

INTERNATIONAL BACCALAUREATE
Mathematics: analysis and approaches
MAA

EXERCISES [MAA 3.4]
ARCS AND SECTORS
Compiled by Christos Nikolaidis

O. Practice questions

1. [Maximum mark: 6] **[without GDC]**

(a) Express the following angles in radians; give your answer in terms of π .

- (i) 20° (ii) 18° (iii) 540° [3]

(b) Express the following angles in degrees

- (i) $\frac{\pi}{18}$ rad (ii) $\frac{\pi}{5}$ rad (iii) 2.5π rad. [3]

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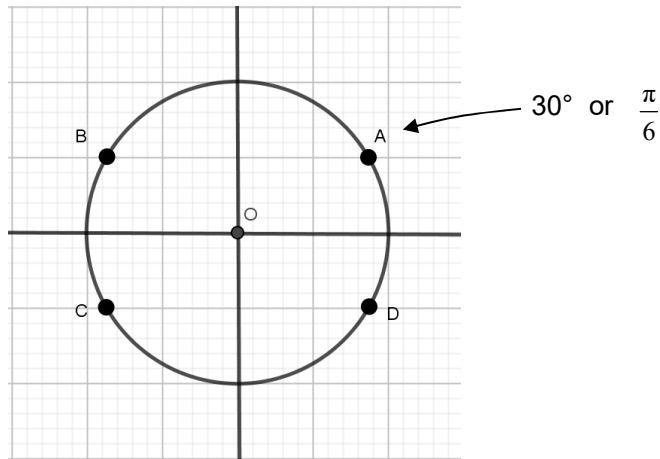
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2. [Maximum mark: 12] **[without GDC]**

In the diagram below, the point A represents the angle of 30° , or otherwise $\frac{\pi}{6}$ rad, on the unit circle.



(a) Complete the table below with the angle values corresponding to the symmetric points B, C and D,

(i) in degrees within the interval $[0^\circ, 360^\circ]$

(ii) in radians within the interval $[0, 2\pi]$

[6]

	A	B	C	D
in degrees	30°			
in radians	$\frac{\pi}{6}$			

Suppose now that C represents the angle of 220° , or otherwise $\frac{11\pi}{9}$ rad, in the third quadrant. Let A, B and D be the symmetric points of C on the unit circle, as shown in the diagram above.

(b) Complete the table below in a similar way as in (a).

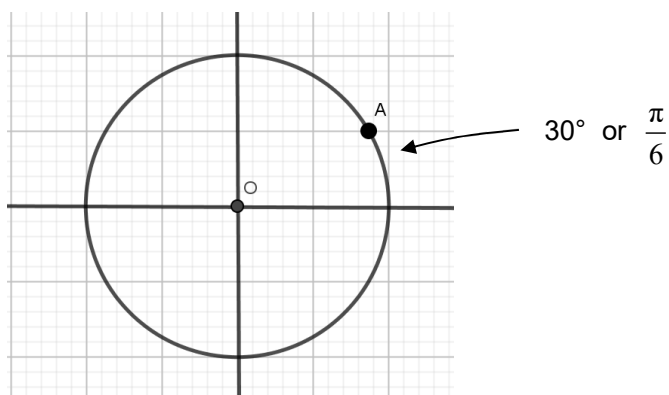
[6]

	A	B	C	D
in degrees			220°	
in radians			$\frac{11\pi}{9}$	

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3. [Maximum mark: 8] **[without GDC]**

In the diagram below, the point A represents the angle of 30° , or otherwise $\frac{\pi}{6}$ rad, on the unit circle.



The general formula for the angles corresponding to point A is

in **degrees**: $30^\circ + 360^\circ k$ $k \in \mathbb{Z}$

in **radians**: $\frac{\pi}{6} + 2k\pi$ $k \in \mathbb{Z}$

Determine the values of the angle at point A within the following intervals:

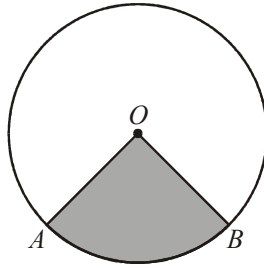
	In degrees	
2 nd period backwards	$-720^\circ \leq \theta < -360^\circ$	
1 st period backwards	$-360^\circ \leq \theta < 0^\circ$	
1 st period	$0^\circ \leq \theta < 360^\circ$	30°
2 nd period	$360^\circ \leq \theta < 720^\circ$	
3 rd period	$720^\circ \leq \theta < 1080^\circ$	

	in radians	
	$-4\pi \leq \theta < -2\pi$	
	$-2\pi \leq \theta < 0$	
	$0 \leq \theta < 2\pi$	$\frac{\pi}{6}$
	$2\pi \leq \theta < 4\pi$	
	$4\pi \leq \theta < 6\pi$	

[8]

4. [Maximum mark: 20] **[without GDC]**

The diagram shows a circle center O radius of 10 cm.



The minor sector is shaded while the major sector is unshaded.

- (a) Given that the size of $\hat{A}OB$ is 1.5 rad, complete the table.

[10]

Length of	the minor arc AB	
	the major arc AB	
Area of	the minor sector	
	the major sector.	
Perimeter of	the minor sector	
	the major sector	

- (b) Given that the size of $\hat{A}OB$ is $\frac{\pi}{2}$ rad, complete the table.

[10]

Length of	the minor arc AB	
	the major arc AB	
Area of	the minor sector	
	the major sector.	
Perimeter of	the minor sector	
	the major sector	

A. Exam style questions (SHORT)

7. [Maximum mark: 6] **[with GDC]**

The following diagram shows a circle of centre O , and radius 15 cm . The arc ACB subtends an angle of 2 radians at the centre O .

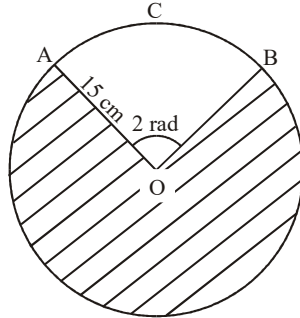


Diagram not to scale

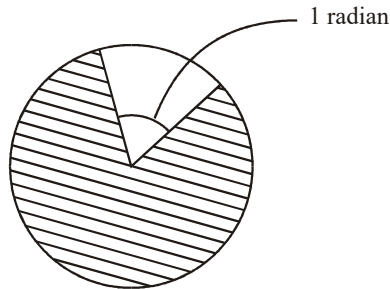
$\widehat{AOB} = 2$ radians
 $OA = 15\text{ cm}$

- (a) Find the length of the arc ACB ; [2]
 (b) Find the area of the shaded region. [4]

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8. [Maximum mark: 4] **[with / without GDC]**

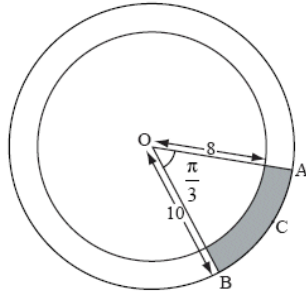
The diagram shows a circle of radius 5 cm . Find the perimeter of the shaded region.



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9. [Maximum mark: 6] **[without GDC]**

The diagram shows two concentric circles with centre O.



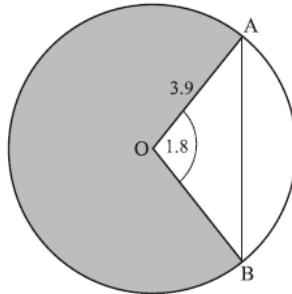
The radius of the smaller circle is 8 cm, the radius of the larger circle is 10 cm. Points A, B and C are on the circumference of the larger circle such that $\angle AOB$ is $\frac{\pi}{3}$ radians

- (a) Find the length of the arc ACB. [2]
 (b) Find the area of the shaded region. [4]

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10. [Maximum mark: 7] **[with GDC]**

The circle shown has centre O and radius 3.9 cm. Points A and B lie on the circle and angle AOB is 1.8 radians.

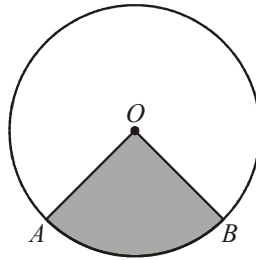


- (a) Find AB. [3]
 (b) Find the area of the shaded region. [4]

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11. [Maximum mark: 4] **[with GDC]**

O is the centre of the circle which has a radius of 5.4 cm.



The area of the shaded sector OAB is 21.6 cm^2 . Find the length of the minor arc AB .

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12. [Maximum mark: 6] **[with GDC]** [diagram as above]

The diagram (above) shows a circle of centre O , and radius r . The shaded sector OAB has an area of 27 cm^2 . Angle $\widehat{AOB} = \theta = 1.5$ radians.

- (a) Find the radius r . [4]
- (b) Calculate the length of the minor arc AB . [2]

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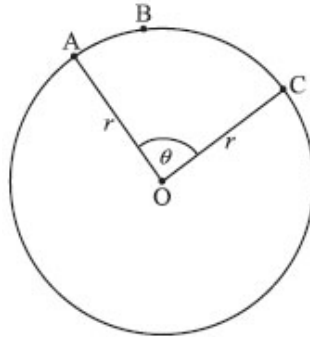
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14. [Maximum mark: 6] **[without GDC]**

The following diagram shows a circle with radius r and centre O . The points A , B and C are on the circle and $\hat{AOC} = \theta$.



The area of sector $OABC$ is $\frac{4}{3}\pi$, the length of arc ABC is $\frac{2}{3}\pi$.
Find the value of r and of θ .

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15. [Maximum mark: 6] **[with GDC]** [diagram as above]

The area of the sector OAC is 180 cm^2 , the length of the arc AC is 24 cm . Find the value of r and of θ .

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16. [Maximum mark: 5] **[with GDC]**

The diagram below shows a sector AOB of a circle of radius 15 cm and centre O. The angle θ at the centre of the circle is 2 radians.

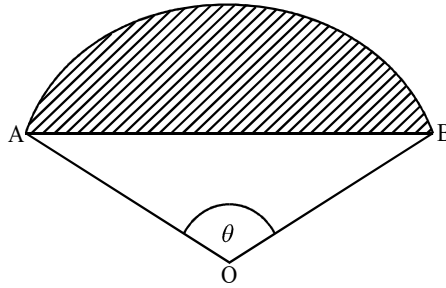


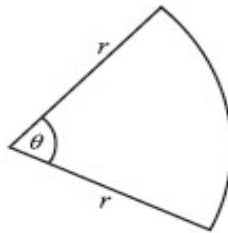
Diagram not to scale

- (a) Calculate the area of the sector AOB. [2]
 (b) Calculate the area of the shaded region. [3]

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17. [Maximum mark: 6] **[with GDC]**

The following diagram shows a sector of a circle of radius r cm, and angle θ at the centre. The perimeter of the sector is 20 cm.

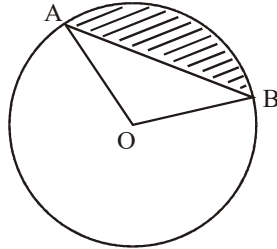


- (a) Show that $\theta = \frac{20-2r}{r}$. [2]
 (b) The area of the sector is 25 cm^2 . Find the value of r . [4]

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18. [Maximum mark: 6] **[with GDC]**

The diagram shows a circle centre O and radius $OA=5\text{ cm}$. The angle $AOB = 135^\circ$.



Find the area of the shaded region.

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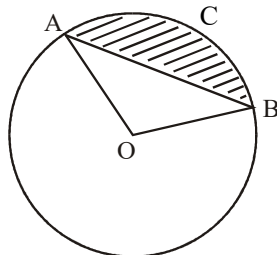
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19. [Maximum mark: 6] **[with GDC]**

The diagram shows a circle centre O and radius r . The length of the arc ACB is $2r$.



The area of the shaded segment may be expressed as kr^2 . Find the value of k .

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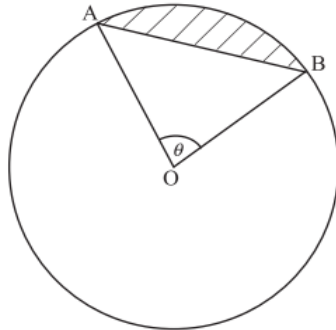
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20*. [Maximum mark: 6] **[with GDC]**

The diagram shows a circle centre O and radius 1, with $\widehat{AOB} = \theta$, $\theta \neq 0$. The area of $\triangle AOB$ is three times the shaded area.



- (a) Show that $3\theta = 4 \sin \theta$. [4]
- (b) Find the value of θ . [2]

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21*. [Maximum mark: 6] **[without GDC]**

The diagram above [Ex. 20] shows a circle centre O and radius r , with $\widehat{AOB} = \theta$.

The ratio of the shaded area to the area of $\triangle AOB$ is 2:5. Show that $5\theta = 7 \sin \theta$.

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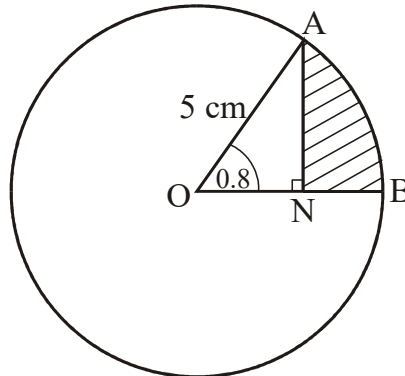
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23*. [Maximum mark: 6] **[with GDC]**

The diagram below shows a circle of radius 5 cm with centre O. Points A and B are on the circle, and \widehat{AOB} is 0.8 radians. The point N is on [OB] such that [AN] is perpendicular to [OB].



Find the area of the shaded region.

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34*. [Maximum mark: 20] **[with GDC]**

The following diagram shows the triangle AOP, where $OP = 2\text{ cm}$, $AP = 4\text{ cm}$ and $AO = 3\text{ cm}$

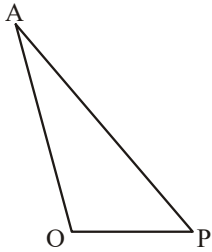


diagram not to scale

(a) Calculate \hat{AOP} , giving your answer in radians. [3]

The following diagram shows two circles which intersect at the points A and B. The smaller circle C_1 has centre O and radius 3 cm, the larger circle C_2 has centre P and radius 4 cm, and $OP = 2\text{ cm}$. The point D lies on the circumference of C_1 and E on the circumference of C_2 . Triangle AOP is the same as triangle AOP in the diagram above.

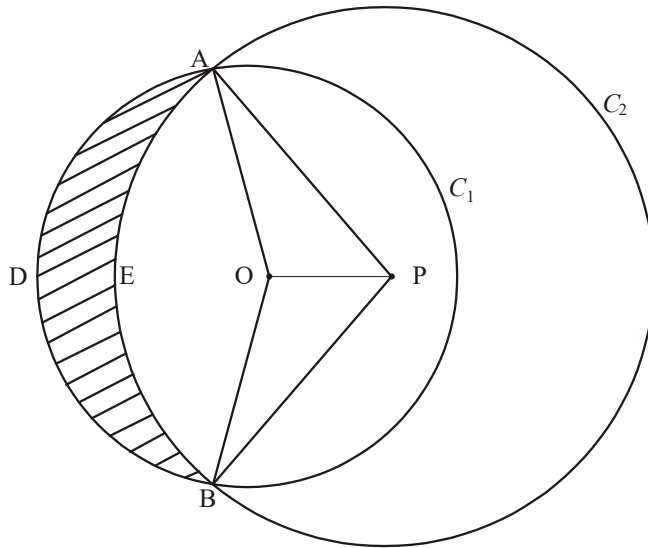


diagram not to scale

(b) Find \hat{AOB} , giving your answer in radians. [2]

(c) Show that $\hat{APB} \cong 1.63$ radians. [3]

(d) Calculate the area of
 (i) sector PAEB;
 (ii) sector OADB. [5]

(e) Show that the area of the quadrilateral AOBP $\cong 5.81\text{ cm}^2$. [3]

(f) Find the areas of
 (i) AOBE.
 (ii) the shaded region AEBD. [4]

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