Polynomial Overview

Polynomial Operations and Factoring

(Including the miscellaneous facts not usually presented.)

When doing algebra problems, one must work with many forms of polynomials—algebra problems are built from them. A monomial, the simplest form of a polynomial, consists of a coefficient (the numeric value) or the product of a coefficient multiplied by one or more non-constant variable(s). Here, we will discuss the different polynomial formats starting with the **monomial**, the building block for all polynomial (algebraic) forms.

$$\boxed{x^0 = 1, if\ x \neq 0;\ 0^0\ is\ undefined!}$$

1. Monomials (These are building blocks of all polynomials.)

Definitions: (a) A monomial (term), it can also be called a power product, is an algebraic expression consisting more or less of a coefficients (unique numeric values), variables, and exponents (exponents **must be** whole numbers*, not variables); (b) a polynomial which has only one term. (c) A monomial is a product of coefficient and/or various variables: i.e.,

51, 5n, 2x, 158, π, 3xy, $14x^3y$, -43, 0, xy^3z, x^0y^2z, |-7|, …

A Monomial

coefficient exponent

$$7x^5$$

base variable

A coefficient is the number of times a base and its exponent are add together.

An exponent is the number of times the base is repeatedly multiplied.

Remember all single variables have a coefficient of 1 and an exponent of 1 that are not normally displayed.

$$x = 1 \cdot x^1$$

Unwritten coefficients or exponents have a value of **one, 1**

Monomials can only factor into their relatively prime or prime parts:
$14x^3y$ = 2•7•x•x•x•y, 5•n, -1•43, 1•π, or x•y•y•y•z, …

There are some things you ***cannot*** have within a monomial: you can't add or subtract any parts; divide by a non-constant factor (variable); raise a non-constant factor to a negative or variable power; and you cannot take absolute values of non-constant factors. Examples of non-monomials:

$$\boldsymbol{x + 7,\ x - 3,\ \frac{1}{x},\ y^{-2} \text{ or } \frac{1}{y^2},\ -15\,x^{-2}y,\ 25\frac{xy}{z},\ \frac{x^3}{x-3},\ |x|,\ x^y}$$

Definition:

$$|b| = \begin{cases} b, b \geq 0 \\ -b, b < 0 \end{cases}$$

Absolute Value is a **distance**. It strips the condition modifier signs from all values. The values are neither positive nor negative.

$|^+7| = 7$ and $|^-7| = 7$

If a number has a sign (+ or –), it is a directed number.

Directed numbers are a distance from zero on the number line.

Absolute values have no direction.

Like Monomials have the SAME variables and exponents of those variables. The coefficients on the terms can be different.

$$3z^5yx^2 + 8yx^2z^5 = (3+8)x^2yz^5 = 11x^2yz^5$$

Like monomials will simplify to single monomial term.

The Standard Form for a monomial is when there is a single numeric value, and all variables with exponents are written in alphabetical order.

The Degree of a Monomial is the total value of the exponents of all its variable elements. The degree of the monomial $3xy^5b^3$ is 9 (= 0 + 1 + 5 + 3). Or count the variables in factored form 3•x•y•y•y•y•y•b•b•b. (Note: $3b^3xy^5$ is standard form.)

Polynomials are formed when adding and/or subtracting monomial terms in the simplified form.

https://examples.yourdictionary.com/examples-of-monomials.html
http://content.nroc.org/DevelopmentalMath/COURSE_TEXT2_RESOURCE/U11_L2_T1_text_final.html

Natural (Counting) numbers: N ={1, 2, 3, …}
***Whole numbers**: W = {0, 1, 2, 3, …}
Integers: Z = {… , ⁻3, ⁻2, ⁻1, 0, 1, 2, 3, …}

∅ denotes the empty set, the set with no members
Q denotes the set of rational numbers
R denotes the set of real numbers

Rational numbers are all possible fractions including integers, $4 = \frac{4}{1}$, include terminating and repeating decimals.

Irrational numbers have decimal values which do not repeat or terminate, π, $\sqrt{2}, \sqrt{3}, \sqrt{13}$, 2.121122111222333….
Real numbers include all Rational and all Irrational numbers.

There is an additional set of numbers called the Imaginary Numbers (**not HSE tested**); $i = \sqrt{-1}$ is a factor of all.
The Real Numbers plus the Imaginary numbers are called the **C**omplex Numbers.

Circle the valid monomials.

$9s^4t^2u^2$ $\quad$ $2gh + g$ $\quad$ $-x^5y^2z^2$ $\quad$ $-x^{5}y^{-2}z^{2}$ $\quad$ $-4x^4 - x^2$ $\quad$ $|-5|x^5y^2z^2$ $\quad$ $-5x^3y|z|$

Write the degree of each monomial.

1) $x^5y^2z^2$ _____ $\quad$ 2) $-9p^2q^2$ _____ $\quad$ 3) $2g^2h^3$ _____

4) $-6c^3$ _____ $\quad$ 5) $-5st^3u^3$ _____ $\quad$ 6) $8m^5n^0$ _____

Monomials: [1, 3, 6]
1) 9
2) 4
3) 5
4) 3
5) 7
6) 5

Not all sums are monomials, state this when found. Add the following monomials. If there is a “,” replace with a “+”.

1) $-y^4 + 4y^5$ $\quad$ 2) $11n + 9n$ $\quad$ 3) $8w^5, 8w^5$

4) $2s, -9s$ $\quad$ 5) $-12pq^3r^2, 12pq^3r^2$ $\quad$ 6) $5ab^2, 4a^2b$

7) $-13pqr,\ \frac{6}{7}pqr$ $\quad$ 8) $24g^3h^3, -4g^3h^3$ $\quad$ 9) $3ab^2,\ -5b^2a$

Addition
1) $4y^5 - y^4$
2) $20n$
3) $16w^5$
4) $-7s$
5) 0
6) $4a^2b + 5ab^2$
7) $-12\frac{1}{7}pqr$
8) $20g^3h^3$
9) $-2ab^2$

Subtraction
1) $ab^5c - 7b$
2) $4v^3w^2$
3) $-33q^4r^3$
4) $-13b^2c^4d$
5) $-24pq^3r^2$
6) $-4a^2b + 5ab^2$
7) $-13\frac{6}{7}pqr$
8) $28g^3h^3$
9) $8ab^2$

Not all differences are monomials, state this when found. Subtract the following monomials. If there is a “,” replace with a “–”.

1) $(-7b) - (-ab^5c)$ $\quad$ 2) $15v^3w^2 - 11v^3w^2$ $\quad$ 3) $q^4r^3 - 34q^4r^3$

4) $(-16b^2c^4d) - (-3b^2c^4d)$ $\quad$ 5) $-12pq^3r^2, 12pq^3r^2$ $\quad$ 6) $5ab^2, 4a^2b$

7) $-13pqr,\ \frac{6}{7}pqr$ $\quad$ 8) $24g^3h^3, -4g^3h^3$ $\quad$ 9) $3ab^2,\ -5b^2a$

Multiply the following. When multiplying like variable, add exponents ($d^3 \bullet d^2 = d^5$).

1) $\frac{1}{6}q^6rs \times 72q^5$

2) $-7y^2z^3 \times 7y^3z^2$

3) $\frac{5}{8}rs^3t^2 \times 64r^3st^3$

4) $-\frac{1}{2}uv^4 \times \frac{3}{4}u \times -\frac{8}{3}v^3$

5) $\frac{2}{7}c^5d \times \frac{7}{4}cd^6$

6) $-45r^3s \times \frac{7}{9}s$

7) $5m^2n^3 \times \frac{3}{-4}mn^7$

8) $-4c^3 \times \frac{2}{3}cf^5$

9) $12b^4c^3 \div 3b^4c^3$

Find the missing monomials.

1) ________ $\times$ $-8x^4y$ $=$ $-96x^7y^2$

2) $-4v^7$ $\times$ ________ $=$ $68v^{12}$

3) ________ $\times$ $5m^3$ $=$ $-35m^7n$

4) $8p^6q^5$ $\times$ ________ $=$ $-64p^{12}q^{12}$

5) $4m^3n$ ____ $\times$ ____ mn^5p $=$ $-32m^4n^6p^4$

Multiplication
1) $12q^{11}rs$
2) $-49y^5z^5$
3) $40r^4s^4t^5$
4) u^2v^7
5) $\frac{1}{2}c^6d^7$
6) $-5r^3s^2$
7) $-\frac{15}{4}m^3n^{10}$
8) $-\frac{8}{3}c^4f^5$
9) 4

Missing Monmials
1) $12x^3y$
2) $-17v^5$
3) $-7m^4n$
4) $-8p^6q^7$
5) p^3 and -8

Definition of a Real Polynomial Expression

A polynomial expression has the form: $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$
where the coefficients $a_n, a_{n-1}, a_{n-2}, \ldots, a_3, a_2, a_1, a_0$ are real constants and *n* is a whole number.
{Expressions do not have any relation symbols like =, <, >, ≤, ≥, or ≠.} $4x^3 - 2x^2 + 3x - 5$

Definition of a Real Polynomial Equation

A polynomial equation has the form: $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$
where the coefficients $a_n, a_{n-1}, a_{n-2}, \ldots, a_3, a_2, a_1, a_0$ are real constants and *n* is a whole number. {All polynomial equations can become functions by getting all terms on one side of an equal sign, =. No function crosses the y-axis more than once. All functions have the relation symbol, =. Note: y = is in place of the f(x) = .} $f(x) = 4x^3 - 2x^2 + 3x - 5$

Expressions

Examples of sum(s) and/or difference(s):

Monomial Expressions have no + or – signs.

Binomial Expressions:

x + y, sum of monomials

x – y, difference of monomials

Trinomial Expressions:

x + y + 3, all sums

x + y – 3, a sum and a difference

x – y + 3, a difference and a sum

x – y – 3, all differences

An expression has n – 1 operators for its n-terms.

Equations and Inequalities

An **equation** has an expression = to another expression.

An **inequality** has an inequality sign replacing the =.

Definition of a Real Polynomial Inequality

A polynomial equation has the form: $f(x) < a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$

Similar to equations but with any of the inequality signs <, >, ≤, ≥, or ≠.

Definition: Degree of a Monomial

The degree of a monomial is the total value of the exponents of all its variables.

Definition: Degree of a Polynomial

The degree of a polynomial is determined by the monomial with the highest degree of all the individual monomial terms in the polynomial. If all (or group) are equal, alpha order is used: ($3x^2 + 4y^2 + 3xy$ each term has degree 2, correct order is $3x^2 + 3xy + 4y^2$.)

Key Points

A polynomial is of the form: $a_n x^n + a_{n-1}x^{n-1} + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$

- The degree of a monomial is the total value of all exponents of its factors.
- A polynomial is the sum and/or difference of monomials.
- The degree of a polynomial is the highest degree of the highest degreed monomial.
- When finding the degree of more intricate polynomials, we do not need to expand all the brackets. Instead, consider what the highest exponent would be as a result of the expansion.
  $(2xy^4)^5 = 2^5x^5y^{20} = 32x^5y^{20}$ has a degree of 25.

Monomial is a polynomial of one term.

Binomial is a polynomial sum or difference two terms.

Trinomial is a polynomial with the sum(s) and/or difference(s)[£] three terms.

Polynomial of n terms is used for all other sum(s) and/or difference(s) terms.

Coefficient Concepts

The coefficient is the number of times a value is added in a list of the same values.

3x = x + x + x
5x = x + x + x + x + x
3x + 5x = x + x + x + x + x + x + x + x
3x + 5x = 8x
5x - 3x = x + x + x + x + x
5x - 3x = 2x

Remember all single variables have a coefficient of 1 and an exponent of 1 that are not normally displayed.

$$x = 1 \bullet x^1$$

An expression has n – 1 operators for its n-terms.

Definitions used in this document:

Constant: any numeric value that does not change

Constant Expression: an algebraic expression with no variables in it.

Coefficients: the constant value of a monomial; number in front of variable(s); if you do not see a value, the value is 1. If -x, the coefficient is -1.

Exponent Concepts

An exponent is the number of times an element is multiplied in a list of the same elements.

$x^3 = x \bullet x \bullet x$
$x^5 = x \bullet x \bullet x \bullet x \bullet x$
$x^3 \bullet x^5 = x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x$
$x^3 \bullet x^5 = x^{3+5} = x^8$
$x^5 \div x^3 = \frac{x \bullet x \bullet x \bullet x \bullet x}{x \bullet x \bullet x} = x^2$
$x^5 \div x^3 = x^{5-3} = x^2$

Degree: the sum of the exponents of all variables in a monomial; strength of exponents

Exponents: powers of numbers or variables; for variables without an exponents, it is a one, 1.

Even vs Odd Functions:$^{\nabla}$ are determined by the degree of function.

Odd functions cross x-axis (1, 2, 3, … , n)

Even functions may have {0, 1,2, … ,n} crossings of the x-axis depending on the degree of the function, **n**. The value of the degree of a function equates to the number of solutions of the function.

Examples on HSE exams: most linear functions are odd—1 solution; all quadratic functions are even—0, 1, or 2 solutions. These are the only types of functions tested.

Imaginary Numbers: numbers which occur when you need to get the square root of a negative number (not tested on GED®, but it is important knowledge.) $\sqrt{-1} \stackrel{\text{def}}{=} i$

Integer: A number that is neither a fraction nor a decimal; the counting numbers, the negatives of the counting numbers, and 0. {…, -2, -1, 0, 1, 2, …} integers

Irrational Numbers: cannot be written as a ratio, their decimal parts never repeat a pattern

Like Terms: terms having same variables-exponents patterns

Negative Integers: An integer less than zero {…, -3, -2, -1} negative integers

Non-Negative Integers: An integer that is not negative; 0 and all positive integers {0, 1, 2, 3, …} a.k.a., whole numbers

Non-Zero Integers: all of the integers except zero {…, -2, -1, 1, 2, …}

Positive Integers: An integer greater than zero {1, 2, 3, …} natural numbers

Rational Numbers: numbers which can be written as a ratio (fraction), the numerator can be any integer and the denominator can be any non-zero integer. $\frac{integer}{non-zero\ integer}$

Rational Expression: the ratio of two polynomials (denominator ≠ 0) $\frac{polynomial\ p}{polynomial\ q}$

Standard Form: Writing polynomials in such a way that the highest degree comes first then in descending order by degree. Variables in the same term are written in alphabetical order. Also, the writing in descending order eliminates the highest initial variable terms before looking for the next highest degreed term to write. (Probably not formally tested.)

Terms: an alternate name for monomials elements

Variables: Any letters used to hold the place of numbers, non-constant factor.

Zeroes of the Function: The locations where the function crosses the x-axis (x-intercepts).

$^{\nabla}$On HSE tests, the **only** odd functions are either linear (1 solution) or cubic functions (0, 1, 2, or 3 solutions); the **only** even function is a quadratic function (0, 1, or 2 solutions).

Natural (Counting) numbers: N ={1, 2, 3, …}
*__Whole numbers__: W = {0, 1, 2, 3, …}
Integers: Z = {… , ⁻3, ⁻2, ⁻1, 0, 1, 2, 3, …}

∅ denotes the empty set, the set with no members
Q denotes the set of rational numbers
R denotes the set of real numbers

Rational numbers are all possible fractions including integers, $4 = \frac{4}{1}$.

Real numbers include all Rational and all Irrational numbers.

*https://dewwool.com/difference-between-rational-and-irrational-numbers/

Factoring Monomials

1) Which of the following are the factors of $2x^2$?

a) 2 b) $4x$ c) x d) x^2 e) $2x^5$

2) Which of the following are the factors of $30a^3b$?

a) $6ab$ b) $10a^2$ c) a^3 d) $4a^2$ e) b^2

3) Which of the following are the factors of $8m^5n^3$?

a) mn^3 b) $4m^3n$ c) $3m$ d) $2n^2$ e) m^5

4) Which of the following are the factors of $16x^2yz$?

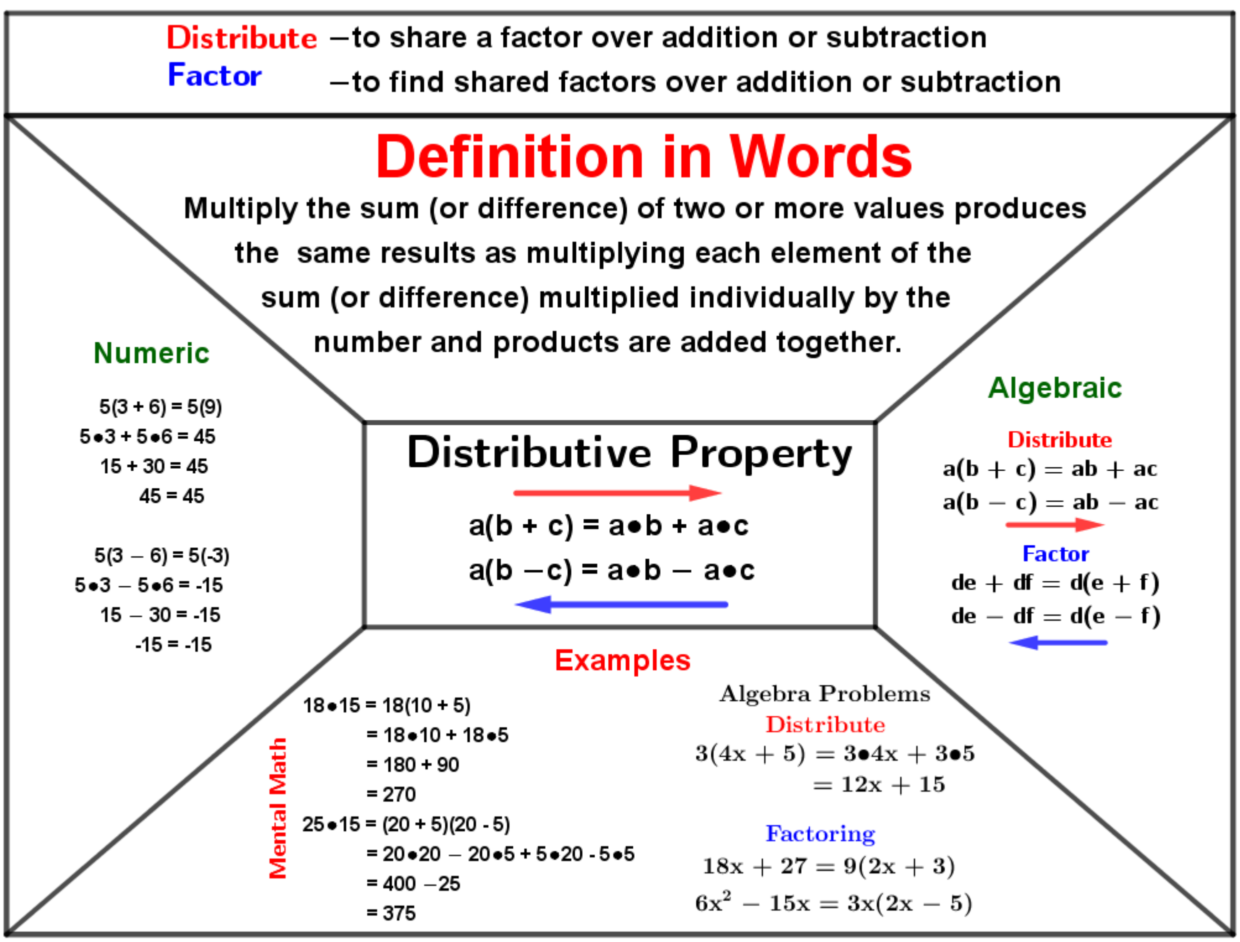
a) $4xyz$ b) $2xy$ c) xz d) 12 e) xyz

5) Which of the following are the factors of $20pr^3$?

a) $2p$ b) $6pr$ c) $5r^2$ d) $8r$ e) pr^2

6) List all possible factors of $6a^2b$; there are many possible factors.

1) a, c, d
2) a, b, c
3) a, b, d, e
4) a, b, c, e
5) a, c, e
6) 6: {1, 2, 3, 6}
a^2: {a, a^2} and {b}
{2a, 3a, 6a}
{2ab, 3ab, 6ab}
{$2a^2$, $3a^2$, $6a^2$}
{$2a^2b$, $3a^2b$, $6a^2b$}
{2b, 3b, 6b}
More?



2. **Binomials** (<u>Multiplying</u> binomials are in Section 11. Simple binomial <u>factors</u> are below.)

A binomial is an algebraic expression of the sum and/or the difference of two terms (monomials); a polynomial that is the sum/difference of two terms, each of which is a monomial, i.e., $5b + 4$, $3xy + 4x^3y$, $-\sqrt{131} + xy^3z$, $2x^3 - 8xy^7$

<u>Standard Form</u> for a binomial is when the highest degree term is first. $5b + 4$ is in standard form. The other three would be: $4x^3y + 3xy$, $xy^3z - \sqrt{131}$, and $-8xy^7 + 2x^3$. If two monomials have same degree, follow alphabetic order. Always use standard form in final answers.

Binomial forms can be factored if they have <u>like factors</u> in each terms:

- $5b + 4$ is not factorable.
- $3xy + 4x^3y$ has some common terms, they are x and y. The factors x and y can be factored: $4x^3y + 3xy = x\,y\,(4x^2 + 3)$.
- This binomial $xy^3z - \sqrt{131}$ is not factorable as the terms have no common factor.
- This binomial $2x^3 - 8xy^7$ is factorable as $2x(x^2 - 4y^7)$; in standard form is $-8xy^7 + 2x^3$ and factors as $2x(-4y^7 + x^2)$.
- Knowledge of factor list or factor pairs in each coefficient is needed to expedite the factoring: For #1 knowing that largest shared factor is 3 in each coefficient tells us what to factor out of each one. 6: {1, 2, 3, 6} and 9: {1, 3, 9}

Factoring Binomials (This is using the inverse of the distributive property to find the common factor in both terms.)

1) $6x + 9$
   $3(2x + 3)$

2) $-20y - 5z$

3) $3x + 6y$

4) $-18b - 48$

5) $2n - 2$

6) $-64m - 40$

7) $44p + 11q$

8) $42 + 35w$

9) $70q + 14r$

10) $5v - 5$

11) $45r^3s^2u - 15rs^2u^3$

12) $144x^{12}y^6z^3 + 168x^9y^4z^8$

1. $3(2x + 3)$
2. $-5(4y - z)$
3. $3(x + 2y)$
4. $-6(3b + 8)$
5. $2(n - 1)$
6. $-8(8m + 5)$
7. $11(4p + q)$
8. $7(5w + 6)$
9. $14(5q + r)$
10. $5(v - 1)$
11. $15rs^2u(3r^2 - u^2)$
12. $24x^9y^4z^3(6x^3y^2 + 7z^5)$

Section 13 covers binomial factoring of the difference of two squares.

3. Trinomials Find the Common Factors (See Section 14 Factoring Trinomials)

A trinomial is an algebraic expression of the sum and/or of the difference of three terms (monomials); a polynomial of three terms or monomials:

i.e., $5n - 2x + 158$, $3xy - 4x^3y + -43$, $-\sqrt{131} - xy^3z + x$

<u>Standard Form</u> of a trinomial is when the terms are written in descending degree order. The only trinomial above in standard form is: $5n - 2x + 158$. The other two would look as follows: $-4x^3y + 3xy + -43$ and $-xy^3z + x - \sqrt{131}$. If two or more monomials have same degree, follow alphabetic order.

Trinomials can have common factors, these <u>like factors</u> in each of the three terms can be factored out to simplify them, none of these examples above are factorable.

In $3x^3y + 6x^2y^2 - 9xy^3$, each term has the monomial $3xy$ as a factor, it factors as follows:

$$3x^3y + 6x^2y^2 - 9xy^3 = 3\,x\,y\,(x^2 + 2xy - 3y^2)$$

The factoring of common factors from all terms many times simplifies algebraic expressions for other factoring methods. Factoring trinomials which are the product of two binomials is covered in Section 14. The sum and difference of the coefficient's method is a tool to find the solutions to factorable equations. A second tool for trinomial called the quadratic formula:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. These and other methods will be discussed later.

Factoring Trinomials by finding the common factors in all 3 terms, write final answer in Standard Form.

1) $39u - 52v + 13$

2) $40b - 80c - 40d$

3) $49u - 63v + 28w$

4) $66g + 22h - 44$

5) $12r^2s - 15rs^2 + 18rs$

6) $121x^3y^6z^9 + 44x^2y^4z^6 - 55x^6y^3z^4$

1) $13(3u - 4v + 1)$
2) $40(b - 2c - d)$
3) $7(7u - 9v + 4w)$
4) $11(6g + 2h - 4)$
5) $3rs(4r - 5d + 6)$
6) $11x^2y^3z^4(11xy^3z^5 + 4yz^2 - 5x^4)$

4. Polynomials of n terms

A polynomial is an expression consisting of variables and coefficients, that involves only the operations of addition, subtraction, multiplication, and non-negative integer exponents. of variables. If division by a variable is indicated in any term, it is NOT a polynomial. Sections 1-3 describe the simple polynomials. While there is no limit to the number of terms in a polynomial, a polynomial is said to be "**<u>simplified</u>**" if all like terms are combined and there are no parenthesis or other grouping symbols in use.

5. Degree of a Polynomial (Review and Problems)

The **degree of a monomial** is the sum of the exponents of all its variables.

The degree of the monomial 75 is 0 (constants have degree 0).
The degree of the monomial 144m is 1 (= 0 + 1 since the power of m is 1).
The degree of the monomial $3xy^5b^3$ is 9 (= 0 + 1 + 5 + 3).

The degree of polynomials of more than one term is the degree of the monomial with the highest degree. This monomial is written first.

There are some things you ***cannot*** have within a monomial: you can't add or subtract any parts; divide by a non-constant factor (variable); raise a non-constant factor (variable) to a negative or variable power; and you cannot take absolute values of non-constant factors (variables). Examples of non-monomials:

$x + 7$, x - 3, $\frac{1}{x}$, y^{-2} or $\frac{1}{y^2}$, -15 x^{-2} y, 25 $\frac{xy}{z}$, $\frac{x^3}{x-3}$, |b|, x^y

Definition:
$$|b| = \begin{cases} b, b \geq 0 \\ -b. b < 0 \end{cases}$$

Some of these terms are <u>not monomials</u>, identify and circle them first. Then write the degree of each monomial (*no variable has a value of 0*).

1. $2u^3v^5w$ __________
2. -125x __________
3. 478 __________
4. $-x^2$ __________
5. $6s^2t^7u^0$ __________
6. $-\sqrt{136}$ __________
7. $|2x^2|$ __________
8. m __________
9. 5^{12} __________
10. $bc^{-3}d^3$ __________

1) 9
2) 1
3) 0
4) 2
5) 9
6) 0
7) Invalid
8) 1
9) 0
10) invalid

6. Standard Form of Polynomial Expressions

The Standard Form of the polynomial of ‘n’ is:

$$a_n x^n + a_{n-1} x^{n-1} \ldots + a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

$$y^5 - 3y^4 + y^3 - y$$

How to get a random polynomial in standard form:

- **Step 1:** Write out the terms. {Ensure every term is a valid monomial.}
- **Step 2:** Group all the like terms and simplify them.
- **Step 3:** Locate the exponents in the terms.
- **Step 4:** Write the term with the highest exponent first.
- **Step 5:** Write the rest of the terms in descending order of exponents, once all terms with that variable, start with the next highest term.
- **Step 6:** Write the constant term (a number with no variable) in the end.

Place the polynomial in standard form. If not a polynomial, indicate this.

$$7x^2yz - x^2 - y^2 + 8x^6yz + xy$$

____

$$a^3bc + bc^4 + 9a^7c - 2c - 6a$$

____

$$p^3 - q^2 - \frac{1}{p} + q^4$$

____

$$7uv - v^3w + \frac{u}{\sqrt{8}}$$

____

$$s^4t - 3tu^3 + 9s^6$$

____

While these are not testable items on the HSE exam, learning these ideas can help match to the correct solutions on multiple choice questions.

1) $8x^6yz + 7x^2yz - x^2 + xy - y^2$
2) $9a^7c + a^3bc - 6a + bc^4 - 2c$
3) Invalid
4) $-v^3w + 7uv + \frac{u}{\sqrt{8}}$
5) $9s^6 + s^4t - 3tu^3$

7. Adding and Subtracting Monomials

Monomials can be added/subtracted if they are like monomials (compatible), like 18 and 32 are the same type. 18 + 32 = 50, 18 – 32 = -14, 32 – 18 = 14, etc. Other monomials have variable(s) multiplied by the number, as long as each monomial has the exact same variables (they do not need to be in the same order.) 3x + 5x = 8x, 15xy – 5yx = 10xy (this is possible since 5xy = 5yx since the difference is an example of the Commutative Property of Equality, and each monomials is a power products.) **Note**: *In the end, it is common to write all monomial variables listed in alphabetical order for easier understanding or matching. When there are mixed powers, the monomial parts are written in descending order of the powers of the variable with the greatest value.*

Some answers may not have monomials solutions, circle the statement. Add these monomials. (Replace the "," with the operator "+", when needed.)

1) $2u + -7u$ 2) $-a^2 + 3a^2$ 3) $6x + 14$

4) $-4m, -5m$ 5) $-3c^3, c^3$ 6) $9p^4, 2p^6$

7) $-t, -2t$ 8) $17v, -17v$ 9) $7z^5, 4z^5$

10) $-21, -8k$ 11) $14c, 15c, -9c$ 12) $7z^5, -4z^5, 17z^5, -8z^5$

1) -5u
2) $2a^2$
3) Invalid, 6x + 14
4) -9m
5) $32c^3$
6) Invalid, $2p^6 + 9p^4$
7) -3t
8) 0
9) $11z^5$
10) Invalid, -8k – 21
11) 20c
12) $12z^5$

Some answers may not have monomials solutions, circle the statement. Subtract these monomials. (Replace the "," with the operator "–", when needed.)

1) $2u - (-7u)$

2) $-a^2 - 3a^2$

3) $6x - 14$

4) $-4m, -5m$

5) $-3c^3, c^3$

6) $9p^4, 2p^6$

7) $-t, -2t$

8) $17v, -17v$

9) $7z^5, 4z^5$

10) $-21, -8k$

11) $14c - (15c - -9c)$

12) $(7z^5 - -4z^5) + (-17z^5 - -8z^5)$

1) $5u$
2) $-4a^2$
3) Invalid, $6x - 14$
4) W
5) $-4c^3$
6) $-2p^6 + 9p^4$
7) t
8) $34v$
9) $3z^5$
10) Invalid, $8k - 21$
11) $-10c$
12) $2x^5$

8. Adding and Subtracting Polynomials

Polynomials are made up of monomials to create various expressions which can then be added or subtracted (or even multiplied.) If you have polynomial with compatible monomial terms, you can simplify the sums or differences. Note the different underling pattern of the like (compatible) terms. {When there is + in front parenthesis, the terms inside do not change signs; however, if the parenthesis has a leading minus sign, all signs within that parenthesis change to their inverse values.}

$$a - b = a + (-b)$$

$$5 - 6 = 5 + (-6)$$

$(3a + 5b - 3) + (5a + 15b - 12) = 8a + 20b - 15$

$3a + 5b + (-3) + 5a + 15b + (-12)$ (clear parenthesis)

$3a + 5a + 5b + 15b + (-3) + (-12)$ (reorder terms, commute order)

$8a + 20b + (-15) = 8a + 20b - 15$ (simplify like terms)

$(4xy - 6x + 3) - (2yx + 6y - 3) = 2xy - 6x - 6y + 6$ (show the steps like above)

1) $(2x^2 + 4x + 3) - (-6x^3 + 3 - x^2)$

2) $(-7c^4 + 14c) + (-2c + 4c^4)$

3) $(-10m^3 + 5m + 3m^2) - (5 + 6m^3 - 2m)$

4) $(-7b^3 + 11b) - (-b + 2b^3)$

5) $(9a^5 - a + 2a^3) - (5a^4 + 3a^5 + 2a)$

6) $-4b^3 + 5c^3 - (-5c^3 + 6b^3)$

1) $6x^3 + 3x^2 + 4x$
2) $-3c^4 + 12c$
3) $-16m^3 + 3m^2 - 7m - 5$
4) $-9b^3 + 12b$
5) $6a^5 - 5a^4 + 2a^3 - 3a$
6) $-10b^3 + 10c^3$

9. Multiplying Monomials

When you multiply monomials, first multiply the coefficients and then multiply the variables by adding the exponents. Note that when you multiply monomials with same base, you can add their exponents. This is called the <u>Product of Powers Rule</u>.

$(3xy)\ (4x^3y) = 3 \bullet 4 \bullet x \bullet x^3 \bullet y \bullet y$

$= 12x^4y^2$

Multiply these monomials. (Replace the "," with the operator "•".

1) $2u(-7u)$ 2) $(-a^2)3a^2$ 3) $(6x)(14)$

4) $-4m, -5m$ 5) $-3c^3, c^3$ 6) $9p^4, 2p^6$

7) $-t, -2t$ 8) $17v, -17v$ 9) $7z^5, 4z^5$

1) $-14u^2$
2) $-3a^4$
3) $84x$
4) $20m^2$
5) $-3c^6$
6) $18p^{10}$
7) $2t^2$
8) $-289v^2$
9) $28z^{10}$

<u>Methods of Multiplication</u>: Traditional, Box, and Distribution (FOIL).

$$\begin{array}{r} 25 \\ \underline{\times\ 43} \\ 75 \\ \underline{+100\ \ } \\ 1075 \end{array}$$

25 × 43 Box Method

×	40	3	43
20	800	60	860
5	200	15	215
25	1000	75	**1075**

Here is another algorithm using distributive property sometimes called the FOIL method but without the arrows:

25 × 43
(20 + 5) (40 + 3)
20(40 + 3) + 5(40 + 3) distribute 20 and 5 to the (40 + 3) expression
20×40 + 20×3 + 5×40 + 5×3 (F.O.I.L.)
800 + 60 + 200 + 15 = 1075

The values are reminiscent of the box method. Also, use this method with any multiplication you want:

348 × 1,459

(300 + 40 + 8)(1000 + 400 + 50 +9)

300×(1000 + 400 + 50 +9) + 40×(1000 + 400 + 50 +9) + 8×(1000 + 400 + 50 +9)

300,000 + 120,000 + 15,000 + 2,700 + 40,000 + 16,000 + 2,000 + 360 + 8,000 + 3,400 + 400 + 72

507,732

BTW, this methodology applies to the multiplication of all polynomial expressions.

10. Multiplying a Monomial with a Binomial or Trinomial

When you **multiply monomials**, first **multiply** the coefficients, and then **multiply** the variables by adding the exponents. Note that when you **multiply monomials** with same base, you can add their exponents. This is called the Product of Powers Property.

$$(12x^2y^3)\,(-3x^3 + 6y)$$

$$(12 \bullet -3 \bullet x^2 \bullet x^3 \bullet y^3) + (12 \bullet 6 \bullet x^2 \bullet y^3 \bullet y)$$

$$-36 \bullet x^{2+3} \bullet y^3 + 72 \bullet x^2 \bullet y^{3+1}$$

$$-36x^5y^3 + 72x^2y^4$$

$$3x^2y\,(2x^2 - \sqrt{5}xy + 3y^2)$$

$$3x^2y \bullet 2x^2 - 3x^2y \bullet \sqrt{5}xy + 3x^2y \bullet 3y^2$$

$$3 \bullet 2x^2\; x^2y - 3 \bullet \sqrt{5}\; x^2x\, y\, y + 3 \bullet 3\; x^2y\, y^2$$

$$6x^{2+2}y - 3\sqrt{5}\; x^{2+1}\; y^{1+1} + 9x^2\; y^{1+2}$$

$$6x^4y - 3\sqrt{5}\; x^3y^2 + 9x^2y^3$$

$$2xy\,(5x^2 + \sqrt{6}xy - 3y^2)$$

$$2xy \bullet 5x^2 + 2xy \bullet \sqrt{6}xy - 2xy \bullet 3y^2$$

$$2 \bullet 5x\; x^2y - 2 \cdot \sqrt{6} \cdot x \cdot x\, y\, y - 2 \bullet 3xy\; y^2$$

$$10x^{1+2}y - 2\sqrt{6}\; x^{1+1}\; y^{1+1} - 6x\; y^{1+2}$$

$$10x^3y - 2\sqrt{6}\; x^2y^2 - 6xy^3$$

$$-4x^3y\,(2x^4 - 12xy + 8y^3)$$

$$-4x^3y \bullet 2x^4 - (-4x^3y) \bullet 12xy + (-4x^3y) \bullet 8y^3$$

$$-4 \bullet 2x^3\; x^4y - (-4) \bullet 12\; x^3x\, y\, y$$

$$+ (-4) \bullet 8\; x^3y\; y^3$$

$$-8x^{3+4}y - (-48)\; x^{3+1}\; y^{1+1} + (-32)x^3\; y^{1+3}$$

$$-8x^7y + 48\; x^4y^2 - 32x^3y^4$$

Notice the use of the **box method** to multiply 46•39!

The **box method** can be used to expand and simplify a binomial product: (4x + 6)(3x + 9)

Calculate 46 × 39

×	40	6	
30	1200	180	1380
9	360	54	414
			1794

(40 + 6)(30 + 9)

= 1200 + 180 + 360 + 54

= 1794

$$\begin{array}{rrrr} & & 4 & 6 \\ & \times & 3 & 9 \\ \hline & 4 & 1 & 4 \\ 1 & 3 & 8 & 0 \\ \hline 1 & 7 & 9 & 4 \end{array}$$

Expand and Simplify (4x + 6)(3x + 9)

×	4x	6	
3x	$12x^2$	18x	$12x^2 + 18x$
9	36x	54	36x + 54
			$12x^2 + 54x + 54$

(4x + 6)(3x + 9)

= $12x^2 + 18x + 36x + 54$

= $12x^2 + 54x + 54$

$$\begin{array}{rrrrr} & 4x & + & 6 \\ \times & 3x & + & 9 \\ \hline & 36x & + & 54 \\ 12x^2 + & 18x & & \\ \hline 12x^2 + & 54x & + & 54 \end{array}$$

And if x was equal to 10 then…

11. Multiplying two Binomials

Many teachers teach the **FOIL** Method. However, FOIL **only** works for the product of two binomials. A preferred method is to use the <u>Distributive Property of Equality</u>, illustrated on the previous page by multiplying a monomial times each binomial term; one method for all products. This table shows the different possibilities of the binomial products. Binomial products can have 2, 3, or 4 terms.

Basic Distribution	Common Examples	Box Method
(a + b)(c + d) = a(c + d) + b(c + d) = **a·c** + **a·d** + **b·c** + **b·d** = **ac** + **ad** + **bc** + **bd** **F. O. I. L.**	(x + 3)(x + 5) = x (x + 5) + 3(x + 5) = x^2 + 5x + 3x + 15 **F. O. I. L.** = x^2 + 8x + 15	(a + b)(c + d)
(a - b)(c - d) = a(c - d) - b(c - d) = a·c - a·d - b·c + b·d = **ac** - **cd** - **bc** + **bd**	(x - 3)(x - 5) = x (x - 5) - 3(x - 5) = x^2 - 5x - 3x + 15 = x^2 - 8x + 15	
(a + b)(c - d) = a(c - d) + b(c - d) = a·c - a·d + b·c - b·d = **ac** - **cd** + **bc** - **bd**	(x + 3)(x - 5) = x (x - 5) + 3(x - 5) = x^2 - 5x + 3x - 15 = x^2 - 2x - 15	(a - b)(c + d)
(a - b)(c + d) = a(c + d) - b(c + d) = a·c + a·d - b·c - b·d = **ac** + **cd** - **bc** – **bd**	(x - 3)(x + 5) = x (x + 5) - 3(x + 5) = x^2 + 5x - 3x - 15 = x^2 + 2x – 15	
The color coding above matches that for **F.O.I.L.** below.	The color coding above matches that for **F.O.I.L.** below.	

(a + b)(c + d)

×	c	d	
a	**a·c**	**a·d**	**a·c** + **a·d**
b	**b·c**	**b·d**	**b·c** + **b·d**
			ac + **ad** + **bc** + **bd**

(a - b)(c + d)

×	c	d	
a	**a·c**	**a·d**	**a·c** + **a·d**
-b	**-b·c**	**-b·d**	**-b·c** - **b·d**
			ac + **ad** - **bc** - **bd**

Reversing the process above is called **factoring**. On the HSE exams, factoring a trinomial product into its original binomial factors is a common requirement. A deep examination these examples allows us to find a method to accomplish the common feat.

F.O.I.L. Method can make a "Happy Face", only for binomial factors.

$$(3x + 5)(4x + 7) = \underset{F}{3x\bullet 4x} + \underset{O}{3x\bullet 7} + \underset{I}{5\bullet 4x} + \underset{L}{5\bullet 7}$$

$$= \mathbf{12x^2 + 21x + 20x + 35} \quad ¥$$

$$= 12x^2 + \mathbf{41}x + 35$$

F.O.I.L. can be done without making the "Happy Face" using distributive property (always works).

$$(3x + 5)(4x + 7)$$
$$3x(4x + 7) + 5(4x + 7)$$
$$3x \cdot 4x + 3x \cdot 7 + 5 \cdot 4x + 5 \cdot 7)$$
F O I L
$$\mathbf{12x^2 + 21x \ + 20x + 35}$$
$$12x^2 + \mathbf{41x} + 35$$

¥ **Note:** **12** * **35** = 420, two other factors of 420 are **21** and **20**, whose sum is **41**.
This is a clue as to how to factor trinomials which are binomial products. The ratio of 'a' to each of these addends yields the coefficient and constant of each factor. (Page 17)

Factor pairs of 420:

{1, 420}
{2, 210}
{3, 140}
{5, 85}
{6, 70}
{7, 60}
{10, 42}
{12, 35} values of 'a' and 'c', × 460
{14, 30}
{15, 28}
{20, 21} addends for 'b', add 41

Look for a factor pair which adds to **41** (middle term). The ratio of 'a' to each of the pair provides the coefficient and constant of the binomial factors.

- **Factors of 420:**
  {1, 2, 3, 4, 5, 6, 7, 10, 12, 14, 15, 20 , 21, 28, 30, 35, 42, 60, 70, 84, 105, 140, 210, 420}

  {(1, 420), (2, 210), (3, 140), (4, 105), (5, 84), (6, 70), (7, 60), (10, 42), (<u>12, 35</u>), (14, 30), (15, 28), (<u>20, 21</u>)}

The ratio of coefficient 'a' to each of the sum addends provide the coefficient and constant term of each factor.

$$\frac{a}{addend\#1} \text{ and } \frac{a}{addend\#2}$$
$$\frac{12}{20} = \frac{3}{5} \text{ and } \frac{12}{21} = \frac{4}{7}$$
(3x + 5)(4x + 7)

- **Prime Factorization of 420 : 2 × 2 × 3 × 5 × 7 = 2^2 × 3 × 5 × 7**
  Most binomial multiplications result in trinomials; however, there is a binomial product which results in new binomial (Page 19). These binomial products are identified by being the **difference between squares**, it is the product of the **sum and difference of two values**.

$$(x + a)(x - a) = x^2 - ax + ax + a^2$$
$$x^2 - a^2 = (x + a)(x - a)$$

Find the product of the following binomials.

1. $(x + 3)(x + 3)$
2. $(x - 3)(x - 3)$
3. $(x + 3)(x - 3)$
4. $(2x + 3)(x + 4)$
5. $(3x - 4)(x - 8)$
6. $(5x + 8)(5x - 8)$
7. $(2x^2 + 1)(x + 12)$
8. $(x^3 - 2)(x^2 - 5)$
9. $(x - 3)(x + 3)$
10. $(x + 3)^2$
11. $(x - 3)^2$
12. $(2x + 3)(x - 3)$
13. $(5x + 2)(5x + 8)$
14. $(5x - 2)(5x - 8)$
15. $(5x + 2)(5x - 8)$

1) $x^2 + 6x + 9$
2) $x^2 - 6x + 9$
3) $x^2 - 9$
4) $2x^2 + 11x + 12$
5) $3x^2 - 28x + 32$
6) $25x^2 - 64$
7) $2x^2 + 25x + 12$
8) $x^5 - 5x^3 - 2x^2 - 10$
9) $x^2 - 9$
10) $x^2 + 6x + 9$
11) $x^2 - 6x + 9$
12) $2x^2 - 3x - 9$
13) $25x^2 + 50x + 16$
14) $25x^2 - 50x + 16$
15) $25x^2 - 30x - 16$

12. Multiplying Other Polynomials

Binomial times a Trinomial

$(4x^2 - 3)(5x^2 + 7x - 3)$

×	$5x^2$	$7x$	$- 3$
$4x^2$	$4x^2 \cdot 5x^2$	$4x^2 \cdot 7x$	$- 4x^2 \cdot 3$
$- 3$	$- 3 \cdot 5x^2$	$- 3 \cdot 7x$	$- (- 3 \cdot 3$

$4x^2(5x^2 + 7x - 3) - 3(5x^2 + 7x - 3)$

$4x^2 \cdot 5x^2 + 4x^2 \cdot 7x - 4x^2 \cdot 3 - 3 \cdot 5x^2 + - 3 \cdot 7x - (- 3 \cdot 3)$

F. _____ O. I. ____ L.

(The ___s represents × 1st and last terms with middle terms.)

$20x^{2+2} + 28x^{2+1} - 12x^2 - 15x^2 - 21x + 9$

$20x^4 + 28x^3 - 27x^2 - 21x + 9$

Trinomial times a Trinomial

$(x^2 - 4x + 3)(5x^2 + 10x - 5)$

$x^2(5x^2 + 10x - 5) - 4x(5x^2 + 10x - 5) + 3(5x^2 + 10x - 5)$

$x^2 \cdot 5x^2 + x^2 \cdot 10x - x^2 \cdot 5 - 4x \cdot 5x^2 - 4x \cdot 10x - (-4x \cdot 5) + 3 \cdot 5x^2 + 3 \cdot 10x - 3 \cdot 5$

$5x^{2+2} + 10x^{2+1} - 5x^2 - 20x^{2+1} - 40x^{1+1} + 20x + 15x^2 + 30x - 15$

$5x^4 + 10x^3 - 5x^2 - 20x^3 - 40x^2 + 20x + 15x^2 + 30x - 15$

$5x^4 + 10x^3 - 20x^3 - 5x^2 - 40x^2 + 15x^2 + 20x + 30x - 15$

$5x^4 - 10x^3 - 30x^2 + 50x - 15$

This method of distributing elements of one polynomial to the entire second polynomial works for all polynomial multiplications and reduces confusion. For these examples, no mental math was used in initial steps for emphasis.

A. $(3x^2 + 5)(6x^3 - 3x^2 - 5x + 1)$
$3x^2(6x^3 - 3x^2 - 5x + 1) + 5(6x^3 - 3x^2 - 5x + 1)$
…
$18x^5 - 9x^4 + 15x^3 - 12x^2 - 25x + 5$

B. $(3x^2y - 2xy^2 + y^3)(2xy^3 + 3x^2y^2 - 4x^3y)$
$3x^2y(2xy^3 + 3x^2y^2 - 4x^3y) - 2xy^2(2xy^3 + 3x^2y^2 - 4x^3y) + y^3(2xy^3 + 3x^2y^2 - 4x^3y)$
…
$-6x^3y^5 + 2xy^6 + x^2y^5 + 2x^3y^4 + 17x^4y^3 - 12x^5y$

HINT: Students need to pay attention to how the simplifications are done as the next level of exercises is doing the reverse of the above procedures. This process in called factoring. *A student's knowledge of basic multiplication facts (1-16), squares (1-25), cubes (1-10), their inverse operation, and prime factorization will assist here, greatly.*

Expand these:

A. $(3x^2 + 5)(6x^3 - 3x^2 - 5x + 1)$

B. $(3x^2y - 2xy^2 + y^3)(2xy^3 + 3x^2y^2 - 4x^3y)$

13. Factoring the Binomial Difference of Two Squares[¥]

Binomials are polynomial **factors** with exactly two terms (monomials). Finding **binomial factors** can be easily solved by looking at the factors of each term of binomial. The identical factors are the ones that are the same in each term. This can be applied to every polynomial to reduce the efforts in factoring.

3ab + 6ac = 3a•b + 3a•2c the shared factors are 3a in each term.
= 3a (b + 2c) This is the factored form.

The special binomial which known as the **difference of two squares** factors as the sum and difference of the square roots of the terms: $x^2 - a^2 = (x - a)(x + a)$.

$(x - a)(x + a)$
$x^2 + ax - ax - a^2$
$x^2 - a^2$

Examples:

$x^2 - 25 = (x - 5)(x + 5)$ $\quad$ $9x^2 - 25 = (3x - 5)(3x + 5)$

$x^2 - 5 = (x - \sqrt{5})(x + \sqrt{5})$ $\quad$ $5x^2 - 15 = (\sqrt{5}x - \sqrt{15})(\sqrt{5}x + \sqrt{15})$

Factor each of the following binomials which are the difference of two squares.

1. $x^2 - 1$ 2. $64x^2 - 4$ 3. $36 - 100x^2$

4. $1 - 36x^2$ 5. $49 - 4x^2$ 6. $81x^2 - 9$

Simplify, then factor.

7. $2x^2 - 6 + 4x^2 + 3x^2 - 10$ 8. $-32 + 60x^2 - 4 + 4x^2$

9. $75x^2 - 8(4) - 25x^2$ 10. $4x^2 - 7$

1) $(x - 1)(x + 1)$
2) $(8x - 2)(8x + 2)$
$4(4x - 1)(4x + 1)$
3) $(6 - 10x)(6 + 10x)$
$4(3 - 5x)(3 + 5x)$
4) $(1 - 6x)(1 + 6x)$
5) $(7 - 2x)(7 + 2x)$
6) $(9x - 3)(9x + 3)$
$9(3x - 1)(3x + 1)$
7) $(3x - 4)(3x + 4)$
8) $(8x - 6)(8x + 6)$
$4(4x - 3)(4x + 3)$
9) $2(5x - 4)(5x + 4)$
10) $(2x - \sqrt{7})(2x + \sqrt{7})$

Items in **bold** are fully factored expressions.

[¥]Factoring the **Binomial Sum of Two Squares** results in imaginary roots ($\sqrt{-1}$), and it is not an HSE topic.

14. Factoring Trinomials¥

When factoring trinomials, there several ways in which trinomials factor:

1) Whenever polynomials have common factors in every term, factor the term out,
2) Some trinomials are the result of the product of two binomials,
3) Some trinomials are not factorable by any of the above methods. This method requires the quadratic formula (discriminant) to solve.

To **factor** a **trinomial** in the form $ax^2 + bx + c$, find two integers, r and s, whose product is c and whose sum is "b". Rewrite the **trinomial** as $ax^2 + rx + sx + c$ and then use grouping and the distributive property to **factor** the polynomial. The resulting **factors** will be $(x + r)$ and $(x + s)$. If a trinomial is part of an equation the factored trinomial can assist in find a solution to the equation. (See ¥ in Binomial Multiplication, Page 16.) **Factoring** a polynomial is the first step to finding its roots.

Keys to factoring polynomials:

1. Understand monomial, binomial, and polynomials multiplication.
2. Be able to find all factors a value.
3. Recognize Difference of Squares.
4. Familiar with binomial products.

Leading coefficient a = 1

Factor: $s^2 + 13st + 30t^2$ [Expression]

1 x 30 = 30

Factors of 30: {1, 2, 3, 5, 6, 10, 15, 30}

adds to 13

Factor pairs of 30: {(1, 30), (2, 15), **(3, 10)**, (5, 6)}

$$s^2 + 13st + 30t^2 = s^2 + 3st + 10st + 30t^2$$

$$= s(s + 3t) + 10t(s + 3t) = (s + 3t)(s + 10t)$$

Factor: $z^2 - 57 = 16z$, the **equation** must be written in **standard form**. **[Equation]**

$z^2 - 16z - 57 = 0$ [A function must be equal to zero to be factored.]

1 • –57 = –57

Negative Factor sets of –57: {1, 3, –19, –57} or {–1, –3, 19, 57}

adds to –16

Factor pairs of -57: {(1, –57), **(3, –19)**, (–1, 19), (–3, 57)}

$z^2 - 19z + 3z - 57 = 0$

$z(z - 19) + 3(z - 19) = 0$

$(z + 3)(z - 19) = 0$, since this is an equation, so z has two solutions zero products property.

$z + 3 = 0$	and	$z - 19 = 0$
$z + 3 - 3 = 0 - 3$	and	$z - 19 + 19 = 0 + 19$
$z = -3$	and	$z = 19$

Note: If you cannot find factors to add to "**b**", the trinomial is "**not factorable**"! These problems are done by quadratic formula, Part 15 Page 27.

¥Introduced in Section 11, Multiplying Binomials

Factor each trinomial.

1. $x^2 - 9x + 14$	2. $a^2 - 9a - 36$	3. $x^2 + 2x - 15$
4. $n^2 + 8n + 15$	5. $b^2 + 22b + 21$	6. $c^2 + 2c - 3$
7. $x^2 - 5x - 24$	8. $n^2 - 8n + 7$	9. $m^2 - 10m - 39$
10. $z^2 + 15z + 36$	11. $s^2 - 13st - 30t^2$	12. $y^2 + 2y - 35$
13. $r^2 + 3r - 40$	14. $x^2 + 5x - 6$	15. $x^2 - 4xy - 5y^2$
14. $r^2 + 16r + 63$	15. $v^2 + 24v - 52$	18. $k^2 - 27jk - 90j^2$

Factor each trinomial. (Solutions)

1. (x - 2) (x - 7)	2. (a - 12) (a + 3)	3. (x - 3) (x + 5)
4. (n - 5) (n - 3)	5. (b + 1) (b + 21)	6. (c - 1) (c + 3)
7. (x + 3) (x – 8)	8. (n – 1) (n – 7)	9. (m – 13) (m + 3)
10. (z + 3) (z + 12)	11. (s + 2t) (s - 15t)	12. (y + 7) (y - 5)
13. (r + 5) (r - 8)	14. (x - 1) (x + 6)	15. (x – 5y) (x + 1y)
16. (r + 7) (r + 9)	17. (v - 2) (v + 26)	18. (k + 3j) (k – 30j)

Additional Examples

$x^2 + 6x + 8$

Find the coefficients (a, b, c):

a=1, b=6, c=8

For factoring we examine a·b

1 · 8 = 8

Examine the factor of 8 to find two factors whose sum is b: 6.

8: {1,2,4,8}

1 x 8 2 x 4

1 + 8 **2 + 4**

$x^2 + 6x + 8$

$x^2 + 4x + 2x + 8$

$x^2 + 4x + 2x + 8$

$x(x + 4) + 2(x + 4)$

(x+4)(x+2)

$x^2 + 4x - 5$

Find the coefficients (a, b, c):

a=1, b=4, c=-5

For factoring we examine a·b

1 · -5 = -5

Examine the factor of -5 to find two factors whose sum is b: 4.

-5: {1,-5} or {-1,5}

1 x -5 -1 x 5

1 + -5 **-1 + 5**

$x^2 + 4x - 5$

$x^2 + 5x - x - 5$

$x^2 + 5x - x - 5$

$x(x + 5) - (x + 5)$

(x+5)(x-1)

$4x^2 - 8x + 3$

Find the coefficients (a, b, c):

a=4, b=⁻8, c=3

For factoring we examine a•b: 4•3 = 12

Examine the factors of 12 to find two factors whose sum is b: 4.

12: {1,2,3,4,6,12}

12: {1,2,3,4,6,12}, or

12: {-1,-2,-3,-4,-6,-12}

Since, b = -8 or -2 + -6 or -6 – 2

$4x^2 - 8x + 3$

$4x^2 - 6x - 2x + 3$

$4x^2 - 6x - 2x + 3$

$2x(2x - 3) - (2x - 3)$

(2x – 3)(2x - 1)

$x^2 - 3x - 4 = 0$

Find the coefficients (a, b, c):

a=1, b=-3, c=-4

For factoring we examine:

a•b = 1• -4 = -4

Examine the factors of -4 to find two factors whose sum is b, -3.

-4: {-1,-2,2,4} -4: {1,2,-2,-4}

3 **-3**

$x^2 - 3x - 4 = 0$

$x^2 + x - 4x - 4 = 0$

$x^2 + x - 4x - 4 = 0$

$x(x + 1) - 4(x + 1) = 0$

(x+1)(x-4)=0

Find the solution:

x + 1 = 0 or x - 4 = 0

x+1-1=0-1 or x-4+4=0+4

x = -1 or x = 4 Set notation **{-1, 4}**

The three without an equal sign are expressions. The fourth one has an equal sign and is called a function, it requires additional steps to complete. If faction does not seem to work, check if the discriminant < 0 which shows the expression is NOT factorable. (**D = b^2 – 4ac**)

Solve each equation. Check your solutions, the Zeroes of the Function.

19. $a^2 + 3a - 4 = 0$

20. $x^2 - 8x - 2 = 0$

21. $b^2 + 11b + 24 = 0$

22. $y^2 + y - 42 = 0$

23. $k^2 + 2k - 24 = 0$

24. $r^2 - 13r - 48 = 0$

25. $n^2 - 9n = -18$

26. $2z + z^2 = 35$

27. $-20x + 19 = -x^2$

28. $10 + a^2 = -7a$

29. $z^2 - 57 = 16z$

30. $x^2 = -14x - 33$

31. $22x - x^2 = 96$

32. $-144 = q^2 - 26q$

33. $x^2 + 84 = 20x$

Solve each equation. (Solutions)

19. (a - 1) (a + 4) = 0; {-4, 1}
20. (x + 2) (x - 10) = 0; {-2, 10}
21. (b + 8) (b + 3) = 0; {-8, -3}
22. (y - 6) (y + 7) = 0; {-7, 6}
23. (k + 6) (k - 4) = 0; {-6, 4}
24. (r + 3) (r - 16) = 0; {-3, 16}
25. (n – 3) (n – 6) = 0; {3, 6}
26. (z + 7) (z - 5) = 0; {-7, 5}
27. (x – 1) (x – 19) = 0; {1, 19}
28. (a + 2) (a + 5) = 0; {-2, -5}
29. (z + 3) (z - 19) = 0; {-3, 19}
30. (x + 11) (x + 3) = 0; {-11, -3}
31. (x - 6) (x - 16) = 0; {6, 16}
32. (q - 8) (q - 18) = 0; {8, 18}
33. (x - 4) (x - 16) = 0; {4, 16}

https://starless-suite.blogspot.com/2015/03/30-factoring-x2-bx-c-worksheet-answers.html

When the leading coefficient a ≠ 1

When the leading coefficient is different from one, most of the same instructions apply, however, you may be required to factor elements of the sum of the product of first coefficient and constant terms to spread across the coefficients of the terms of the factored expression. See below.

$3x^2 \underline{- 8x} + 5$
$= 3x^2 \underline{- 3x - 5x} + 5$
$= 3x(x - 1) - 5(x - 1)$
$= (x - 1)(3x - 5)$

If we multiply a and c (3 • 5), the product is 15. Looking at both factor sets for pair to add to **–8**, the b-value.

15: {1, 3, 5, 15} or {-1, -3, -5, -15} or {(1, 15), (3, 5), (-1, -15), (-3, -5)}

-3 + -5 = -8 is where we get the middle term, b, of the quadratic equation.

When the product of two binomials is in an equation with a sum is 0, one or both factors must be 0. a × b, either a = 0 or b = 0 or both are equal 0. (**Zero Products Rule**)

So, when $3x^2 - 8x + 5 = 0$, factor the left-side first, then set each factor to 0. Solve each equation for its unknown value. It is a good idea to check your results.

Solve for the x-intercepts:

$3x - 5 = 0$ and $x - 1 = 0$
$3x - 5 + 5 = 0 + 5$ and $x - 1 = 0$
$3x = 5$ and $x - 1 + 1 = 0 + 1$
$\frac{3}{3}x = \frac{5}{3}$ and $x + 0 = 1$
$x = \frac{5}{3}$ and $x = 1$

Checking you work for all x-intercepts:

$0 = 3x^2 - 8x + 5$, let $x = 1$
$0 = 3(1^2) - 8(1) + 5$
$0 = 3 - 8 + 5$
$0 = -5 + 5$
$0 = 0$

$0 = 3x^2 - 8x + 5$, let $x = \frac{5}{3}$
$0 = 3\left(\frac{5}{3}\right)^2 - 8\left(\frac{5}{3}\right) + 5$
$0 = 3\left(\frac{25}{9}\right) - \left(\frac{40}{3}\right) + 5$
$0 = \frac{25}{3} - \frac{40}{3} + 5$
$0 = -\frac{15}{3} + 5$
$0 = -5 + 5$
$0 = 0$

https://www.youtube.com/watch?v=CuolpIqSdd0 Leading Coefficient
https://www.youtube.com/watch?v=KUMhpKGwpCY Factor Polynomials

More detail on Factoring Quadratic Equations:

Factoring when a = 1: $x^2 + 5x + 6$
Multiply a•c and call the product m•n

1. Find the factor set of m•n
2. Select the factors of m•n which add to b
3. Example: 1•6 = 6 : {1, **2**, **3**, 6} or {(1, 6), (**2**, **3**)}
   a. m + n = 2 + 3 (b-value)
   b. $x^2 + 2x + 3x + 6$
   c. $x(\boldsymbol{x + 2}) + 3(\boldsymbol{x + 2})$
   d. $(x + 2)(x + 3)$

Alternately,

a. $m_f + n_f = 2 + 3$ (the b-value of 5)
b. $\frac{a}{n_f} = \frac{\mathbf{1}}{\mathbf{2}}$ and $\frac{a}{m_f} = \frac{\mathbf{1}}{\mathbf{3}}$
   $(x + \mathbf{2})(x + \mathbf{3})$

Factoring when a ≠ 0: $5x^2 + 12x + 4$
Multiply a•c and call the product m•n

1. Find the factor set of m•n
2. Select the factors of m•n which add to b
3. Example: 5•4 = 20: {1, 2, **4**, **5**, 10, 20}
   {(1, 20), (2, 10), (**4**, **5**)}
   a. $m_f + n_f = 10 + 2$ (the b-value of 12)
   b. $5x^2 + 10x + 2x + 4$
   c. $5x(x + 2) + 2(x + 2)$
   d. $(x + 2)(5x + 2)$

Alternately,

a. $m_f + n_f = 10 + 2$ (the b-value of 12)
c. $\frac{a}{n_f} = \frac{\mathbf{5}}{\mathbf{10}}$ *or* $\frac{\mathbf{1}}{\mathbf{2}}$ and $\frac{a}{m_f} = \frac{5}{2}$
d. $(\mathbf{1}x + \mathbf{2})(\mathbf{5}x + \mathbf{2})$ **{1x ≡ x}**

In the alternative factoring method above, the $\boldsymbol{m_f}$ and $\boldsymbol{n_f}$ denominators' numerator is the coefficient of the first binomial factor's unknown (a).

Factor this quadratic expression: $5x^2 - 3x + 8$; where a = 5, b = -3, c = 8; a•c = 5•8 or 40

The factors of 40 are {1, **2**, 4, **5**, ***-8***, -10, **-20**, -40} or {-1, **-2**, -4, ***-5***, ***8***, 10, **20**, 40}. Examine each factor pair for a match.

Since the difference 5 – 8 = -3 we get: $5x^2 + 5x - 8x + 8$, which we can factor.

Or, 5 + -8 = -3 ➔ $\frac{a}{m_f}, \frac{a}{n_f}$ or a:mfactor, a:nfactor $\frac{5}{-8}, \frac{5}{5} = \frac{1}{1}$; 5:-8 ➔ (5x – 8) and 1:1 ➔ (x + 1)

Using a = 5 as the numerator and each factor as a denominator gives factored form:

(5x – 8)(x + 1)

Factoring Trinomial Squares with Leading Coefficient Different from 1.

Factor each completely. Every multiple of 3 is a function, so you need to find the zeros of the function. If an expression is un-factorable, state that in your answer.

1) $7m^2 + 6m - 1$

2) $3k^2 - 10k + 7$

3) $5x^2 - 36x - 81 = 0$

4) $2x^2 - 9x - 81$

5) $3n^2 - 16n + 20$

6) $2r^2 + 7r - 30 = 0$

7) $5k^2 + 8k + 80$

8) $5x^2 - 14x + 8$

9) $7p^2 - 20p + 12 = 0$

10) $3v^2 + 14v - 49$

11) $7x^2 - 26x - 45$

12) $5p^2 - 52p + 20 = 0$

13) $5x^2 - 43x + 24$

14) $5x^2 + 26x + 24$

15) $3r^2 + 40r + 100 = 0$

16) $2x^2 - 3x - 5$

17) $5p^2 + 19p + 12$

18) $2m^2 + 3m - 27 = 0$

19) $3n^2 + 10n - 8$

20) $2a^2 + 7a - 7$

21) $10n^2 - 21n - 49 = 0$

22) $6x^2 + 41x + 70$

23) $9x^2 + 9x - 40$

24) $8n^2 + 71n - 90 = 0$

25) $4m^2 - 4m - 63$

26) $6r^2 + 37r + 45$

27) $4x^2 - 35x + 24 = 0$

28) $10m^2 + 23m + 6$

29) $6k^2 - 10k + 50$

30) $6r^2 - 17r + 12 = 0$

Answers to Factoring Trinomial Squares with Leading Coefficient Different from 1

1) $(7m - 1)(m + 1)$	2) $(3k - 7)(k - 1)$	3) $(5x + 9)(x - 9)$ $\{-\frac{9}{5}, 9\}$	4) $(2x + 9)(x - 9)$
5) $(3n - 10)(n - 2)$	6) $(2r - 5)(r + 6)$ $\{\frac{5}{2}, -6\}$	7) Not factorable	8) $(5x - 4)(x - 2)$
9) $(7p - 6)(p - 2)$ $\{\frac{6}{7}, 2\}$	10) $(3v - 7)(v + 7)$	11) $(7x + 9)(x - 5)$	12) $(5p - 2)(p - 10)$ $\{\frac{2}{5}, 10\}$
13) $(5x - 3)(x - 8)$	14) $(5x + 6)(x + 4)$	15) $(3r + 10)(r + 10)$ $\{-\frac{10}{3}, -10\}$	16) $(2x - 5)(x + 1)$
17) $(5p + 4)(p + 3)$	18) $(2m + 9)(m - 3)$ $\{-\frac{9}{2}, 3\}$	19) $(3n - 2)(n + 4)$	20) Not factorable
21) $(5n + 7)(2n - 7)$ $\{-\frac{7}{5}, \frac{7}{2}\}$	22) $(3x + 10)(2x + 7)$	23) $(3x - 5)(3x + 8)$	24) $(n + 10)(8n - 9)$ $\{-10, \frac{9}{8}\}$
25) $(2m + 7)(2m - 9)$	26) $(3r + 5)(2r + 9)$	27) $(x - 8)(4x - 3)$ $\{8, \frac{3}{4}\}$	28) $(m + 2)(10m + 3)$
29) Not factorable	30) $(2r - 3)(3r - 4)$ $\{\frac{3}{2}, \frac{4}{3}\}$	https://www.lavc.edu/math/study-material2/Math-113-114-115.aspx	

15. Using Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

When a function is un-factorable, the quadratic formula is used to find the x-intercepts if they exist. (HSE test only find **real solutions**.)

$$ax^2 + bx + c = 0$$

$$x = \frac{-\boxed{} \pm \sqrt{\boxed{}^2 - 4 \cdot \boxed{} \cdot \boxed{}}}{2 \cdot \boxed{}}$$

$$f(x) = -x^2 + 4x + 6$$

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x_1 = \frac{-4 + \sqrt{4^2 - 4 * -1 * 6}}{2 * -1}$$

$$x_1 = \frac{-4 + \sqrt{40}}{-2} = \frac{-4 + 2\sqrt{10}}{-2}$$

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$x_2 = \frac{-4 - \sqrt{4^2 - 4 * -1 * 6}}{2 * -1}$$

$$x_2 = \frac{-4 - \sqrt{40}}{-2} = \frac{-4 - 2\sqrt{10}}{-2}$$

$$\textbf{Or } x_1 = 2 - \sqrt{10} \text{ or } x_2 = 2 + \sqrt{10}$$

Intro to Quadratic Equations https://www.geogebra.org/m/mEs37yMj#chapter/737388 This is an interactive website where you can practice Using the Quadratic Formula and other quadratic lessons.

YouTube link about quadratic formula: https://youtu.be/p0T_Z3PcR6Y

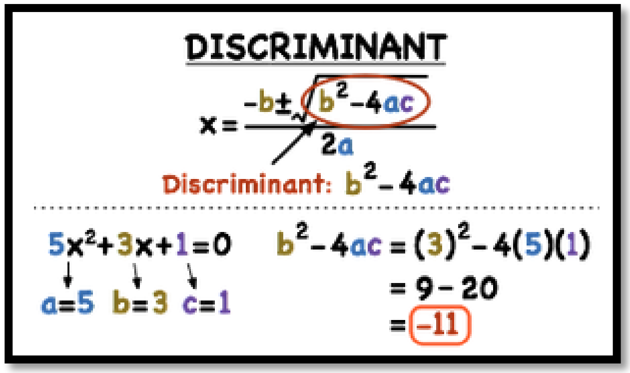
Try solving these problems using the quadratic formula.

1. $5k^2 + 10k + 3 = 0$
2. $2a^2 + 7a - 7 = 0$
3. $5k^2 - 25k + 10 = 0$

1) $x = \frac{-10 \pm \sqrt{10^2 - 4 \cdot 5 \cdot 3}}{2 \cdot 5} = \frac{-5 \pm \sqrt{10}}{5}$

2) $x = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 2 \cdot -7}}{2 \cdot 2} = \frac{-7 \pm \sqrt{105}}{4}$

3) $x = \frac{-(-25) \pm \sqrt{(-25)^2 - 4 \cdot 5 \cdot 10}}{2 \cdot 5} = \frac{5 \pm \sqrt{17}}{2}$



The discriminant of a quadratic function is the quantity inside the radical sign: $b^2 - 4ac$

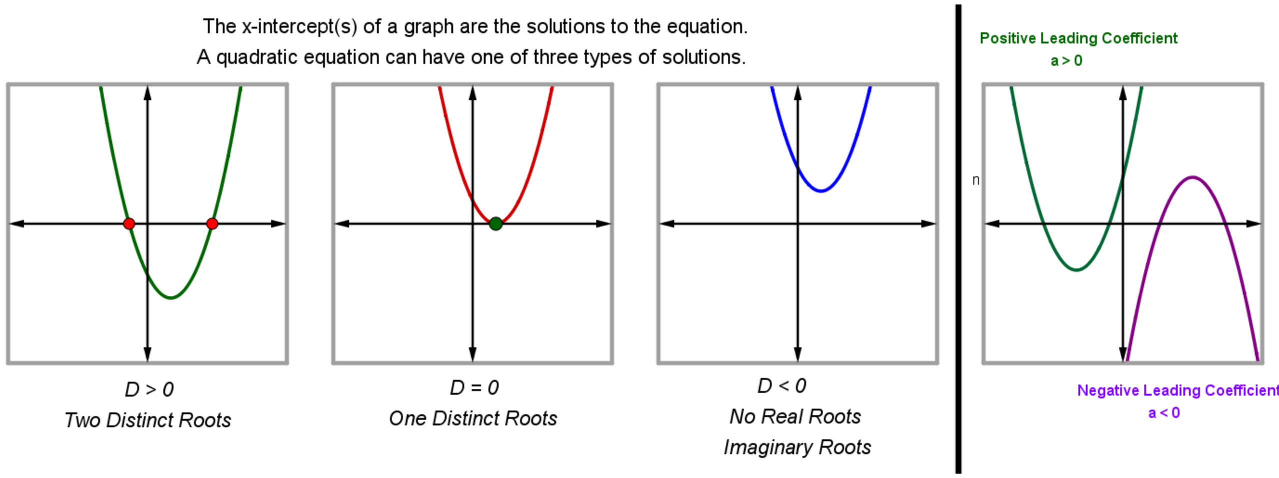
Find the, , the, and the following:

	Problem	Discriminant	Number of solutions	Nature of the roots	Up/Down
1	$x^2 - 6$				
2	$x^2 + 8x + 13$				
3	$x^2 - x + 1$				
4	$x^2 - 8x - 17$				
5	$-x^2 - 8x + 17$				
6	$x^2 - 8x + 16$				

Properties of Quadratic Equations

Given the quadratic equation: $y = ax^2 + bx + c$ The discriminant $D = b^2 - 4ac$ tells the type of roots the equation.

The x-intercept(s) of a graph are the solutions to the equation.
A quadratic equation can have one of three types of solutions.



The roots are **rational** whenever the square root of the discrimanent is a whole number, fraction, or repeating decimal.
The roots are **irrational** whenever the square root of the discrimanent cannot be turned into a rational number.
This occurs whenever the discriminant is not a perfect square.

1. 24, 2, irrational, up
2. 38, 2, rational, up
3. -4, 2, imaginary, up
4. 132, 2, rational, up
5. 132, 2, rational, down
6. 0, 1, rational, up

Complete steps a-c for each quadratic equation.

a. Find the value of the Discriminant, $b^2 - 4ac$.
b. Describe the number and type of roots. {rational or irrational}
c. Find the exact solutions by using the Quadratic Formula.

$$x = \frac{-\square \pm \sqrt{\square^2 - 4 \cdot \square \cdot \square}}{2 \cdot \square}$$

$$x = \frac{-\square \pm \sqrt{\square^2 - 4 \cdot \square \cdot \square}}{2 \cdot \square}$$

1. $x^2 - 8x + 16 = 0$

2. $x^2 - 11x - 26 = 0$

1) 0, 1, rational, 4
2) 225, 2, rational, 13 & –2
3) 4, 2, rational, $\frac{2}{3}$ & 0
4) 289, 2, rational, $\frac{1}{4}$ & $-\frac{3}{5}$
5) 120, 2, irrational, $\pm\frac{\sqrt{30}}{5}$
6) 21, 2, irrational, $\frac{1\pm\sqrt{21}}{10}$
7) 72, 2, irrational, $1 \pm 3\sqrt{2}$
8) 2304, 2, rational, $\frac{9}{4}$ & $-\frac{3}{4}$
9) 109, 2, irrational, $\frac{-7\pm\sqrt{109}}{10}$
10) 609, 2, irrational, $\frac{15\pm\sqrt{609}}{6}$

3. $3x^2 - 2x = 0$

4. $20x^2 + 7x - 3 = 0$

5. $5x^2 - 6 = 0$

6. $5x^2 - x - 1 = 0$

7. $x^2 - 2x - 17 = 0$

8. $16x^2 - 24x – 27 = 0$

9. $5x^2 + 7x – 3 = 0$

10. $3y^2 – 15y = 32$

12. Rational Expressions

A rational expression is created when you have polynomials written in rational form. The above lessons are a precursor to simplifying Rational Expressions.

Examples: $\frac{3a}{46b}$, $\frac{x+1}{3}$, $\frac{5}{y-6}$, $\frac{3x-4}{4x+3}$, $\frac{2x^3}{x-3}$, $\frac{3x^3-2x^2-1}{4x^2-6}$, $\frac{4x^2-6}{4x^3-2x^2-3}$, $\frac{3}{4}$, $\frac{21}{301}$

Some rational expressions can be reduced, the ones above cannot be. Whenever you have a rational expression as part of a solution, it needs to be reduced to simplest form.

Just like in basic arithmetic where you can add, subtract, multiply, and divide, these and other arithmetic operations can be applied to all polynomial expressions and terms.

Let's add these two expressions and simplify:

$$\frac{15}{x-6}+\frac{7}{x+6}$$

Reducing a Rational Expression

$$\frac{2x^2-6x}{4x^3-11x-3}$$

$$\frac{2x\cancel{(x-3)}}{\cancel{(x-3)}(4x+1)}$$

$$\frac{2x}{4x+1}$$

Where 4x+1 ≠ 0, or $x \neq -\frac{1}{4}$.

Also, x ≠ 3.

As when adding numeric fractions, we need a common denominator. Since these denominators are relatively prime, we will multiply the original numerator and denominator by the other fraction's denominator (Note: it is shown as multiplying these by the equivalent of **1**.

$$\frac{15}{x-6}\cdot\frac{\boldsymbol{x+6}}{\boldsymbol{x+6}}+\frac{7}{x+6}\cdot\frac{\boldsymbol{x-6}}{\boldsymbol{x-6}}$$

Resulting in:

$$\frac{15(x+6)}{(x-6)(x+6)}+\frac{7(x-6)}{(x+6)(x-6)}$$

Or,

$$\frac{15x+90+7x-42}{x^2-36}=\frac{22x+48}{x^2-36}$$

If the numerator has any shared factors with denominator, the result needs to be simplified. The example below shows the difference if fractions are subtracted.

$$\frac{15}{x-6}-\frac{7}{x+6}=\frac{15x+90-(7x-42)}{x^2-36}=\frac{8x+132}{x^2-36}$$

If we multiply the fractions, this is the result:

$$\frac{15}{x-6}\times\frac{7}{x+6}=\frac{105}{x^2-36}$$

If we divide the fractions, this is the work and result:

$$\frac{15}{x-6}\div\frac{7}{x+6}=\frac{15}{x-6}\times\frac{x+6}{7}=\frac{15x+90}{7x-42}$$

It is a good idea to factor any factorable expression as you may find out that there are common factors which may divide out and simplify your work. **Recall, with all rational expressions, no factors in the denominator can have value of 0.**

Add these rational expressions and simplify if possible:

1) $\frac{5}{x+5}+\frac{8}{x+5}$

2) $\frac{x-5}{x+5}+\frac{x+5}{x+5}$

3) $\frac{2x}{x-5}+\frac{8}{x-5}$

4) $\frac{2x+3}{5x}+\frac{3x+2}{5x}$

5) $\frac{5x-3y}{6x+5}+\frac{5x+3y}{6x+5}$

6) $\frac{3x^2+x}{x^2-1}+\frac{x+3}{x^2-1}$

Subtract these rational expressions:

1) $\frac{5}{x+5}-\frac{8}{x+5}$

2) $\frac{x-5}{x+5}-\frac{x+5}{x+5}$

3) $\frac{2x}{x-5}-\frac{8}{x-5}$

4) $\frac{2x+3}{5x}-\frac{3x+2}{5x}$

5) $\frac{5x-3y}{6x+5}-\frac{5x+3y}{6x+5}$

6) $\frac{3x^2+x}{x^2-1}-\frac{x+3}{x^2-1}$

Add these rational expressions, remember common denominators are required:

1) $\frac{5}{x+5}+\frac{8}{x-5}$

2) $\frac{x-5}{x+5}+\frac{x+5}{x-5}$

3) $\frac{2x}{x+5}+\frac{8}{x-5}$

4) $\frac{3x-2}{x+1}+\frac{x-1}{3x+4}$

5) $\frac{x^2-4}{x+2}+\frac{x^2-9}{x-3}$

6) $\frac{x^2+3x+2}{x-2}+\frac{x^2-3x+2}{x+2}$

Subtract these rational expressions, remember common denominators are required:

1) $\frac{5}{x+5}-\frac{8}{x-5}$

2) $\frac{x-5}{x+5}-\frac{x+5}{x-5}$

3) $\frac{2x}{x+5}-\frac{8}{x-5}$

4) $\frac{3x-2}{x+1}-\frac{x-1}{3x+4}$

5) $\frac{x^2-4}{x+2}-\frac{x^2-9}{x-3}$

6) $\frac{x^2+3x+2}{x-2}-\frac{x^2-3x+2}{x+2}$

Multiply these rational expressions, simplify.

1) $\frac{5}{x+5} \bullet \frac{8}{x+5}$

2) $\frac{x-5}{x+5} \bullet \frac{x+5}{x+5}$

3) $\frac{2x}{x-5} \bullet \frac{8}{x-5}$

4) $\frac{2x+3}{5x} \bullet \frac{3x+2}{5x}$

5) $\frac{5x-3y}{6x+5} \bullet \frac{5x+3y}{6x+5}$

6) $\frac{3x^2+x}{x^2-1} \bullet \frac{x+5}{x^2-4}$

7) $\frac{y^2-1}{x^2-1} \bullet \frac{x+1}{y-1}$

8) $\frac{6x^2+5x-6}{2x^2+13x+15} \bullet \frac{x^2+8x+15}{3x^2+7x-6}$

9) $\frac{4x^2-4}{8x^2-16x-24} \bullet \frac{9x^2-81}{3x^2+6x-9}$

Divide these rational expressions, simplify.

1) $\frac{5}{x+5} \div \frac{8}{x+5}$

2) $\frac{x-5}{x+5} \div \frac{x+5}{x-5}$

3) $\frac{2x}{x-5} \div \frac{8}{x-5}$

4) $\frac{2x+3}{5x} \div \frac{3x+2}{5x}$

5) $\frac{5x-3y}{6x+5} \div \frac{5x+3y}{6x-5}$

6) $\frac{3x^2+x}{x^2-4} \div \frac{x+5}{x^2-1}$

7) $\frac{y^2-1}{x^2-1} \div \frac{y+1}{x-1}$

8) $\frac{x^2+8x+15}{3x^2+7x-6} \div \frac{6x^2+5x-6}{2x^2+13x+15}$

9) $\frac{2x^2-3x-1}{12x^2+54x+54} \div \frac{2x^2-9x-9}{x^2-9}$