INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

MAA

EXERCISES [MAA 2.6]

TRANSFORMATIONS

Compiled by Christos Nikolaidis

O.   Practice questions

1.   [Maximum mark: 9]   ***[without GDC]***

The point A(1, 0.5) lies on the curve $y = f(x)$. Write down the coordinates of the

corresponding point under the following transformations

$y = f(x) + 5$	(1, 5.5)	$y = f(x + 5)$	
$y = f(x) - 5$		$y = f(x - 5)$	
$y = 5f(x)$		$y = f(5x)$	
$y = f(x)/5$		$y = f(x/5)$	
$y = -f(x)$		$y = f(-x)$	

2.   [Maximum mark: 12]   ***[without GDC]***

The point A(-1, 3) lies on the curve $y = f(x)$.

(a)   Write down the coordinates of the corresponding point under the following

transformations   [9]

$y = f(x) + 3$	(-1, 6)	$y = f(x + 3)$	
$y = f(x) - 3$		$y = f(x - 3)$	
$y = 3f(x)$		$y = f(3x)$	
$y = f(x)/3$		$y = f(x/3)$	
$y = -f(x)$		$y = f(-x)$	

(b)   Find the coordinates of the corresponding point under $y = 2f(x-3)+4$.   [3]

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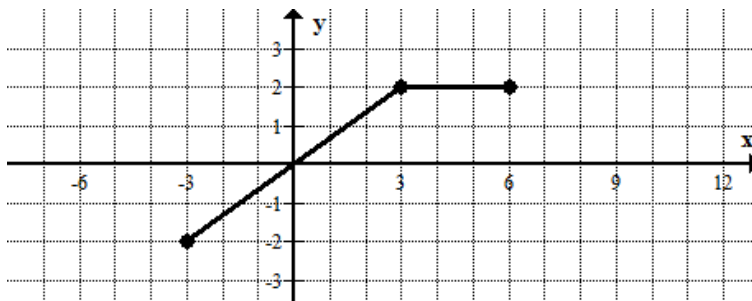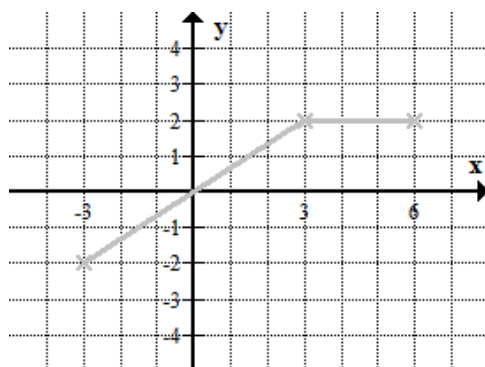
3. [Maximum mark: 10]   *[without GDC]*

The graph of $y = f(x)$ is shown in the diagram.

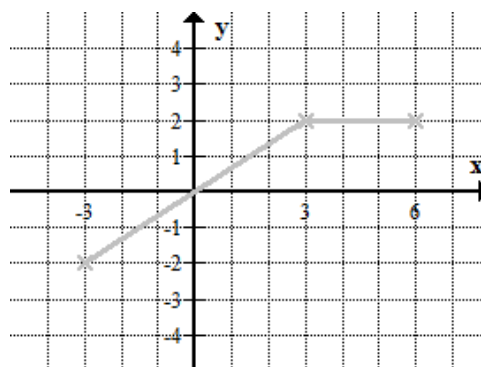On each of the following diagrams draw the required graph.

(a) $y = f(x) + 1$ (b) $y = f(x) - 2$ [1+1]
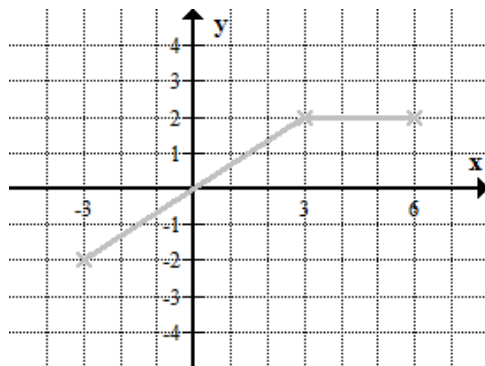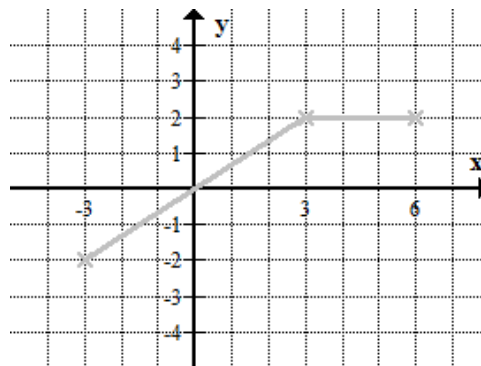
(c) $y = 2f(x)$ (d) $y = \dfrac{1}{2}f(x)$ [1+1]
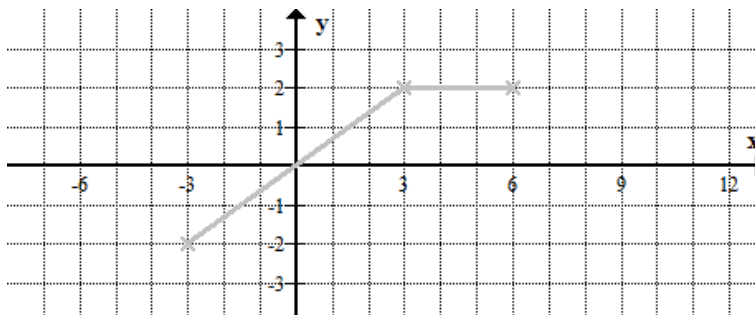
(e) $y = -f(x)$ [1]

(f) $y = f(x+3)$ [1]
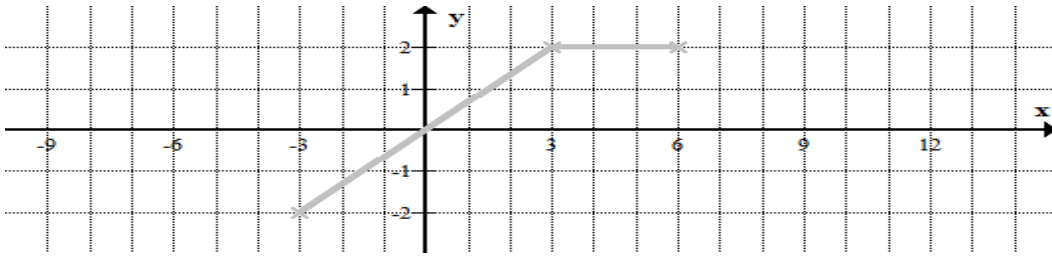
(g) $y = f(x-3)$ [1]
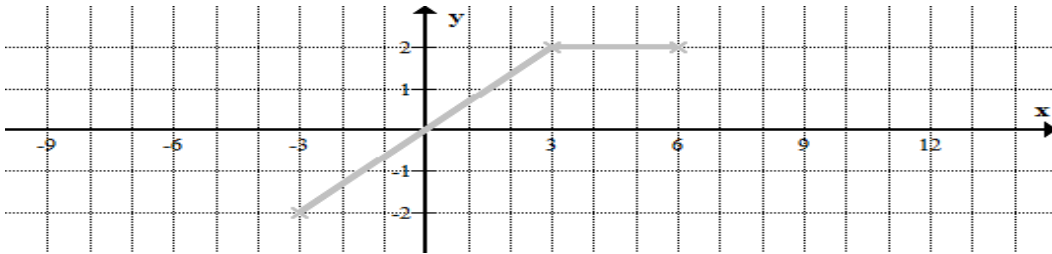
(h) $y = f(3x)$ [1]
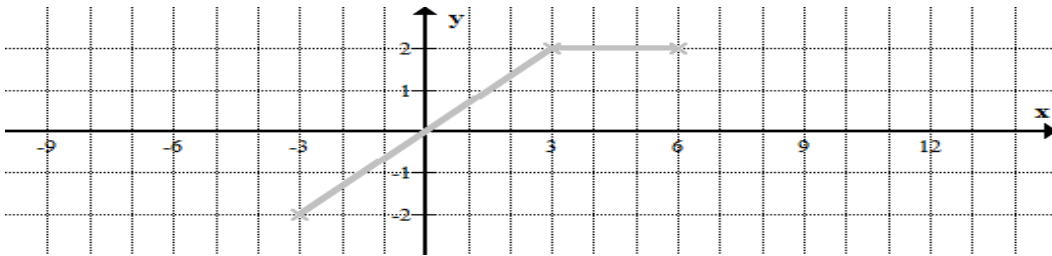
(i) $y = f\left(\dfrac{x}{2}\right)$ [1]
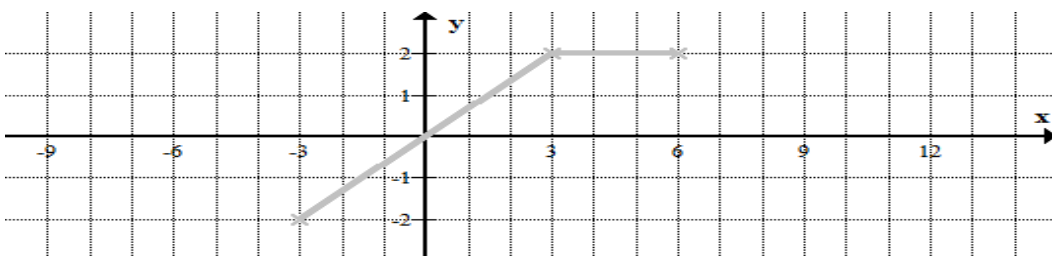
(j) $y = f(-x)$ [1]
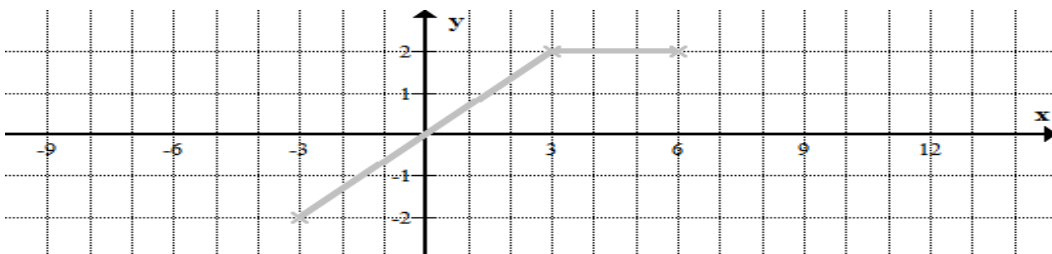
4*.   [Maximum mark: 11]   *[without GDC]*

The sequence of transformations from $y = f(x)$ to $y = 2f(3x) + 4$ is given below

$f(x)$	original
$2f(x)$	vertical stretch with s.f. 2
$2f(3x)$	horizontal stretch with s.f. 1/3 (i.e. shrink))
$2f(3x) + 4$	vertical translation 4 units up

Describe similarly the sequence of transformations, in a correct order, for the following functions

(a) $-f(x-2)+5$ [3]

$f(x)$	original

(b) $-3f(x+2)-1$ [4]

$f(x)$	original

(c) $f(2x-10)$ [2]

$f(x)$	original

(d) $f(2(x-5))$ [2]

$f(x)$	original

A. Exam style questions (SHORT)

5. [Maximum mark: 6]   ***[without GDC]***

The graph of $y = f(x)$ is shown in the diagram.
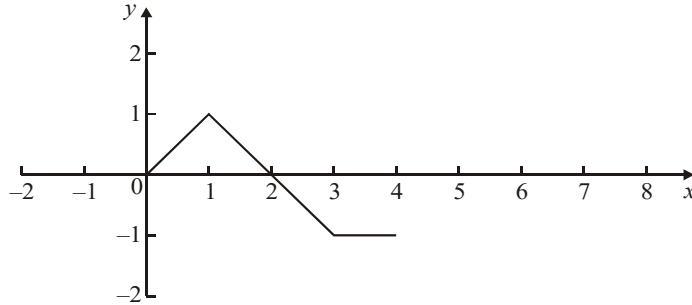
(a) On each of the following diagrams draw the required graph,

(i) $y = 2f(x)$;

(ii) $y = f(x-3)$.

[4]
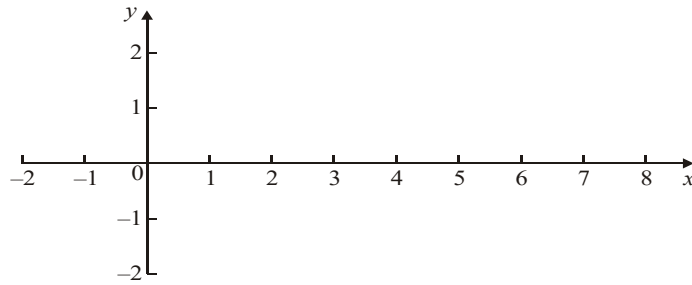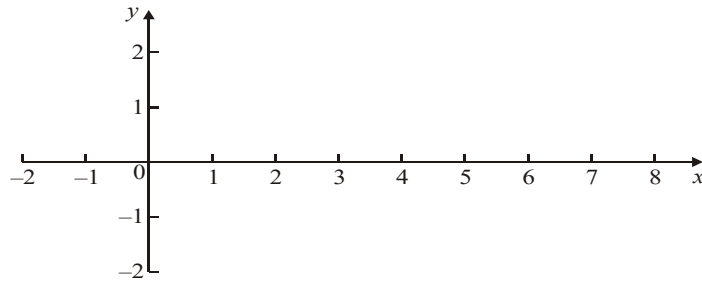
(b) The point A $(3, -1)$ is on the graph of *f*. The point A' is the corresponding point on the graph of $y = -f(x) + 1$.

Find the coordinates of A'. [2]

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6. [Maximum mark: 5] *[without GDC]*

The following diagram shows part of the graph of $f(x)$
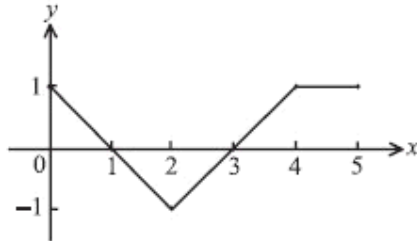
Consider the five graphs in the diagrams labelled A, B, C, D, E below.
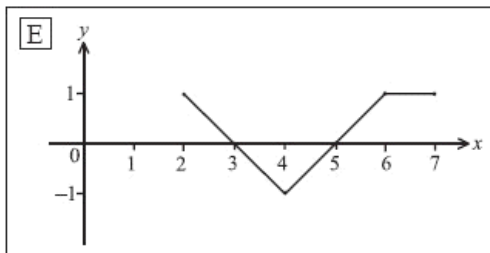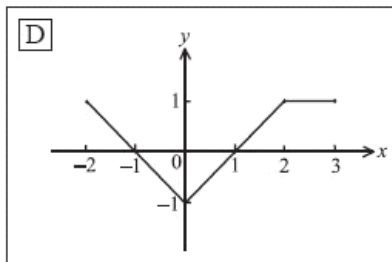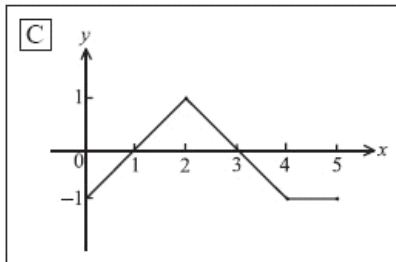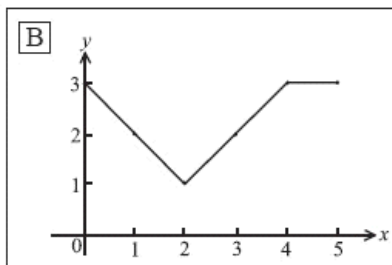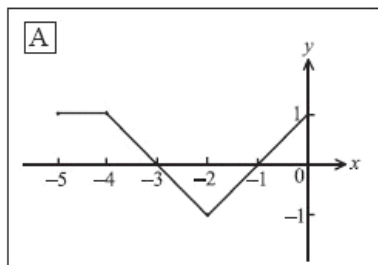
(a) Which diagram is the graph of

(i) $f(x+2)$? (ii) $-f(x)$? (iii) $f(-x)$ [3]

(b) Write down the expression for each of the two remaining graphs [2]

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7. [Maximum mark: 10] **[without GDC]**

The graph of the function $y = f(x)$, $0 \le x \le 4$, is shown below.
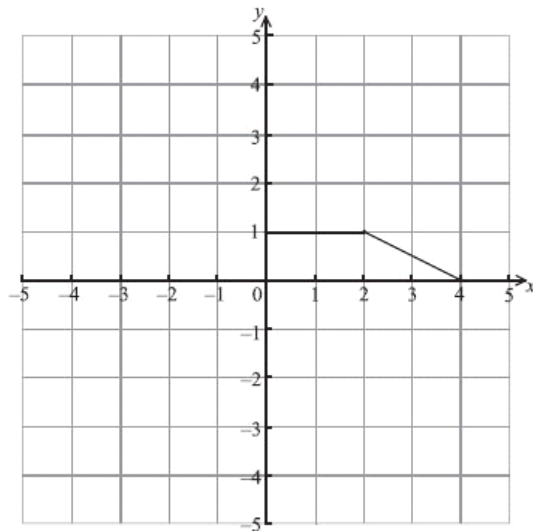
(a) Write down the value of (i) $f(1)$ (ii) $f(3)$. [2]

(b) On the diagrams below, draw the graphs of $y = 3f(-x)$ and $y = f(2x)$. [4]

(c) Write down the domain and the range of $y = 3f(-x)$ and $y = f(2x)$. [4]

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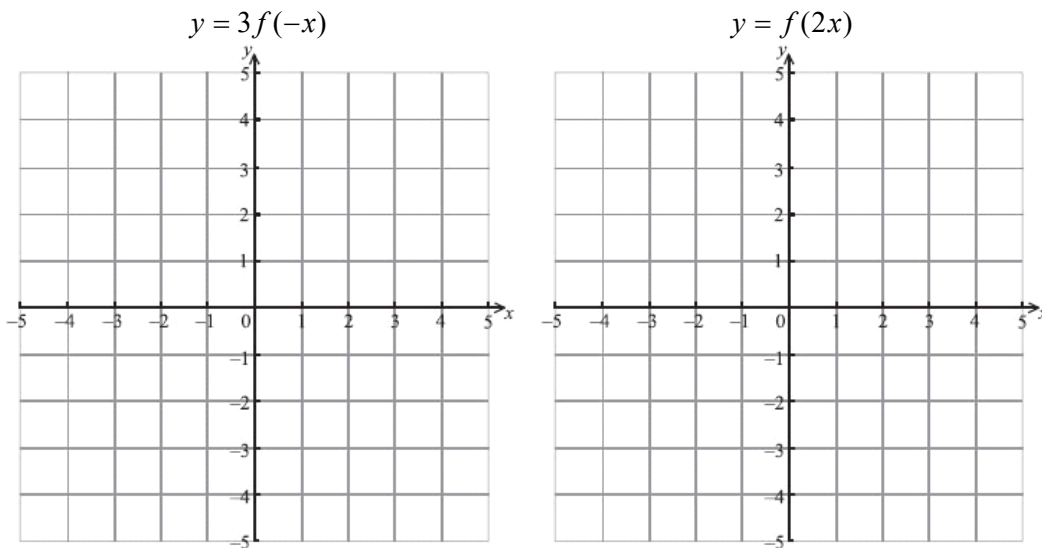
$y = 3f(-x)$ $y = f(2x)$

		$y = f(x)$	$y = 3f(-x)$	$y = f(2x)$
Domain		$0 \le x \le 4$		
Range		$0 \le y \le 1$		

8. [Maximum mark: 5] *[without GDC]*

The diagram below shows the graph of a function $f(x)$, for $-2 \le x \le 4$.
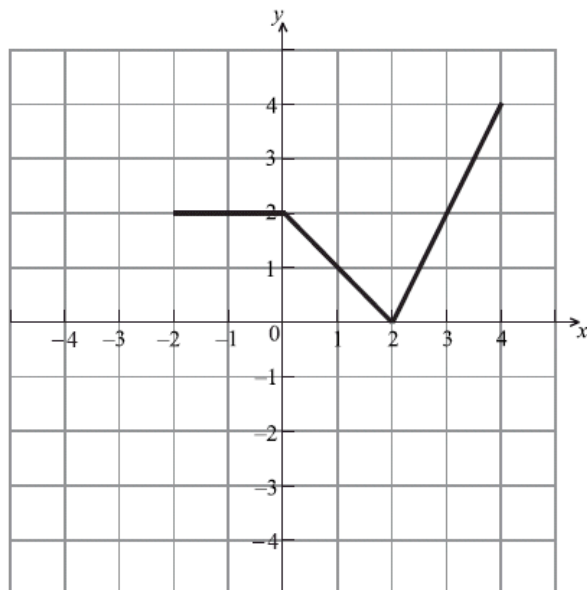
(a) Let $h(x) = f(-x)$. Sketch the graph of h on the grid below.

[2]
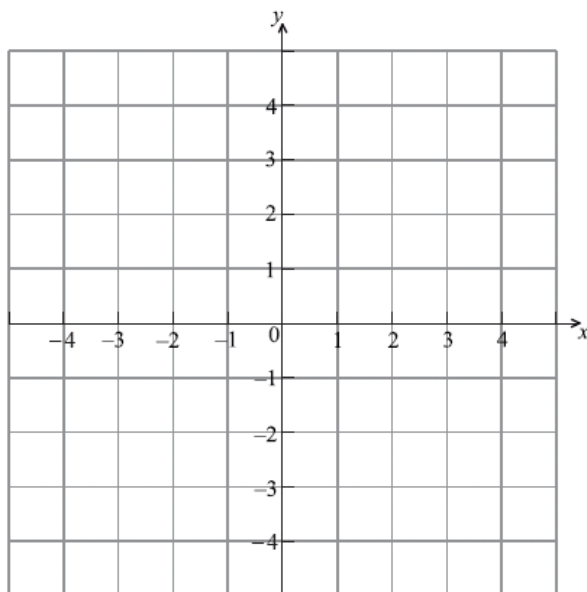
(b) Let $g(x) = \dfrac{1}{2}f(x-1)$. The point A$(3, 2)$ on the graph of f is transformed to the point P on the graph of g. Find the coordinates of P. [3]

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9. [Maximum mark: 6]   *[without GDC]*

Consider the graph of f shown below.
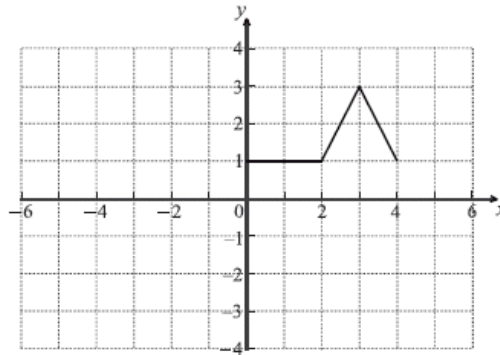
(a) On the **same** grid sketch the graph of $y = f(-x)$. [2]

The following four diagrams show **images** of f under different transformations.

Diagram A
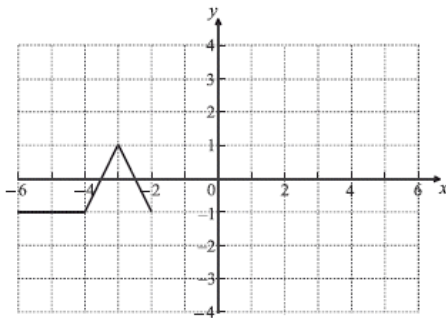
Diagram B
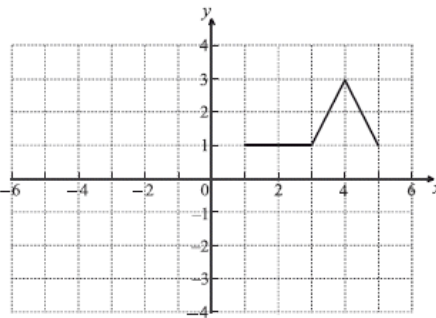
Diagram C
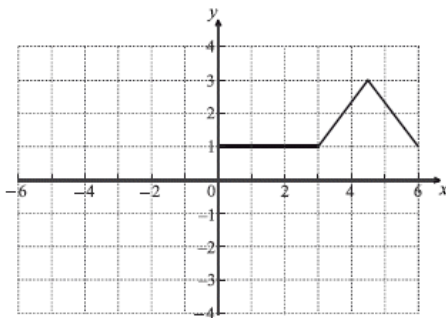
Diagram D
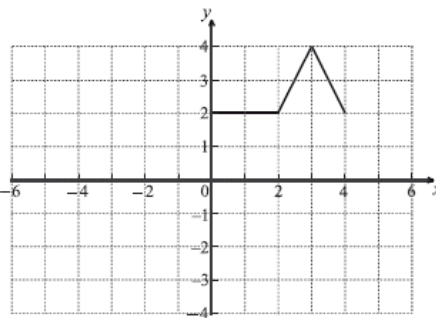
(b) Complete the following table. [2]

Description of transformation	Diagram letter
Horizontal stretch with scale factor 1.5	
Maps $f(x)$ to $f(x)+1$	

(c) Give a full geometric description of the transformation in Diagram A. [2]

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10. [Maximum mark: 8]   *[with GDC]*

Let $f(x) = \dfrac{1}{3}x^3 - x^2 - 3x$. Part of the graph of f is shown below.
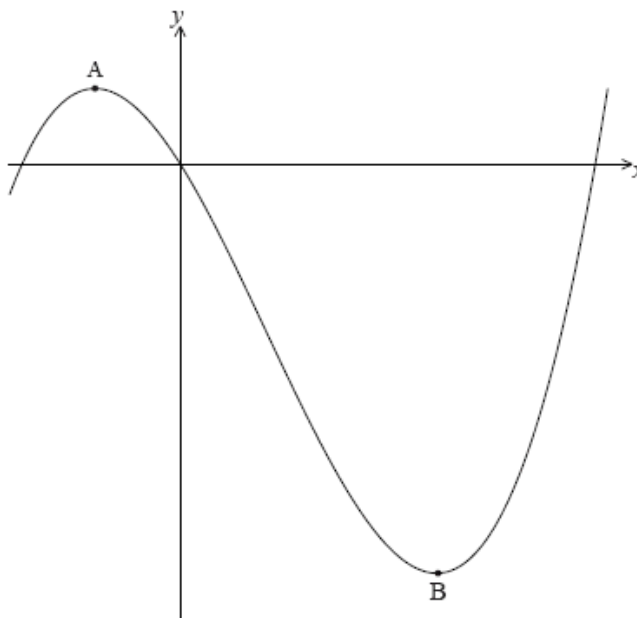
There is a maximum point at A and a minimum point at B(3, –9).

(a)   Write down the coordinates of A.   [2]

(b)   Write down the coordinates of

    (i)   the image of B after a reflection in the y-axis;

    (ii)   the image of B after a translation by –2 units in the x-direction and 5 units in the y -direction).

    (iii)   the image of B after a reflection in the x-axis followed by a horizontal stretch with scale factor $\dfrac{1}{2}$.   [6]

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11. [Maximum mark: 10]   *[without GDC]*

The graph of a function f is shown in the diagram below.
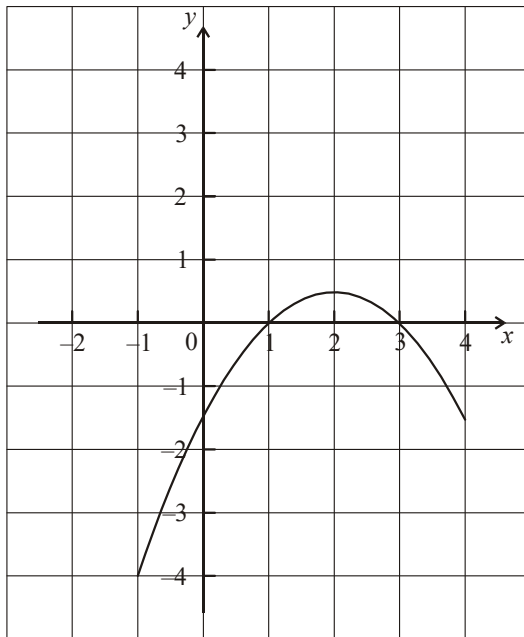
(a)   On the same diagram sketch the graph of $y = -f(x)$.   [2]

(b)   Let $g(x) = f(x+3)$.

(i)   Find $g(-3)$

(ii)   Describe **fully** the transformation that maps the graph of f to the graph of g [4]

(c)   Write down in the table below the domain and the range for each function:

	$y = f(x)$	$y = -f(x)$	$y = f(x+3)$
Domain	$-1 \le x \le 4$		
Range	$-4 \le y \le 0.5$		

[4]

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12. [Maximum mark: 4]   *[without GDC]*

The following diagram shows the graph of $y = f(x)$. It has minimum and maximum points at $(0, 0)$ and $(1, \frac{1}{2})$.
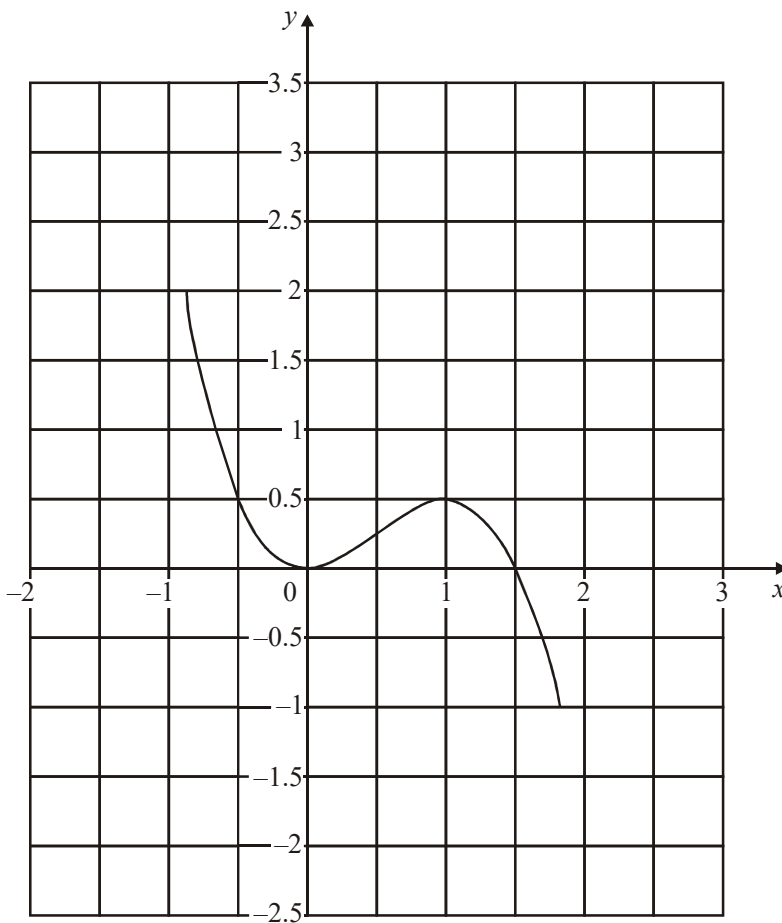
(a)   On the same diagram, draw the graph of $y = f(x-1) + \frac{3}{2}$.

(b)   What are the coordinates of the minimum and maximum points of $y = f(x-1) + \frac{3}{2}$?

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13. [Maximum mark: 6]   *[without GDC]*

The sketch shows part of the graph of $y = f(x)$ which passes through the points

A(−1, 3), B(0, 2), C(l, 0), D(2, 1) and E(3, 5).
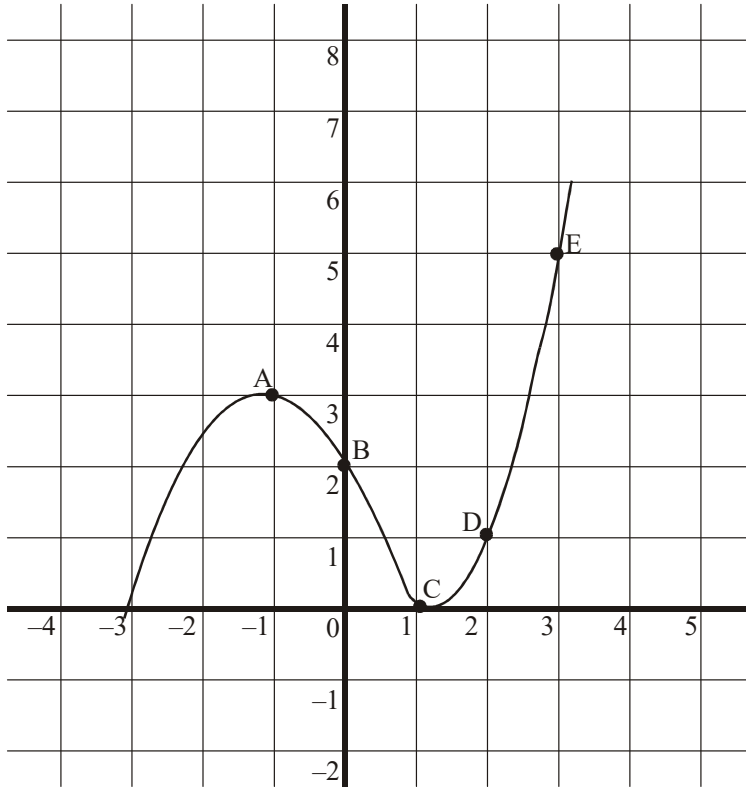
A second function is defined by $g(x) = 2f(x − 1)$.

(a)   Calculate $g(0), g(1), g(2)$ and $g(3)$.   [3]

(b)   On the same axes, sketch the graph of the function $g(x)$.   [3]

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14. [Maximum mark: 8]   *[without GDC]*

Let $f(x) = 2x + 1$.

(a)   On the grid below draw the graph of $f(x)$ for $0 \le x \le 2$.   [2]

(b)   Let $g(x) = f(x+3) - 2$. On the grid below draw the graph of $g(x)$ for $-3 \le x \le -1$.   [4]
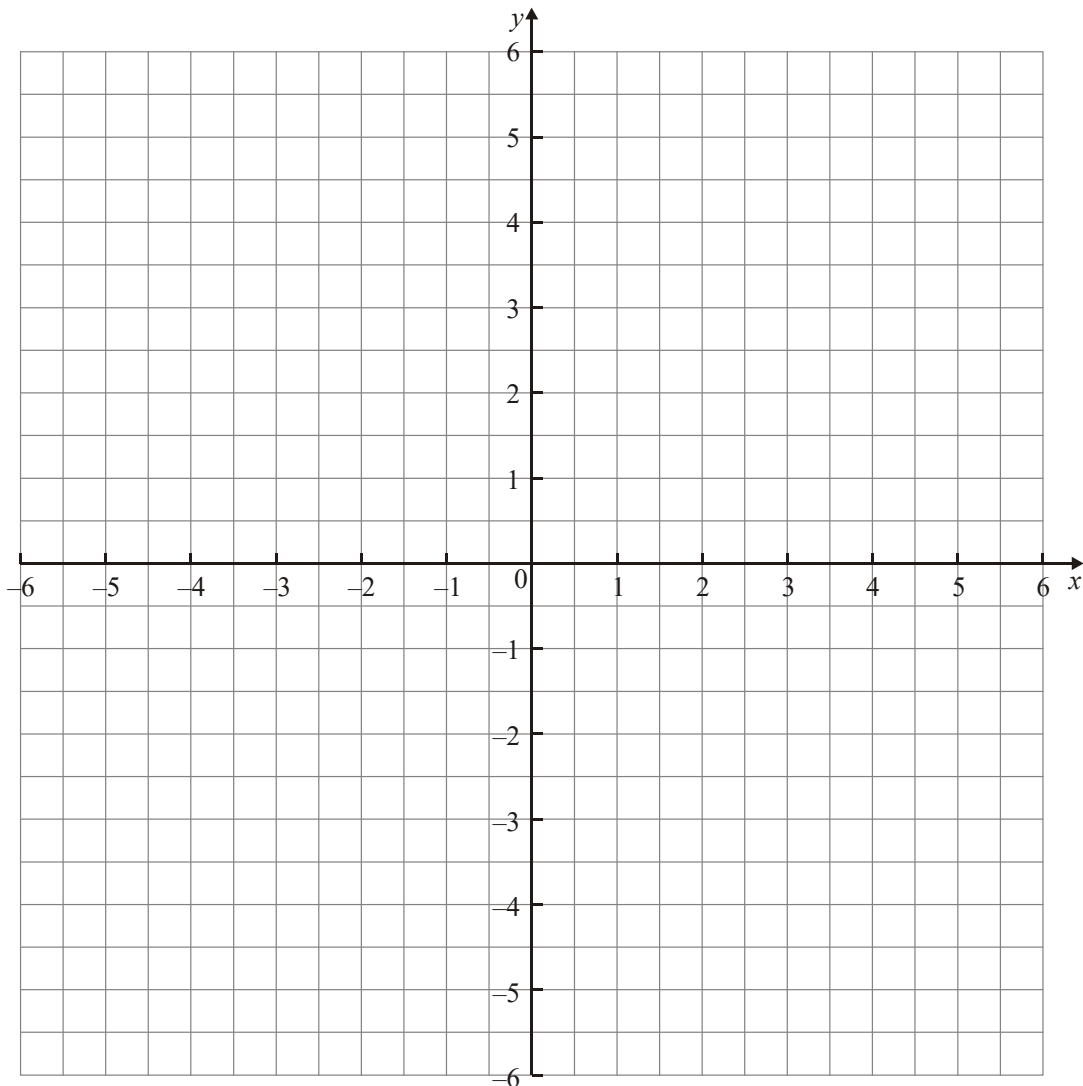
(c)   Write down the range for each function:

	$y = f(x)$	$y = g(x)$
Domain	$0 \le x \le 2$	$-3 \le x \le -1$
Range		

[2]

15. [Maximum mark: 5]   *[without GDC]*

The diagrams show how the graph of $y = x^2$ is transformed to the graph of $y = f(x)$ in three steps.
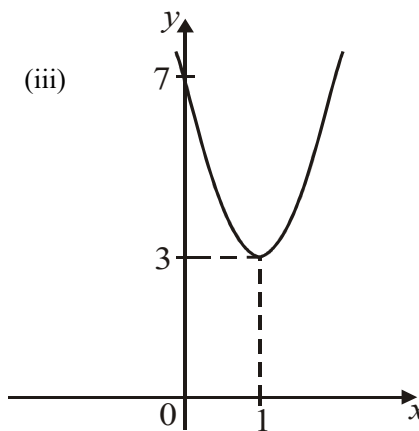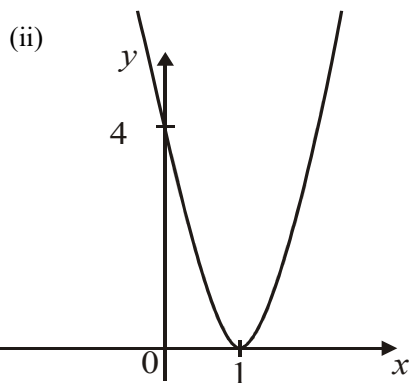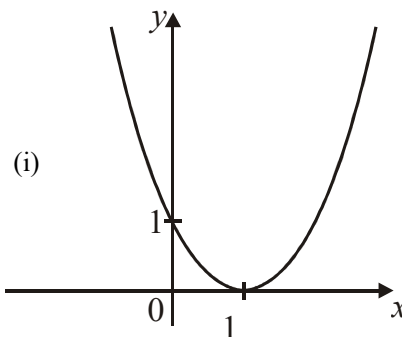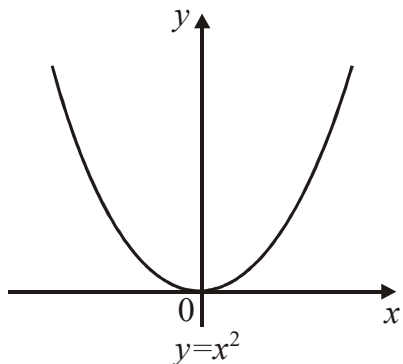
In the table below

(a)   Give the equation of the curve for each diagram   [3]

(b)   Write down the coordinates of the vertex at each step   [2]

	Equation of the curve	Vertex
	$y = x^2$	(0,0)
(i)		
(ii)		
(iii)		

16*. [Maximum mark: 6]   *[without GDC]*

Let $f(x) = x^2$ and $g(x) = 2(x-1)^2$.

(a)   The graph of g can be obtained from the graph of f using two transformations.

Give a full geometric description of each of the two transformations.   [2]

(b)   The graph of g is translated by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ (i.e. 3 units in the x-direction and

–2 units in the y-direction) to give the graph of h. The point (–1, 1) on the graph

of f is translated to the point P on the graph of h. Find the coordinates of P.   [4]

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17.   [Maximum mark: 6]   *[without GDC]*

Let $f(x) = 3x^2$. The graph of f is translated 1 unit to the right and 2 units down.

The graph of g is the image of the graph of f after this translation.

(a)   Write down the coordinates of the vertex of the graph of g.   [2]

(b)   Express g in the form $g(x) = 3(x-p)^2 + q$.   [2]

The graph of h is the reflection of the graph of g in the x-axis.

(c)   Write down the coordinates of the vertex of the graph of h.   [2]

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18. [Maximum mark: 6]   *[without GDC]*

The diagram shows a sketch of part of the graph of $f(x) = x^2$ and a sketch of part of the graph of $g(x) = -x^2 + 6x - 13$.
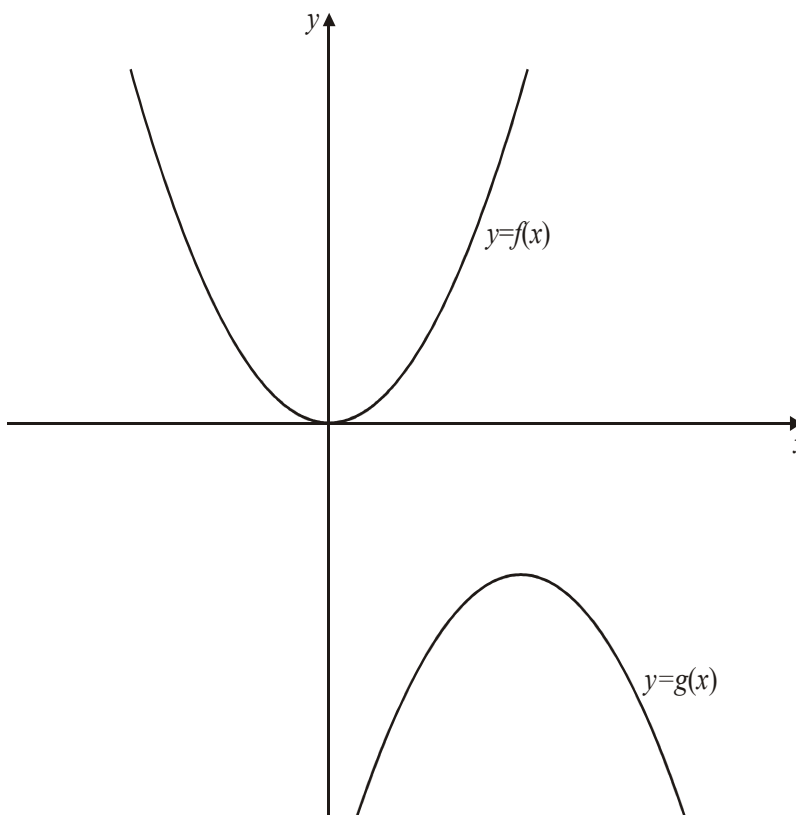
(a)   Write down the coordinates of the maximum point of $y = g(x)$

(b)   Describe fully each of the transformations, which together map the graph of $y = f(x)$ onto the graph of $y = g(x)$ in a correct order,

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19. [Maximum mark: 6]   *[with / without GDC]*

(a)   Express $y = 2x^2 - 12x + 23$ in the form $y = 2(x-c)^2 + d$.   [3]

The graph of $y = x^2$ is transformed into the graph of $y = 2x^2 - 12x + 23$ by the

transformations

a vertical stretch with scale factor k **followed by**

a horizontal translation of p units **followed by**

a vertical translation of q units.

(b)   Write down the value of   (i) k ;   (ii) p ;   (iii) q .   [3]

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20. [Maximum mark: 4]   *[without GDC]*

The diagram shows parts of the graphs of $y = x^2$ and $y = 5 - 3(x-4)^2$.
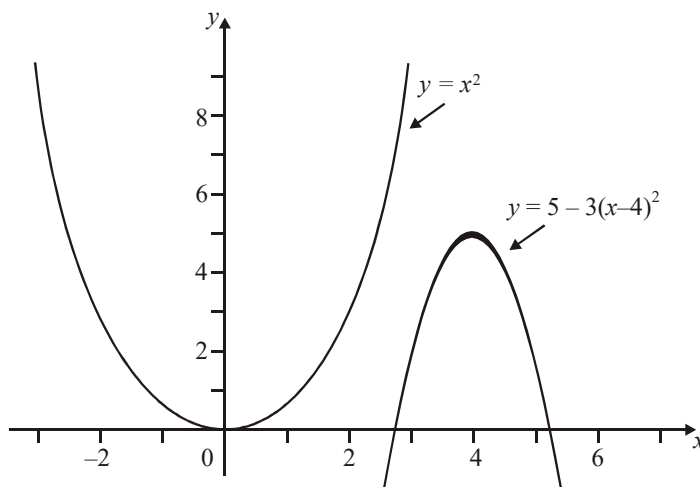
The graph of $y = x^2$ may be transformed into the graph of $y = 5 - 3(x-4)^2$ by these

transformations.

A reflection in the line $y = 0$ **followed by**

a vertical stretch with scale factor k **followed by**

a horizontal translation of p units   **followed by**

a vertical translation of q units.

Write down the value of   (i) k ;   (ii) p ;   (iii) q .

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21. [Maximum mark: 6]   *[without GDC]*

A translation with vector $\begin{pmatrix} h \\ k \end{pmatrix}$ implies a shift by h units horizontally and k units vertically.

The following diagram shows part of the graph of $f(x) = 5 - x^2$ with vertex V(0, 5).

Its image $y = g(x)$ after a translation with vector $\begin{pmatrix} h \\ k \end{pmatrix}$ has vertex T(3, 6).
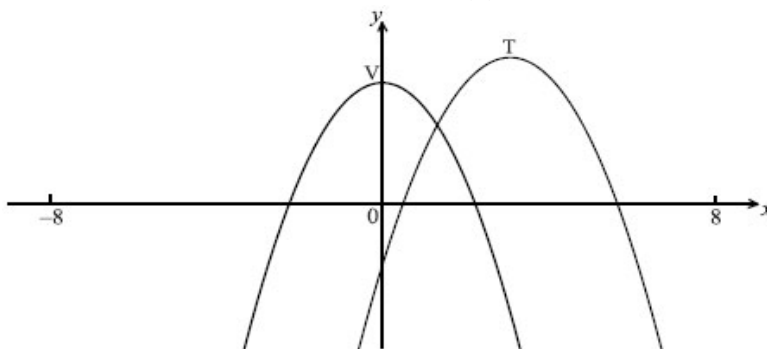
(a) Write down the value of (i) h (ii) k

(b) Write down an expression for $g(x)$.

(c) On the same diagram, sketch the graph of $y = g(-x)$

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22. [Maximum mark: 6]   *[with / without GDC]*

The quadratic function f is defined by $f(x) = 3x^2 - 12x + 11$.

(a) Write f in the form $f(x) = 3(x - h)^2 + k$.

(b) The graph of f is translated 3 units in the positive x-direction and 5 units in the positive y-direction. Find the function g for the translated graph, giving your answer in the form $g(x) = 3(x - p)^2 + q$.

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23. [Maximum mark: 6]   *[without GDC]*

The diagrams below show the graph of $y = f(x)$ which passes through the points A, B, C and D.

Sketch, indicating clearly the images of A, B, C and D, the graphs of

(a) $y = f(x-4)$;   [2]
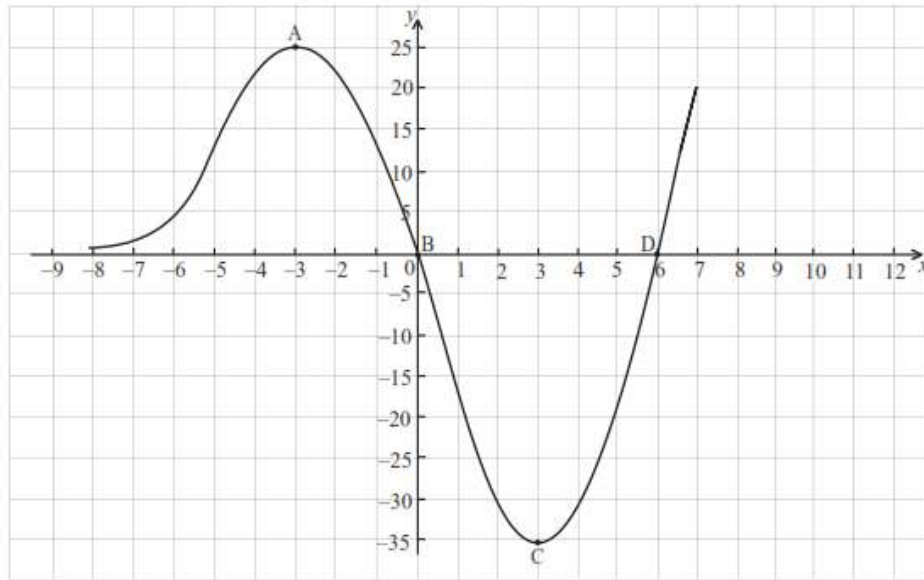
(b) $y = f(-3x)$;   [4]
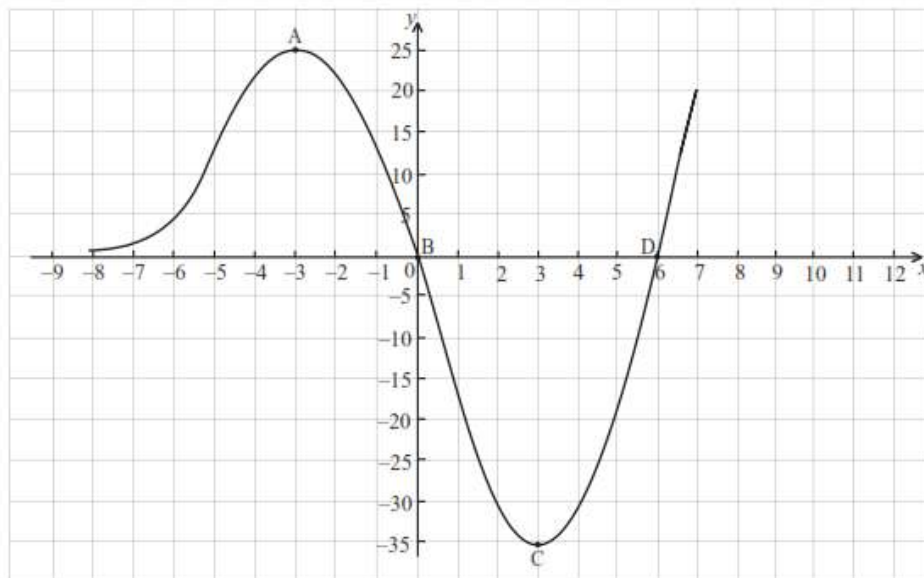
24. [Maximum mark: 6]   *[without GDC]*

The graph of $y = 2x^2 + 4x + 7$ is translated using the vector $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$. Find the equation of

the translated graph, giving your answer in the form $y = ax^2 + bx + c$.

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25. [Maximum mark: 6]   *[without GDC]*

The graph of the function $f(x) = 2x^3 - 3x^2 + x + 1$ is translated to its image, $g(x)$, by

the vector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Write $g(x)$ in the form $g(x) = ax^3 + bx^2 + cx + d$.

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B. Exam style questions (LONG)

26. [Maximum mark: 15]   *[with GDC]*

Let $f(x) = 3(x+1)^2 - 12$.

(a) Show that $f(x) = 3x^2 + 6x - 9$. [2]

(b) For the graph of f

    (i)   write down the coordinates of the vertex;

    (ii)   write down the **equation** of the axis of symmetry;

    (iii) write down the y-intercept;

    (iv) find both x-intercepts. [8]

(c) **Hence** sketch the graph of f. [2]

(d) Let $g(x) = x^2$. The graph of f may be obtained from the graph of g by the two transformations:

             a stretch of scale factor t in the y-direction

             followed by a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

Find $\begin{pmatrix} p \\ q \end{pmatrix}$ and the value of t. [3]

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27*. [Maximum mark: 12]   *[without GDC]*

Let $f(x) = x^2 + 4$ and $g(x) = x - 1$.

(a)   Find $(f \circ g)(x)$. [2]

The graph of $(f \circ g)$ is translated to the graph of h by 3 units horizontally and -1 unit vertically.

(b)   Find the coordinates of the vertex of the graph of h. [3]

(c)   Show that $h(x) = x^2 - 8x + 19$. [2]

(d)   The line $y = 2x - 6$ is a tangent to the graph of h at the point P.
       Find the x-coordinate of P. [5]

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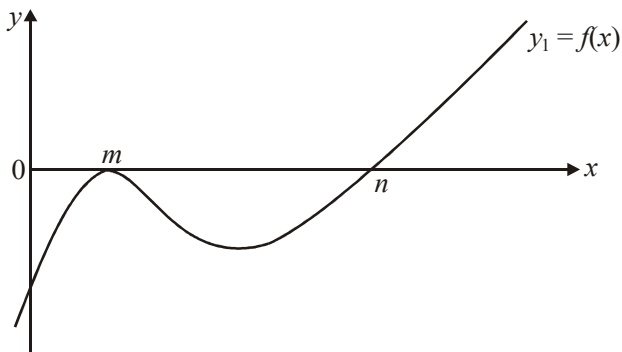
28**. [Maximum mark: 10]   *[without GDC]*

The diagrams below show the graph of $y_1 = f(x)$.

The x-axis is a tangent to $f(x)$ at $x = m$ and $f(x)$ crosses the x-axis at $x = n$.

(a)   On the diagram below sketch the graph of $y_2 = f(2x)$ and indicate the coordinates of the points of intersection of y_2 with the x-axis.

[3]

(b)   On the diagram below sketch the graph of $y_3 = f(x - k)$, where $0 < k < n - m$ and indicate the coordinates of the points of intersection of y_3 with the x-axis.
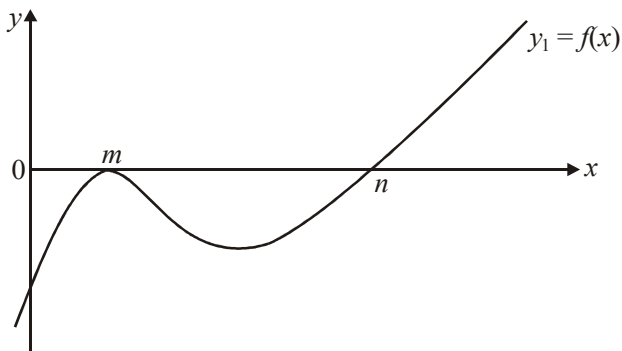
[3]

(c)   Solve the equations (giving the solutions in terms of m and n)

(i) $f(2x) = 0$;   (ii) $f(x - 3) = 0$;   (iv) $f(2x - 3) = 0$.   [4]

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