INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

MAA

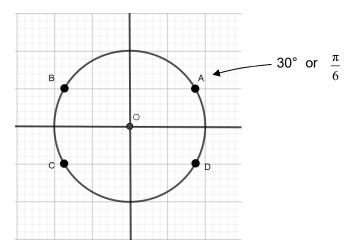
EXERCISES [MAA 3.4] ARCS AND SECTORS

Compiled by Christos Nikolaidis

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<u>U.</u>	Praci	ice que	25110115						
1.	[Max	kimum r	mark: 6]	[with	out GDC]				
	(a)	Expre	ss the foll	owing a	ngles in radi	ans; give y	our answer in teri	ms of π .	
		(i)	20°	(ii)	18°	(iii)	540°		[3]
	(b)	Expre	ss the foll	owing a	ngles in deg	rees			
		(i)	$\frac{\pi}{18}$ rad	(ii)	$\frac{\pi}{5}$ rad	(iii)	2.5π rad.		[3]

2. [Maximum mark: 12] [without GDC]

In the diagram below, the point A represents the angle of 30°, or otherwise $\frac{\pi}{6}$ rad, on the unit circle.



- (a) Complete the table below with the angle values corresponding to the symmetric points B, C and D,
 - (i) in degrees within the interval $[0^{\circ}, 360^{\circ}]$
 - (ii) in radians within the interval $[0,2\pi]$

[6]

[6]

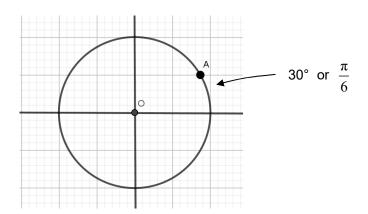
Suppose now that C represents the angle of 220°, or otherwise $\frac{11\pi}{9}$ rad, in the third quadrant. Let A, B and D be the symmetric points of C on the unit circle, as shown in the diagram above.

(b) Complete the table below in a similar way as in (a).

	А	В	С	D
in degrees			220°	
in radians			$\frac{11\pi}{9}$	

3. [Maximum mark: 8] [without GDC]

In the diagram below, the point A represents the angle of 30°, or otherwise $\frac{\pi}{6}$ rad, on the unit circle.



The general formula for the angles corresponding to point A is

in **degrees**: $30^{\circ} + 360^{\circ}k$ $k \in \mathbb{Z}$

in **radians**: $\frac{\pi}{6}$ + $2k\pi$ $k \in Z$

Determine the values of the angle at point A within the following intervals:

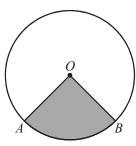
	In degree	es
2 nd period backwards	$-720^{\circ} \le \theta < -360^{\circ}$	
1 st period backwards	$-360^{\circ} \le \theta < 0^{\circ}$	
1 st period	0° ≤ θ < 360°	30°
2 nd period	360° ≤ θ < 720°	
3 rd period	720° ≤ θ < 1080°	

in radia	ans
$-4\pi \le \theta < 2\pi$	
$-2\pi \le \theta < 0$	
$0 \le \theta < 2\pi$	$\frac{\pi}{6}$
$2\pi \le \theta < 4\pi$	
$4\pi \le \theta < 6\pi$	

[8]

4. [Maximum mark: 20] [without GDC]

The diagram shows a circle center O radius of 10 cm.



[10]

[10]

The minor sector is shaded while the major sector is unshaded.

(a) Given that the size of AÔB is 1.5 rad, complete the table.

 Length of
 the minor arc AB

 the major arc AB

 Area of
 the minor sector

 the major sector.

 Perimeter of
 the minor sector

 the major sector

(b) Given that the size of AÔB is $\frac{\pi}{2}$ rad, complete the table.

 Length of
 the minor arc AB

 the major arc AB

 Area of
 the minor sector

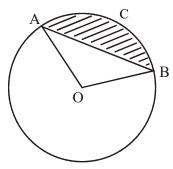
 the major sector.

 Perimeter of
 the minor sector

 the major sector

5. [Maximum mark: 11] [with GDC]

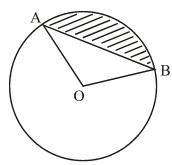
The diagram shows a circle centre O and radius r=8 . The angle \hat{AOB} is $\theta=1.3$ radians.



a)	Find the length of the arc ACB.	[2]
b)	Find the area of the sector OACB.	[2]
c)	Find the length of the chord AB.	[3]
d)	Find the area of the triangle OAB.	[2]
e)	Find the area of the shaded segment.	[2]

6*. [Maximum mark: 14] [with GDC]

The diagram shows a circle centre $\,{\rm O}\,$ and radius $\,r\,.$ The angle $\,{\rm A\hat{O}B}\,$ is $\,\theta\,$ radians.



Find the value of θ , in each of the following three cases

(b) Given that ratio of the lengths of the two arcs (minor AB to major AB) is 1:5. Given that the ratio of the shaded area to the non-shaded area is 1:5.	(a)	Given that the area of the triangle OAB is equal to the shaded area.	[4]
	(b)	Given that ratio of the lengths of the two arcs (minor AB to major AB) is 1:5.	[4]
	(c)	Given that the ratio of the shaded area to the non-shaded area is 1:5.	[5]

A. Exam style questions (SHORT)

7. [Maximum mark: 6] [with GDC]

The following diagram shows a circle of centre O, and radius 15 cm. The arc ACB subtends an angle of 2 radians at the centre O.

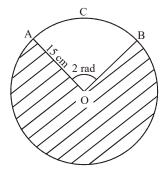


Diagram not to scale

$$A\hat{O}B = 2 \text{ radians}$$

 $OA = 15 \text{ cm}$

(a) Find the length of the arc ACB;

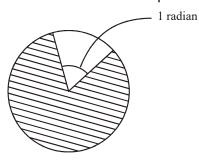
[2]

(b) Find the area of the shaded region.

[4]

8. [Maximum mark: 4] [with / without GDC]

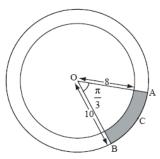
The diagram shows a circle of radius 5 cm. Find the perimeter of the shaded region.



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9. [Maximum mark: 6] [without GDC]

The diagram shows two concentric circles with centre O.



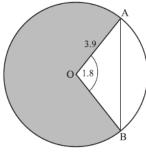
The radius of the smaller circle is 8 cm, the radius of the larger circle is 10 cm. Points A, B and C are on the circumference of the larger circle such that \hat{AOB} is $\frac{\pi}{3}$ radians

(a)	Find the length of the arc ACB.	[2]
(a)	Tilla the length of the are ACD.	[4]

b)	Find the area of the shaded region.	[4]

10. [Maximum mark: 7] **[with GDC]**

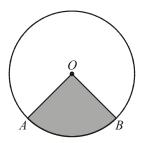
The circle shown has centre O and radius 3.9 cm. Points A and B lie on the circle and angle AOB is 1.8 radians.



- (a) Find AB. [3]
- (b) Find the area of the shaded region. [4]

11. [Maximum mark: 4] [with GDC]

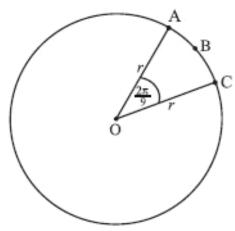
O is the centre of the circle which has a radius of 5.4 cm.



[Maximum mark: 6] [with GDC] [diagram as above]					
has an area of 27 cm ² . Angle $\hat{AOB} = \theta = 1.5$ radians.					
(a) Find the radius r.(b) Calculate the length of the minor arc AB.					

13. [Maximum mark: 6] **[with GDC]**

The diagram below shows a circle centre O, with radius r.

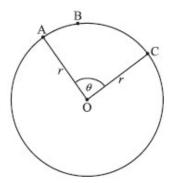


The length of arc ABC is 3π cm and $\triangle AOC = \frac{2\pi}{9}$.

(a)	Find the value of r .	[2]
(b)	Find the perimeter of sector OABC.	[2]
(c)	Find the area of sector OABC.	[2]

14. [Maximum mark: 6] [without GDC]

The following diagram shows a circle with radius r and centre O. The points A, B and C are on the circle and $\hat{AOC} = \theta$.



The area of sector OABC is $\frac{4}{3}\pi$, the length of arc ABC is $\frac{2}{3}\pi$. Find the value of r and of θ .

 	•••••	•	•••••	

15. [Maximum mark: 6] **[with GDC]** [diagram as above]

The area of the sector OAC is $180~\rm{cm^2}$, the length of the arc AC is $24~\rm{cm}$. Find the value of r and of θ .

16. [Maximum mark: 5] **[with GDC]**

The diagram below shows a sector AOB of a circle of radius 15 cm and centre O. The angle θ at the centre of the circle is 2 radians.

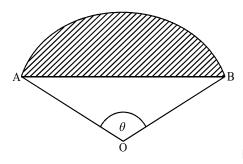


Diagram not to scale

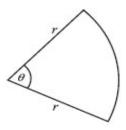
(a)	Calculate the area of the sector AOB.	[2]
(a)	Calculate the area of the sector AOB.	[2]

Calculate the area of the shaded region.	[3]

17. [Maximum mark: 6] *[with GDC]*

(b)

The following diagram shows a sector of a circle of radius r cm, and angle θ at the centre. The perimeter of the sector is 20 cm.

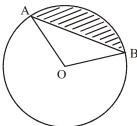


(a)	Show that $\theta = \frac{20-2r}{r}$.	[2]
	r	

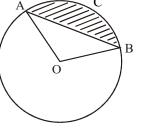
(b)	The area of the sector is 25 cm 2 . Find the value of r .	[4]

18 .	[Maximum	mark: 61	[wit	th GDC]
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The diagram shows a circle centre O and radius $OA=5\,cm$. The angle $AOB=135^\circ$.



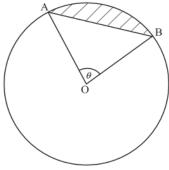
	Find the area of the shaded region.
19.	[Maximum mark: 6] [with GDC]
	The diagram shows a circle centre ${ m O}$ and radius r . The length of the arc ${ m ACB}$ is $2r$.
	A C B



The area of the shaded segment may be expressed as kr^2 . Find the value of k.

20*. [Maximum mark: 6] [with GDC]

The diagram shows a circle centre O and radius 1, with $\hat{AOB} = \theta$, $\theta \neq 0$. The area of $\triangle AOB$ is three times the shaded area.



(a)	Show that $3\theta = 4\sin\theta$.	[4]
(b)	Find the value of $ heta$.	[2]

21*. [Maximum mark: 6] [without GDC]

The diagram above [Ex. 20] shows a circle centre O and radius r, with $\hat{AOB} = \theta$. The ratio of the shaded area to the area of $\triangle AOB$ is 2:5. Show that $5\theta = 7\sin\theta$.

22*. [Maximum mark: 6] [with / without GDC]

In the following diagram, O is the centre of the circle and (AT) is the tangent to the circle at T.

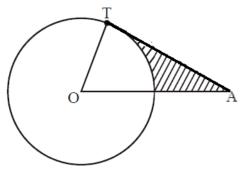
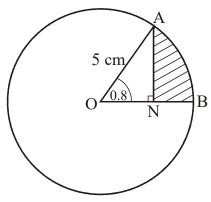


Diagram not to scale

If OA =	= 12 cm, and the circle has a radius of 6 cm, find the are	ea of the shaded region.
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-		
-		
-		
-		
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23*. [Maximum mark: 6] [with GDC]

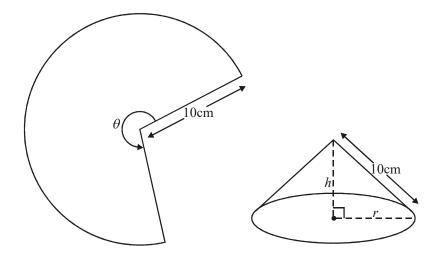
The diagram below shows a circle of radius 5 cm with centre O. Points A and B are on the circle, and \hat{AOB} is 0.8 radians. The point N is on [OB] such that [AN] is perpendicular to [OB]..



Find the area of the shaded region.

24*. [Maximum mark: 6] [without GDC]

The diagrams show a circular sector of radius $10~{\rm cm}$ and angle θ radians which is formed into a cone of slant height $10~{\rm cm}$. The vertical height h of the cone is equal to the radius r of its base.



- (a) Find the value of r. [2]
- (b) Find the angle θ radians. [4]

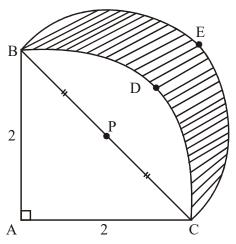
25. [Maximum mark: 6] *[without GDC]*

The diagram below shows a triangle and two arcs of circles.

The triangle ABC is a right-angled isosceles triangle, with AB = AC = 2. The point P is the midpoint of [BC].

The arc BDC is part of a circle with centre A.

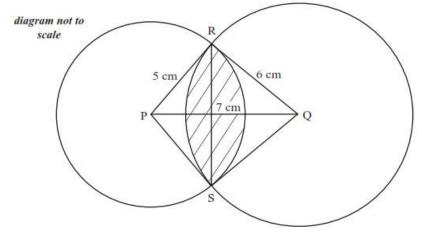
The arc BEC is part of a circle with centre P.



(a)	Calculate the area of the segment BDCP.	[3]
(b)	Calculate the area of the shaded region BECD.	[3]

26*. [Maximum mark: 7] *[with GDC]*

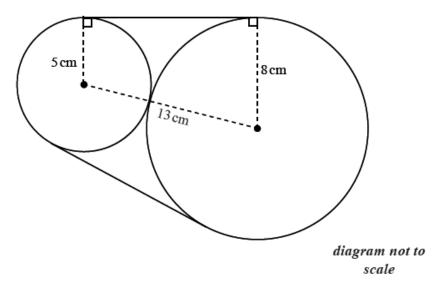
The diagram below shows a pair of intersecting circles with centres at P and Q with radii of $5~\rm cm$ and $6~\rm cm$ respectively. RS is the common chord of both circles and PQ is $7~\rm cm$.



Find the area of the shaded region.

27*. [Maximum mark: 8] [with GDC]

Two discs, one of radius 8 cm and one of radius 5 cm, are placed such that they touch each other. A piece of string is wrapped around the discs. This is shown in the diagram below.

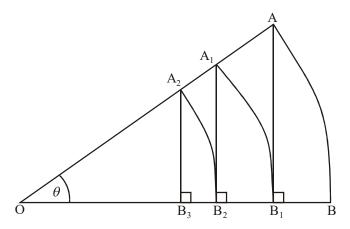


Calculate the length of the string needed to go around the discs.

28*.	[Max	rimum mark: 5] [without GDC]
	A cir	cular disc is cut into twelve sectors whose areas are in an arithmetic sequence.
	(a)	Explain why the angles of the sectors are also in an arithmetic sequence. [2]
	(b)	The angle of the largest sector is twice the angle of the smallest sector. Find the
		size of the angle of the smallest sector. [5

29*. [Maximum mark: 7] [with GDC]

The diagram shows a sector AOB of a circle of radius 1 and centre O, where $\hat{AOB} = \theta$ The lines (AB₁), (A₁B₂), (A₂B₃) are perpendicular to OB. A₁B₁, A₂B₂ are all arcs of circles with centre O.



(a) Calculate, in terms of θ , the sum to infinity of the arc lengths

$$AB + A_1B_1 + A_2B_2 + A_3B_3 + \dots$$
 [5]

(b)	Given that the sum to infinity of the arc lengths is $\frac{2\pi}{3}$, find the value of θ .	[2]

B. Exam style questions (LONG)

(a)

(b)

(d)

30. [Maximum mark: 13] [with GDC]

The following diagram shows a circle with centre O and radius 4 cm.

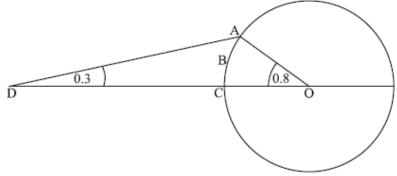


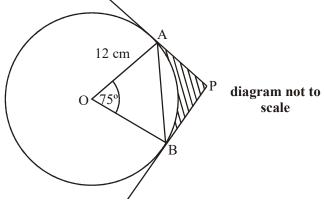
diagram not to scale

The points A, B and C lie on the circle. The point D is outside the circle, on (OC). Angle ADC = 0.3 radians and angle AOC = 0.8 radians.

Find AD.	[3]
Find OD.	[4]
Find the area of sector OABC.	[2]
Find the area of region ABCD.	[4]

31. [Maximum mark: 13] *[with GDC]*

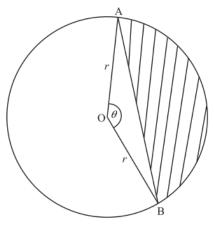
The diagram shows a circle, centre O, with a radius 12cm. The chord AB subtends at an angle of 75° at the centre. The tangents to the circle at A and at B meet at P.



	•	
(a)	Using the cosine rule, show that the length of AB is $12\sqrt{2(1-\cos 75^\circ)}$.	[2]
(b)	Find the length of BP.	[3]
(c)	Hence find (i) the area of triangle OBP; (ii) the area of triangle ABP.	[4]
(d)	Find the area of sector OAB.	[2]
(e)	Find the area of the shaded region.	[2]

32. [Maximum mark: 10] [with GDC]

The following diagram shows a circle centre O, radius r. The angle $A\hat{O}B$ at the centre of the circle is θ radians. The chord AB divides the circle into a minor segment (the shades region) and a major segment.



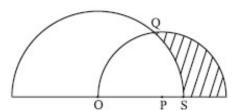
- (a) Show that the area of the minor segment is $\frac{1}{2}r^2(\theta \sin \theta)$. [3]
- (b) Given that the ratio of the areas of the two segments is 2:3, show that

$$\sin \theta = \theta - \frac{4\pi}{5}.$$
 [5]

(c) Hence find the value of θ . [2]

33*. [Maximum mark: 17] [with GDC]

The following diagram shows two semi-circles. The larger one has centre O and radius 4 cm. The smaller one has centre P, radius 3 cm, and passes through O. The line (OP) meets the larger semi-circle at S. The semi-circles intersect at Q.



- (a) (i) Explain why OPQ is an isosceles triangle.
 - (ii) Use the cosine rule to show that $\cos O\hat{P}Q = \frac{1}{9}$.
 - (iii) Hence show that $\sin O\hat{P}Q = \frac{\sqrt{80}}{9}$.
 - (iv) Find the area of the triangle OPQ. [7]
- (b) Consider the smaller semi-circle, with centre P.
 - (i) Write down the size of OPQ.
 - (ii) Calculate the area of the sector OPQ. [3]

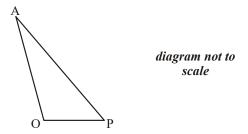
[3]

- (c) Consider the larger semi-circle, with centre O. Calculate the area of the sector QOS.
- (d) Hence calculate the area of the shaded region. [4]

[MAA 3.4] ARCS AND SECTORS

34*. [Maximum mark: 20] *[with GDC]*

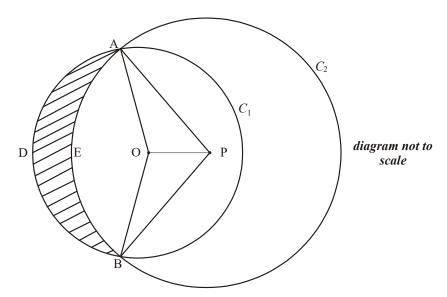
The following diagram shows the triangle AOP, where OP = 2cm, AP = 4cm and AO = 3cm



(a) Calculate \hat{AOP} , giving your answer in radians.

The following diagram shows two circles which intersect at the points A and B. The smaller circle C_1 has centre O and radius 3 cm, the larger circle C_2 has centre P and radius 4 cm, and OP = 2 cm. The point D lies on the circumference of C_1 and E on the circumference of C_2 . Triangle AOP is the same as triangle AOP in the diagram above.

[3]



- (b) Find \hat{AOB} , giving your answer in radians. [2]
- (c) Show that $A\hat{P}B \cong 1.63$ radians. [3]
- (d) Calculate the area of
 - (i) sector PAEB;
 - (ii) sector OADB. [5]
- (e) Show that the area of the quadrilateral AOBP $\cong 5.81 \text{ cm}^2$. [3]
- (f) Find the areas of
 - (i) AOBE.
 - (ii) the shaded region AEBD. [4]

[MAA 3.4] ARCS AND SECTORS

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35. [Maximum mark: 15] *[with GDC]*

The diagram below shows a circle with centre O and radius 8 cm.

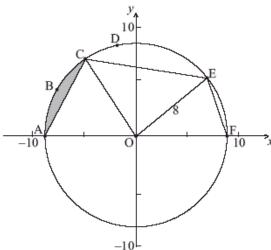


diagram not to scale

The points A, B, C, D, E and F are on the circle, and [AF] is a diameter. The length of arc ABC is 6 cm.

(a) Find the size of angle AOC. [2]
(b) Hence find the area of the shaded region. [6]
The area of sector OCDE is 45 cm².
(c) Find the size of angle COE. [2]
(d) Find EF. [5]