Hans W. Hofmann

1	Tx(zz,ss):=Sequence(Element(Identity(n),If(kk zz kk ss,kk,If(kk=zz,Max(zz,ss),Min(zz,ss)))),kk,1,n);
2	Ex(zle,spl,k_f):=Sequence(Sequence(Element(Identity(n), zz,ss)-1*(zle==spl && zle==zz && spl==ss)*1+
3	A:={{3,0,8,2},{3,-1,6,0},{-2,0,-5,0},{0,0,0,-1}} $\mathbf{A} := \begin{pmatrix} 3 & 0 & 8 & 2 \\ 3 & -1 & 6 & 0 \\ -2 & 0 & -5 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$
4	n:=Length(A) $\mathbf{n := 4}$
5	X:=Take({x1,x2,x3,x4,x5,x6,x7,x8,x9},1,n) $\mathbf{X := \{x1, x2, x3, x4\}}$

Dateianhang ggb classic files - Jordan-Normalform aus Eigenwerten - Jordan-Normaform mit Hauptvektor-Suche

6	A_{-λ}E:= (A - l Identity(n)) $\mathbf{A_{-\lambda}E} := \begin{pmatrix} -\ell+3 & 0 & 8 & 2 \\ 3 & -\ell-1 & 6 & 0 \\ -2 & 0 & -\ell-5 & 0 \\ 0 & 0 & 0 & -\ell-1 \end{pmatrix}$
7	Factor(Determinant(A_{-λ}E)) = 0 $(\ell+1)^4 = 0$
8	EW:=CSolutions($7,l) $\mathbf{EW} := \{-1\}$
9	DimEigenraum:=-Sequence(MatrixRank(A $\mathbf{DimEigenraum} := \{2\}$

10	Sequence({ "λ=" ,EW(k),(A - EW(k) E),Transpose(X)=0} ,k,1,Length(EW)) $\left(\lambda = -1 \quad \begin{pmatrix} 4 & 0 & 8 & 2 \\ 3 & 0 & 6 & 0 \\ -2 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \end{pmatrix} = 0 \right)$
11	LGλi:=Sequence((A - EW(k) E) X,k,1,Length(EW)) $\mathbf{LG\lambda i} := \begin{pmatrix} 4\,x1 + 8\,x3 + 2\,x4 & 3\,x1 + 6\,x3 & -2\,x1 - 4\,x3 & 0 \end{pmatrix}$
12	Aλi:=Sequence(Flatten(Solutions(Element(LGλi,k),X)), k,1,Length(EW)) $\mathbf{A\lambda i} := \begin{pmatrix} -2\,x3 & x2 & x3 & 0 \end{pmatrix}$
13	{Transpose(LGλi)=0,Transpose(X)=Transpose(Aλi)} $\left\{ \begin{pmatrix} 4\,x1 + 8\,x3 + 2\,x4 \\ 3\,x1 + 6\,x3 \\ -2\,x1 - 4\,x3 \\ 0 \end{pmatrix} = 0, \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \end{pmatrix} = \begin{pmatrix} -2\,x3 \\ x2 \\ x3 \\ 0 \end{pmatrix} \right\}$

14	EVi:=Join(Sequence(If(Element(Aλi,k) {},Substitute($EVi := \begin{pmatrix} 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \end{pmatrix}$
15	suche Hauptvektor zu λ1=0 - HVi (Index zum Abgreifen des EW) - Vektoren werden Zeileweise geschrieben!
16	HVi:=1 $HVi := 1$
17	N:=3 Einziger EW -1 benötigt 4 HV - wähle N so, daß 4 HVKandidaten entstehen $N := 3$
18	Suche HV ∈ Ker (A-λE)^N mit dim Ker (A-λE)^N = n ∧ HV ¬∈ Ker (A-λE)^N-1
19	LGHV1:=(A - EW(HVi) E)^N $LGHV1 := \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

20

LHV1:=Solutions(LGHV1 X,X

$$\mathbf{LHV1 := (x1 \quad x2 \quad x3 \quad x4)}$$

21

HVKandidaten1u:=Tranos ∈ Ker (A-λE)^N-1 u3=

$$\mathbf{HVKandidaten1u := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

22

KernHV1:=(A - EW(HVi) E)^(N-1)

$$\mathbf{KernHV1 := \begin{pmatrix} 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 \end{pmatrix}}$$

Dim KernHV1 = 1 ==> 1 Jordanblock

23

HV1u2:=(A - EW(HVi) E) HV u2=

$$\mathbf{HV1u2 := \begin{pmatrix} 4 & 0 & 8 & 2 \\ 3 & 0 & 6 & 0 \\ -2 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}}$$

24	HV1u1:=(A - EW(HVi) E) HV1u2 u1= $\mathbf{HV1u1} := \begin{pmatrix} 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
25	LHV2:=Solutions((A - EW(HVi) E)^(N-1) X,X)
26	HVKandidaten2w:=Transpose(If(LHV2 {},Substitute(Sequence(If(Element(LHV2,1,j) Element(X,j),Flatten(Substitute(LHV2,E
27	HV2w1:=(A - EW(HVi) E) HVKandidaten2w
28	spalte(AA,ss):=Element(Transpose(AA), ss) $\mathbf{spalte(AA, ss) := Element(Transpose(AA), ss)}$
29	u:=4 4. Spalte enthält die gültigen HVs $\mathbf{u := 4}$
30	w:=2 $\mathbf{w := 2}$

Einen EV kombinieren mit u1,u2,u3 [EV=Take(EVI,1,1) oder Take(EVi,2,2)]

31

T:= Transpose(Join(Take(EVi,1,1),{spalte(HV1u1,u)},{spalte(HV1u2,u)},{spalte(HVKandidaten1u,u)}))

$$T := \begin{pmatrix} 0 & 8 & 2 & 0 \\ 1 & 6 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

32

D:=T^(-1) A T

$$D := \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

33

JordanDiagonalization(A)

$$\left\{ \begin{pmatrix} 8 & 2 & 0 & 0 \\ 6 & 0 & 0 & 2 \\ -4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \right\}$$

34

1	Tx(zz,ss):=Sequence(Element(Identity(n),If(kk zz kk ss,kk,If(kk=zz,Max(zz,ss),Min(zz,ss)))),kk,1,n);
2	Ex(zle,spl,k_f):=Sequence(Sequence(Element(Identity(n), zz,ss)-1*(zle==spl && zle==zz && spl==ss)*1+
3	A:={{5,0,1 ,0,0},{0 ,1 , 0 ,0,0 },{-1,0,3,0,0},{0 ,0, 0 ,1 , 0 },{ 0, 0,0 , 0 , 4}} $\mathbf{A} := \begin{pmatrix} 5 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix}$
4	n:=Length(A) $\mathbf{n} := 5$
5	X:=Take({x1,x2,x3,x4,x5,x6,x7,x8,x9},1,n) $\mathbf{X} := \{x1, x2, x3, x4, x5\}$

6	A_{-λ}E:= (A - I Identity(n)) $\mathbf{A_{-\lambda}E} := \begin{pmatrix} -\ell+5 & 0 & 1 & 0 & 0 \\ 0 & -\ell+1 & 0 & 0 & 0 \\ -1 & 0 & -\ell+3 & 0 & 0 \\ 0 & 0 & 0 & -\ell+1 & 0 \\ 0 & 0 & 0 & 0 & -\ell+4 \end{pmatrix}$
7	Factor(Determinant(A_{-λ}E)) = 0 $-(\ell-1)^2\,(\ell-4)^3 = 0$
8	EW:=CSolutions($7,l) $\mathbf{EW} := \{1, 4\}$
9	DimEigenraum:=-Sequence(MatrixRank(A $\mathbf{DimEigenraum} := \{2, 2\}$

10

Sequence({ "λ=" ,EW(k),(A - EW(k) E),Transpose(X)=0} ,k,1,Length(EW))

$$\begin{pmatrix} \lambda = 1 & \begin{pmatrix} 4 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix} & \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \end{pmatrix} = 0 \\ \lambda = 4 & \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & -3 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \end{pmatrix} = 0 \end{pmatrix}$$

11

LGλi:=Sequence((A - EW(k) E) X,k,1,Length(EW))

$$LG\lambda i := \begin{pmatrix} 4\,x1 + x3 & 0 & -x1 + 2\,x3 & 0 & 3\,x5 \\ x1 + x3 & -3\,x2 & -x1 - x3 & -3\,x4 & 0 \end{pmatrix}$$

12

Aλi:=Sequence(Flatten(Solutions(Element(LGλi,k),X)), k,1,Length(EW))

$$A\lambda i := \begin{pmatrix} 0 & x2 & 0 & x4 & 0 \\ -x3 & 0 & x3 & 0 & x5 \end{pmatrix}$$

13

{Transpose(LGλi)=0,Transpose(X)=Transpose(Aλi)}

$$\left\{\begin{pmatrix} 4\,x1+x3 & x1+x3 \\ 0 & -3\,x2 \\ -x1+2\,x3 & -x1-x3 \\ 0 & -3\,x4 \\ 3\,x5 & 0 \end{pmatrix}=0,\begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \end{pmatrix}=\begin{pmatrix} 0 & -x3 \\ x2 & 0 \\ 0 & x3 \\ x4 & 0 \\ 0 & x5 \end{pmatrix}\right\}$$

14

EVi:=Join(Sequence(If(Element(Aλi,k) {},Substitute(Sequence(If(Elem

$$EVi := \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

15

suche Hauptvektor zu λ2=4 - HVi (Index zum Abgreifen des EW)

16

HVi:=2

$$HVi := 2$$

17

N:=2 2.ter EW 4 benötigt 3 HV - wähle N so, daß 3 HVKandidaten entstehen

$$N := 2$$

18	Suche
19	LGHV $\mathbf{LGHV1} := \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
20	Element(LGλi, 2)-Element(EVi, 3) $\{x1 + x3 + 1, -3\,x2, -x1 - x3 - 1, -3\,x4, 0\}$
21	LHV1:=Solutions(LGHV1 X,X) $\mathbf{LHV1} := (\ x1 \quad 0 \quad x3 \quad 0 \quad x5\ )$
22	HVKandidaten1u:=Transp(If(LHV u3= ∈ Ker (A-λE)^N-1 $\mathbf{HVKandidaten1u} := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

23	KernHV1:=(A - EW(HVi) E)^(N - 1) HVKandidaten1u $\mathbf{KernHV1} := \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	Dim KernHV1 = 1 ==> 1 Jordanblock
24	HV1u2:=(A - EW(HVi) E) HVKandidaten1u u2= $\mathbf{HV1u2} := \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	u2 entspricht EW(3)=(-1,0,1,0,0) ersetze EW(3) durch die Hauptvektoren u2,u3
25	HV1u1:=(A - EW(HVi) E) HV1u2 $\mathbf{HV1u1} := \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	
26	LHV2:=Solutions((A - EW(HVi) E)^(N-1) X,X)	

27	HVKandidaten2w:=Transpose(If(LHV2 {},Substitute(Sequence(If(Element(LHV2,1,j) Element(X,j),Flatten(Substitute(LHV2,E
28	HV2w1:=(A - EW(HVi) E) HVKandidaten2w
29	spalte(AA,ss):=Element(Transpose(AA), ss) $\mathbf{spalte(AA, ss) := Element(Transpose(AA), ss)}$
30	u:=2 Spalten u=1,2 enthalten die gültigen HVs $\mathbf{u := 2}$
31	w:=2; EV 1,2 und 4 kombinieren mit u2,u3 [EV=Take(EVI,1,2) oder Take(EVi,4,4)]
32	T:= Transpose(Join(Take(EVi,1,2),Take(EVi,4,4),{spalte(HV1u2,u)},{spalte(HVKandidaten1u,u)})) $\mathbf{T := \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}}$

33	D:=T^(-1) A T $D := \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix}$
34	JordanDiagonalization(A) $\left\{ \begin{pmatrix} 3 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -3 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 4 & 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right\}$
35	

1	A:= {{1, a,b}, {0, 2, c}, {0,0, 2}} $\mathbf{A} := \begin{pmatrix} 1 & a & b \\ 0 & 2 & c \\ 0 & 0 & 2 \end{pmatrix}$
2	n:=Length(A) $\mathbf{n} := 3$
3	X:=Take({x1,x2,x3,x4,x5,x6,x7 $\mathbf{X} := \{x1, x2, x3\}$
4	A_{-λ}E:= (A - I Identity(n)) $\mathbf{A_{-\lambda}E} := \begin{pmatrix} -\ell + 1 & a & b \\ 0 & -\ell + 2 & c \\ 0 & 0 & -\ell + 2 \end{pmatrix}$
5	Factor(Determinant(A_{-λ}E)) = $-(\ell - 1)\,(\ell - 2)^2 = 0$

6

EW:=CSolutions($5,l)

$$\mathbf{EW} := \{1, 2\}$$

7

DimEigenraum:=-Sequence(Matrix

$$\mathbf{DimEigenraum} := \{1, 1\}$$

8

Sequence({ "λ=" ,EW(k),(A - EW(k)

$$\begin{pmatrix} \lambda = 1 & \begin{pmatrix} 0 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} & \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = 0 \\ \lambda = 2 & \begin{pmatrix} -1 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = 0 \end{pmatrix}$$

9

LGλi:=Sequence((A - EW(k) E) X,k

$$\mathbf{LG\lambda i} := \begin{pmatrix} a\,x2 + b\,x3 & c\,x3 + x2 & x3 \\ a\,x2 + b\,x3 - x1 & c\,x3 & 0 \end{pmatrix}$$

10

Aλi:=Sequence(Flatten(Solutions(Element(LGλi,k)

$$A\lambda i := \begin{pmatrix} x1 & 0 & 0 \\ a\,x2 & x2 & 0 \end{pmatrix}$$

11

{Transpose(LGλi)=0,Transpose(X)=Transpose(Aλi)

$$\left\{ \begin{pmatrix} a\,x2 + b\,x3 & a\,x2 + b\,x3 - x1 \\ c\,x3 + x2 & c\,x3 \\ x3 & 0 \end{pmatrix} = 0, \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} x1 & a\,x2 \\ 0 & x2 \\ 0 & 0 \end{pmatrix} \right\}$$

12

EVi:=Join(Sequence(If(Element(Aλi,k) {},Substitu

$$EVi := \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \end{pmatrix}$$

13

suche Hauptvektor zu λ2=2 - HVi (Index zum Abgreifen des EW)

16

HVi:=2

$$HVi := 2$$

17

N:=2

$$N := 2$$ Zum EW 2 alg. Vielfachheit 2 sind 2 HV zu ermitteln - wähle N so, daß 2 HVKandidaten entstehen

18

Suche HV Ker (A-λE)^N mit dim Ker (A-λE)^N = n HV ¬ Ker (A-λE)^N-1

19

LGHV1:=(A - EW(HVi) E)^N

$$LGHV1 := \begin{pmatrix} 1 & -a & a\,c - b \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

20

LHV1:=Solutions(LGHV1 X,X)

$$LHV1 := \begin{pmatrix} -a\,c\,x3 + a\,x2 + b\,x3 & x2 & x3 \end{pmatrix}$$

21

HVKandidaten1u:=Transpose(If(LHV1 {},Substitute(Sequence(If(Element(LHV1,1,j) Element(X,j),Flatten(Su

∈ Ker (A-λE)^N-1 u3=

$$HVKandidaten1u := \begin{pmatrix} a & -a\,c + b \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

21	HV1u2:=(A - EW(HVi) E) HVKandidaten1u u2= $\mathbf{HV1u2} := \begin{pmatrix} \mathbf{0} & \mathbf{a\ c} \\ \mathbf{0} & \mathbf{c} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}$
22	HV1u1:=(A - EW(HVi) E) HV1u2 $\mathbf{HV1u1} := \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}$
23	spalte(AA,ss):=Element(Transpose(AA), ss) $\mathbf{spalte(AA, ss) := Element(Transpose(AA), ss)}$
24	u:=2 $\mathbf{u := 2}$ Die Spalten u=2 enthält den gültigen HV
25	T:=Transpose({ spalte(HV1u2, u), spalte(HV1u2, u)+spalte(HVKandidaten1u, u),Element(EVi,1)}) $\mathbf{T} := \begin{pmatrix} \mathbf{a\ c} & \mathbf{b} & \mathbf{1} \\ \mathbf{c} & \mathbf{c} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \end{pmatrix}$ Kombiniere EW(1) und HVKandidaten1u(2), HV1u2(2) - Fasse zur Vereinfachung HVKandidaten1u(2)+HV1u2(2) zusammen

26	D:=T^(-1) A T $D := \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
27	JordanDiagonalization(A) $\left\{ \begin{pmatrix} a\,c & b & 1 \\ c & c & 0 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right\}$
28	

Rundungsfehler NSolve/NSolutions (numemrische Lösung)

1	A:= {{2, 1,2}, {-1, 2, 1}, {1, 2, 4}} $\mathbf{A} := \begin{pmatrix} 2 & 1 & 2 \\ -1 & 2 & 1 \\ 1 & 2 & 4 \end{pmatrix}$
2	n:=Length(A) $\mathbf{n} := 3$
3	X:=Take({x1,x2,x3,x4,x5,x6,x7,x8,x9},1,n $\mathbf{X} := \{x1, x2, x3\}$
4	A - l Identity(n) $\begin{pmatrix} -\ell + 2 & 1 & 2 \\ -1 & -\ell + 2 & 1 \\ 1 & 2 & -\ell + 4 \end{pmatrix}$
5	Determinant(A - l Identity(n)) = 0 Factor: $-\ell^3 + 8\,\ell^2 - 17\,\ell + 9 = 0$ wir bekommen keine Faktorisierung

6

Eigenwerte:=CSolutions($5,l)

$$\textbf{Eigenwerte} := \{0.801, 2.286, 4.912\}$$

Werte können nicht exakt berechnet werden

7

DimEigenraum:=-Sequence(Ma

$$\textbf{DimEigenraum} := \{0, 0, 0\}$$

Die Eigenraumberechnung scheitert aufgrund von Rundungsfehlern

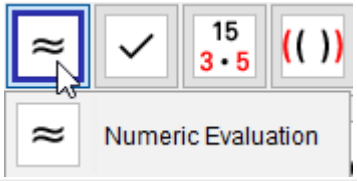
8

Sequence({ "λ=" ,Eigenwerte(k),(A - Eigenwerte(k) Identity(n)),Transpose(X)=0} ,k,1,Length(Eigenwerte))

$$\begin{pmatrix} \lambda = & 0.801 & \begin{pmatrix} 1.199 & 1 & 2 \\ -1 & 1.199 & 1 \\ 1 & 2 & 3.199 \end{pmatrix} & \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = 0 \\ \lambda = & 2.286 & \begin{pmatrix} -0.286 & 1 & 2 \\ -1 & -0.286 & 1 \\ 1 & 2 & 1.714 \end{pmatrix} & \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = 0 \\ \lambda = & 4.912 & \begin{pmatrix} -2.912 & 1 & 2 \\ -1 & -2.912 & 1 \\ 1 & 2 & -0.912 \end{pmatrix} & \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = 0 \end{pmatrix}$$

Aλi:=Sequence(Flatten(Solutions(Element(LGλi,k),X)), k,1,Length(Eigenwerte))

$$\approx \quad A\lambda i := \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



Solve oder Solutions scheitert!
Fehlerbehaftete Werte werden als exakt behandelt:
Alle Rechenschritte müssen über den Button
Numeric Evaluation neu berechnet werden!

9

LGλi:=Sequence((A - Eigenwerte(k) Identity(n)) X,k,1,Length(Eigenwerte))

$$LG\lambda i := \begin{pmatrix} 1.199\,x1 + x2 + 2\,x3 & -x1 + 1.199\,x2 + x3 & x1 + 2\,x2 + 3.199\,x3 \\ -0.286\,x1 + x2 + 2\,x3 & -x1 - 0.286\,x2 + x3 & x1 + 2\,x2 + 1.714\,x3 \\ -2.912\,x1 + x2 + 2\,x3 & -x1 - 2.912\,x2 + x3 & x1 + 2\,x2 - 0.912\,x3 \end{pmatrix}$$

10

Aλi:=Sequence(Flatten(NSolutions(Element(LGλi,k),X)), k,1,Length(Eigenwerte))

$$A\lambda i := \begin{pmatrix} -0.573\,x3 & -1.313\,x3 & x3 \\ 1.454\,x3 & -1.584\,x3 & x3 \\ 0.72\,x3 & 0.096\,x3 & x3 \end{pmatrix}$$

Setze einen numerischen
Lösungsalgorithmus ein
===> NSolve/NSolutions

11

EVi:=Sequence(If(Element(Aλi,k) ≠ {},Substitute(Sequence(If(Element(Aλi,k, j) == Element(X, j), Substitute(Element(Aλi, k) ,

$$EVi := \{(-0.573 \quad -1.313 \quad 1), (1.454 \quad -1.584 \quad 1), (0.72 \quad 0.096 \quad 1)\}$$

12

T:=Transpose(Join((EVi)))

$$T := \begin{pmatrix} -0.573 & 1.454 & 0.72 \\ -1.313 & -1.584 & 0.096 \\ 1 & 1 & 1 \end{pmatrix}$$

13

D:=T^(-1) A T

$$D := \begin{pmatrix} 0.801 & 6.035 \cdot 10^{-14} & 1.201 \cdot 10^{-13} \\ 2.101 \cdot 10^{-13} & 2.286 & -(9.674 \cdot 10^{-14}) \\ -(1.013 \cdot 10^{-12}) & -(6.609 \cdot 10^{-13}) & 4.912 \end{pmatrix}$$

10^{-13} uä. meint fast Null

14

JordanDiagonalization(A)

$$\left\{ \begin{pmatrix} -0.613 & 0.328 & 0.582 \\ 0.668 & 0.751 & 0.078 \\ -0.422 & -0.572 & 0.809 \end{pmatrix}, \begin{pmatrix} 2.286 & 0 & 0 \\ 0 & 0.801 & 0 \\ 0 & 0 & 4.912 \end{pmatrix} \right\}$$

15