}$ as $\mathrm{D}, \mathrm{E}$ and F respectively.


## Step 3

To create a point P that act as a "phantom" of A and snap to the point F, we use the command "DynamicCoordinates".

The format of the command is DynamicCoordinates[ <Point A>, <Number $\mathbf{X}>,<$ Number $\mathbf{Y}>$ ] When the new point moves, A moves with it and the coordinates of the new point are ( $\mathrm{X}, \mathrm{Y}$ ) and usually X and Y depends on the coordinates of A .

At the input area at the bottom of the screen, type $\mathrm{P}=$ DynamicCoordinates[


If[Distance[A, F] < 0.2, x(F), x(A)],


If[Distance[A, F] < 0.2, $\mathrm{y}(\mathrm{F}), \mathrm{y}(\mathrm{A})$ ]

## Step 4

To create a point $P$ that act as a phantom of $A$ and snap to points $\mathbf{E}$ and $\mathbf{F}$, we replace the $x(A)$ and $y(A)$ in the original definition of $P$ by
If[Distance[A, E] < 0.2, $x(E), x(A)]$ and $\operatorname{If}[D i s t a n c e[A, E]<0.2, y(E), y(A)] \quad$ respectively.
Right click on P and edit its definition as
DynamicCoordinates[
A,

If[Distance[A, F] < 0.2, x(F), x $\stackrel{\downarrow}{ }$ )],

If[Distance[A, F] < 0.2, y(F), y(N)]
]
Step 5
Create a point Q that act as a phantom of C and snap to points E .

At the input area, type $\mathrm{Q}=$
DynamicCoordinates[C, If[Distance[C, E] < 0.2, x(E), x(C)], If[Distance[C, E] < 0.2, y(E), $y(C)]$ ]

## Step 6

Hide points A, C, D, E and F.

Rename P as A and Q as C .

Change the color of A, B, and C to black.


## Step 7

Refer to the figure, draw the line segments and create $\angle \mathrm{AOC}, \angle \mathrm{ABC}$ and their reflex angles.

Rename the angles as $\mathrm{p}, \mathrm{q}, \mathrm{r}$ and s as in the figure.

Set the colour of the angles at centre as red and the angles at circumference as blue.


## Step 8

We want to show the pair p and r if $\mathrm{r}<\mathrm{s}$, otherwise show the pair q and s .

Right click on r, select object properties and choose the "Advanced" panel.
In the field "Condition to Show Object", type "r<s", do the same for the angle p

Right click on s, select object properties and choose the "Advanced" panel.
In the field "Condition to Show Object", type "s<r", do the same for the angle q

Step 9
Insert text $\triangle A B C$ to show the sizes of $\angle \mathrm{ABC}$ and $\angle A O C$.

The size of $\angle \mathrm{ABC}$ is r if $\mathrm{r}<\mathrm{s}$, otherwise it is s . This can be written as if[r<s,r,s].

The size of $\angle \mathrm{AOC}$ is the corresponding "partner" of r and s . If $\mathrm{r}<\mathrm{s}$, it is p , otherwise it is $q$, i.e. $\mathrm{if}[\mathrm{r}<\mathrm{s}, \mathrm{p}, \mathrm{q}]$.

Angle at centre $=118.74^{\circ}$
Angle at circumference $=59.37^{\circ}$


## Step 10

Insert text $\triangle A B C$ to show the ratio between $\angle \mathrm{ABC}$ and $\angle \mathrm{AOC}$. To create a fraction, we use the LaTeX command $\backslash$ frac\{numerator\}\{denominatior\}


## Step 11

To change the colour of the numerator and denominator, we enclose the numerator by |red $\{\quad\}$ and the denominator by
\blue\{ \}.
:|frac\{ $\{\operatorname{red}\{\mid \mathrm{If}[\mathrm{r}<\mathbf{s}, \mathrm{p}, \mathrm{q}]\}\}\{\mid$ blue $\{\mid \mathrm{If}[\mathrm{r}<\mathbf{s}, \mathrm{r}, \mathrm{s}]\}\}$ :
$\begin{array}{ll}\text { Angle at centre }=134.57^{\circ} & \angle A O C \\ \text { Angle at circumference }=67.28^{\circ} & \angle A B C\end{array} \frac{134.57^{\circ}}{67.28^{\circ}}=2$

~ End of Task A~

## Task A-1: Angle at Centre and Angle NOT at Circumference

Step 1
Repeat Step 1 of Task A.

Step 2
Create a free point D on the plane. Then, create a line passing through the centre O and D . Mark one of the points of intersection of the line and the circle as E .

Hide the points D, E and the line.

## Step 3

Create a point B with the dynamic coordinates depending on point D , while snapping to point E .
(Exercise)

## Step 4

Follow the rest of steps in Task A to complete the dynamic worksheet to show one constraint of the theorem by the counter-examples.
~ End of Task A-1 ~

## Task A-2: Angle at Centre and Angle at Circumference of an ELLIPSE

Step 1
Create an ellipse. Label the mid-point of the foci as O.

Repeat all the other steps in Task A.
~ End of Task A-2 ~

## Task A-3: Angle at Centre and Angle at "Circumference" of a SQUARE

Step 1
Create a square by using the "Regular Polygon" tool. Label the centre of the square as O.

Step 2
Create $s$ as a list of item holding the four segments of the square using the bracket " $\}$ ". Define points A, B, C as "point[s]".

Repeat all the other steps in Task A.

