

# Contents

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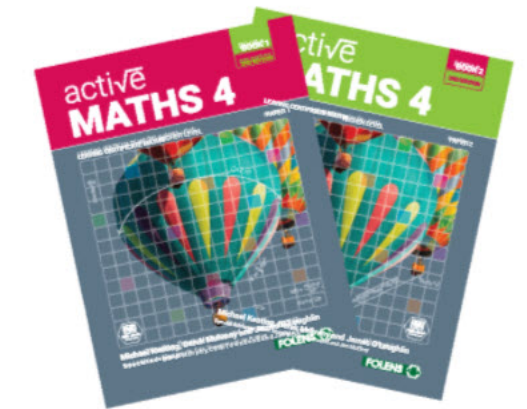


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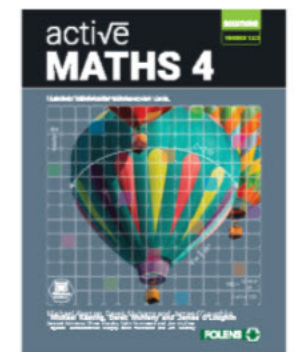
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*Michael Keating, Jim McElroy, Derek Mulvany, Oliver Murphy, James O'Loughlin and Colin Townsend*

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