Activity - Associativity Property – 3D

Learning Goals:

- To understand that the associativity property applies to vectors of dimensions other than two dimensions.
- Express and apply the associativity property in three dimensions with the interactive GeoGebra applet **Associativity Property – 3D**.

By Property 2 of the properties of a Vector Space:

Vector addition is associative, $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$

The applet **Associativity Property – 3D** is at https://www.geogebra.org/m/XnfUWvvp#material/h4fg5hf4

Note, the addition in parentheses is done first, then the other vector is added.

For each side of the equation perform the steps in the applet, taking time to understand what each means.

Consider: $(\vec{u} + \vec{v}) + \vec{w}$ Paste the graphical results of $(\vec{u} + \vec{v}) + \vec{w}$ here:

Consider: $\vec{u} + (\vec{v} + \vec{w})$ Paste the graphical results of $\vec{u} + (\vec{v} + \vec{w})$ here:

Paste the graphical results of $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ here:

Now move the origin of the vectors to any other location in space. Note: the associative property is valid anywhere.

Rest the applet.

If $\vec{u} = \begin{bmatrix} -8 \\ -4 \\ 4 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ 7 \\ 5 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} -4 \\ 3 \\ -5 \end{bmatrix}$, show using algebra that $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$. Use the template below to walk through this process step by step by filling in the blanks.

$$(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$$

$$\left(\begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} + \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}\right) + \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} + \left(\begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} + \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}\right)$$

$$\left(\begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}\right) + \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} + \left(\begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}\right)$$

$$\begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}$$

Using the applet, create these vectors $\vec{u} = \begin{bmatrix} -8 \\ -4 \\ 4 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ 7 \\ 5 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} -4 \\ 3 \\ -5 \end{bmatrix}$ as the given vectors.

Go through each step of the applet. Paste the graphical results of each of these.

$(\vec{u} + \vec{v}) + \vec{w}$

$\vec{u} + (\vec{v} + \vec{w})$

$(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$

Using the above given vectors: $\vec{u} = \begin{bmatrix} -8 \\ -4 \\ 4 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ 7 \\ 5 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} -4 \\ 3 \\ -5 \end{bmatrix}$

Now move the origin to the point (-4, 4, 6).

Paste a copy of the left screen here:

How does this affect the given vectors as compared to the origin being at (0, 0, 0)?

How does it affect the **length/magnitude** of each vector $\vec{u}$, $\vec{v}$, and $\vec{w}$ **?**

How does it affect the vector: $(\vec{u} + \vec{v}) + \vec{w}$?

How does it affect the vector: $\vec{u} + (\vec{v} + \vec{w})$?

How does it affect the equality: $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$