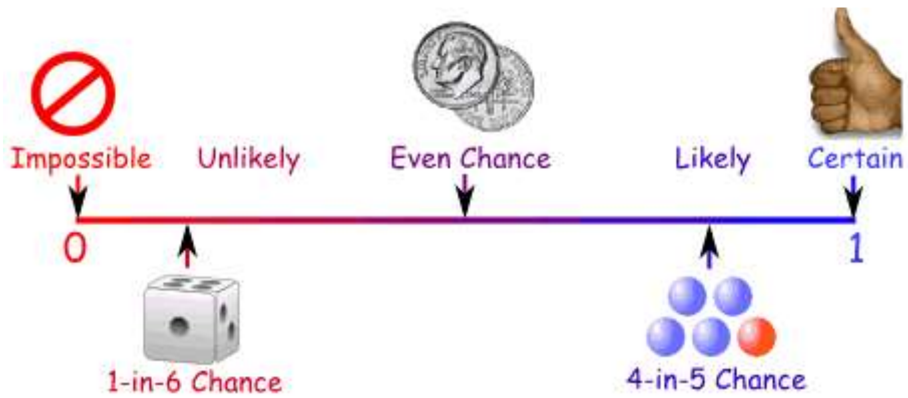
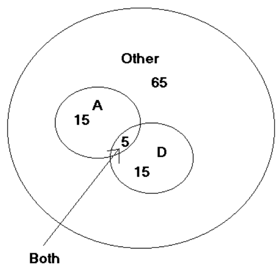
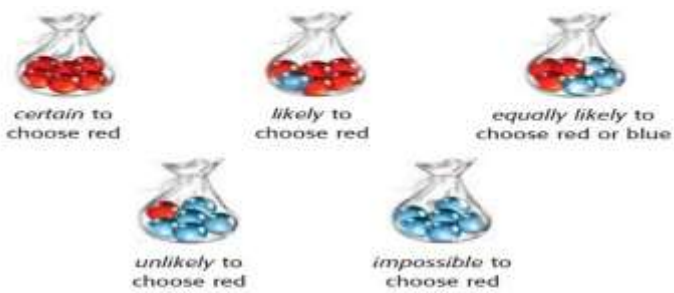
Discrete Probability

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12



Name: Mr. Wain





Probability

1. **Discrete Random Variables:**
   - Conditional Probability
   - Binomial Distribution
2. **Continuous Random Variables:**
   - Probability Density Functions
   - Normal Distribution
3. **Sampling & Estimation**

Introduction to Probability & Sample Space

- Probability assigns a numeric value to the likelihood of an event occurring.
- Probability is concerned with outcomes or results of trials in random experiments.
- A random experiment is one where:
- The possible number of outcomes is finite.
- All outcomes are equally likely.
- The results are uncertain.

- The probability that an event occurs is:

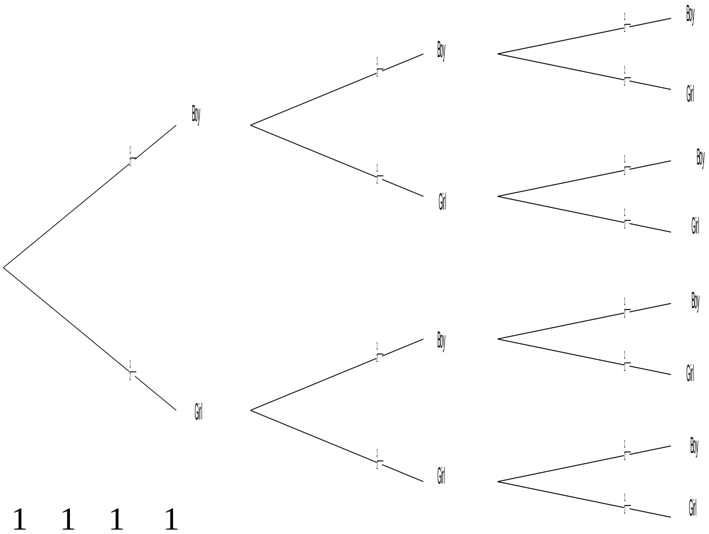
$$\frac{number\ of\ \ outcomes\ \ for\ that\ event}{total\ \ number\ of\ \ all\ \ possible\ outcomes}$$

- If an event is impossible, the probability that this event occurs = 0.
- If an event is certain, the probability that this event occurs = 1.
- So, the probability that any event occurs is between 0 and 1 inclusive.
  - i.e. $0 \le \Pr(event) \le 1$
  - $\Pr(A) = 1 - \Pr(A')$, where A' is the compliment of A
  - $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$ - the addition rule
  - Mutually exclusive: $\Pr(A \cap B) = 0$
  - Independent: $\Pr(A \cap B) = \Pr(A) \times \Pr(B) \quad or \quad \Pr(A \mid B) = \Pr(A)$

- A sample space shows all possible outcomes
- Common sample spaces are Venn diagrams, Tree diagrams and tables.

Example 1:
A family has three children. What is the probability that
(a) they are all boys?
(b) The 1st is a boy and the 2nd and the 3rd are girls?
(c) There is one boy and two girls?
(d) They are not all boys?

Solution:
Sample Space:



(a) Pr (B and B and B) = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

(b) Pr(B and G and G) = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ (specific order)

(c) Pr(one boy only) – order not specific = Pr(BGG)+Pr(GBG)+Pr(GGB)

$$= \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) = \frac{3}{8}$$

(d) Pr(That they are not all boys) = 1 – Pr(all boys) = $1 - \frac{1}{8} = \frac{7}{8}$

- **So**
  - **"and" means X (multiply)**
  - **"or" means + (add)**

Example 2.
A mathematics student calculates his chances of passing the next test according to the results on earlier tests. If he passed the last test he thinks his chances are 0.7 of passing the next test. If he failed the last test he estimates that the probability of passing the next test is 0.5. Draw a probability tree diagram to illustrate the possible results obtained on the next two tests, given that he failed the previous test.

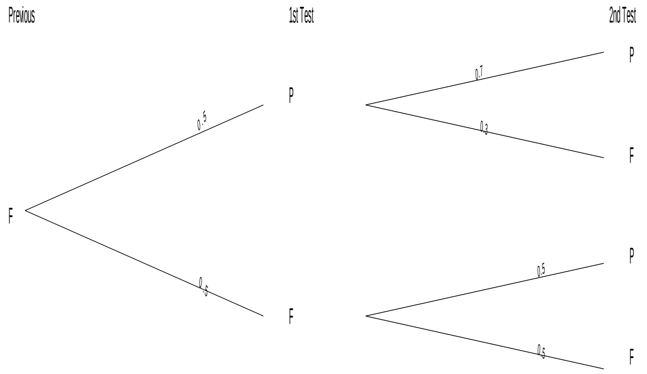
Find the probability that on the next two tests the student will:

(a) pass both;

(b) pass the first but not the second;

(c) fail the first and pass the second;

(d) fail both.



Solution:

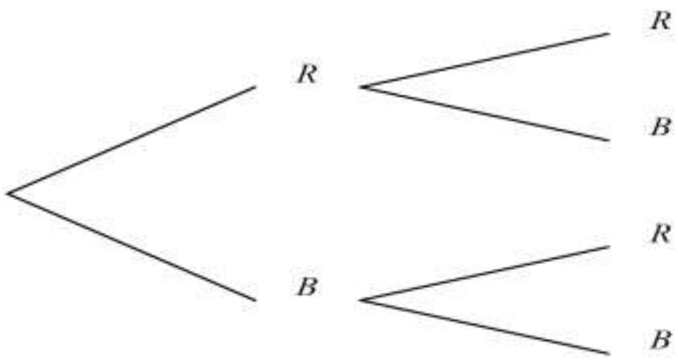
(a) Pr (P and P) = 0.5 x 0.7 = 0.35
(b) Pr (P and F) = 0.5 x 0.3 = 0.15
(c) Pr (F and P) = 0.5 x 0.5 = 0.25
(d) Pr (F and F) = 0.5 x 0.5 = 0.25

Note : the sum of these four answers.

Example 3.
From an urn containing 7 blue and 3 red balls, 2 balls are taken at random

(i) with replacement and
(ii) without replacement

Find the probability that:
(a) both balls are blue;
(b) the first ball is red and the second is blue;
(c) one is red and the other is blue.



Solution: 7 Blue, 3 Red
(i) With Replacement

(a) Pr (B and B) = $\frac{7}{10}\times\frac{7}{10}=\frac{49}{100}$

(b) Pr (R and B) = $\frac{3}{10}\times\frac{7}{10}=\frac{21}{100}$

(c) Pr (R and B or B and R) = $\frac{3}{10}\times\frac{7}{10}+\frac{7}{10}\times\frac{3}{10}=\frac{42}{100}=\frac{21}{50}$

(i) Without Replacement

(a) Pr (B and B) = $\frac{7}{10}\times\frac{6}{9}=\frac{42}{90}=\frac{7}{15}$

(b) Pr (R and B) = $\frac{3}{10}\times\frac{7}{9}=\frac{21}{90}=\frac{7}{30}$

(c) Pr (R and B or B and R) = $\frac{3}{10}\times\frac{7}{9}+\frac{7}{10}\times\frac{3}{9}=\frac{42}{90}=\frac{7}{15}$

Example 4: Simon visits the dentist every 6 months for a checkup. The probability that he will need his teeth cleaned is 0.35, the probability that he will need a filling is 0.1 and the probability that he will need both is 0.05.

a What is the probability that he will not need his teeth cleaned on a visit, but will need a filling?

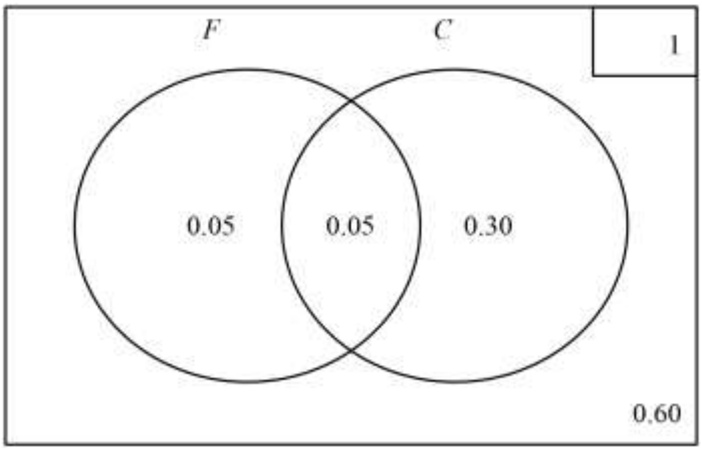
b What is the probability that she will not need either of these treatments?

Solution:

Let F = he will need a filling.

Let C = he will need a clean.

	F	F′	Total
C	0.05	0.30	0.35
C′	0.05	0.60	0.65
Total	0.10	0.90	1



a $\Pr(C' \cap F) = 0.05$

b $\Pr(C' \cap F') = 0.60$

Example 5:

A marksman never misses the target, but he has a lousy aim and his arrows land anywhere on the target. The target comprises three concentric circles with radii 10cm, 30cm, and 50cm with score values of 4 points, 2 points and 1 point respectively.

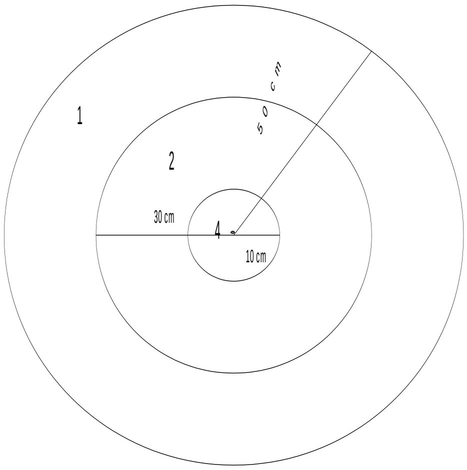
(i) For any single shot what is the probability that the resulting score is:

(a) four? (b) two? (c) one? (d) zero?

(ii) He fires three arrows (one after the other). What is the probability that the resulting score is:

(a) two? (b) three? (c) four? (d) five? (e) > 3?

Solution:



Need the areas of each part of the target.

Area of Target = $\pi \times (50)^2 = 2500\pi$

Area of "4" = $\pi \times (10)^2 = 100\pi$

Area of "2" = $\pi \times (30)^2 - \pi \times (10)^2 = 800\pi$

Area of "1"= $\pi \times (50)^2 - \pi \times (30)^2 = 1600\pi$

Let X = the score from a single shot

(i)

(a) $\Pr(X = 4) = \frac{100\pi}{2500\pi} = \frac{1}{25}$

(b) $\Pr(X = 2) = \frac{800\pi}{2500\pi} = \frac{8}{25}$

(c) $\Pr(X = 1) = \frac{1600\pi}{2500\pi} = \frac{16}{25}$

(d) $\Pr(X = 0)=0$ (he never misses)

Let Y = the score from three shots

(ii)

(a) $\Pr(Y = 2) = 0$

(b) $\Pr(Y = 3) = \Pr(1, 1, 1) = \frac{16}{25} \times \frac{16}{25} \times \frac{16}{25} = \frac{4096}{15625}$

(c) $\Pr(Y = 4) = \Pr(2, 1, 1 \text{ or } 1, 2, 1 \text{ or } 1, 1, 2) = 3 \times \frac{8}{25} \times \frac{16}{25} \times \frac{16}{25} = \frac{6144}{15625}$

(d) $\Pr(Y = 5) = \Pr(1, 2, 2 \text{ or } 2, 1, 2 \text{ or } 2, 2, 1) = 3 \times \frac{8}{25} \times \frac{8}{25} \times \frac{16}{25} = \frac{3072}{15625}$

(e) $\Pr(Y > 3) = 1 - \Pr(Y \le 3) = 1 - \frac{4096}{15625} = \frac{11529}{15625}$

- **Sheet "A", Sheet "B"**
- Sample Space – **Ex 13A** 1, 2, 3, 5, 6, 8, 10, 11, 12, 13, 15, 17

Sheet "A"

1. From a public opinion poll it was found that 2 out of 5 people wanted Sunday shopping. Out of three randomly selected people what is the probability that:
(a) all three wanted Sunday shopping;
(b) none wanted Sunday shopping;
(c) only the first and third wanted Sunday shopping;
(d) the first and second wanted Sunday shopping.

2. Person A is treated with drug X for his particular ailment. Person B with a different complaint is treated with drug Y and person C, with a different complaint again, is treated with drug Z. Drugs X, Y, Z have success rates of 7 in 10, 8 in 9 and 1 in 4 respectively. Find the probability that:
(a) all three are cured; (b) none are cured;
(c) only B is cured; (d) both B and C are cured.

3. Box X contains 3 blue balls and 5 red balls. Box Y contains 4 blue balls and 6 red balls. One ball is randomly selected from *each* of the boxes X and Y. Find the probability that, of the 2 balls taken
(a) both are red; (b) both are blue;
(c) neither are blue; (d) only one is red having come from Box X;
(e) only one is red having come from Box Y;
(f) only one ball is red.

4. Two dice, one white and one blue, are tossed. The white one is a fair die and the blue one is weighted such that the Pr(1) = Pr(2) = 0.2, Pr(3) = 0.3, Pr(4) = Pr(5) = Pr(6) = 0.1. Find the probability that:
(a) a 6 turned up on both dice;
(b) an even number turned up on both dice;
(c) the white die showed an even number and the blue die an odd number;
(d) the white die showed an odd number and the blue die an even number;
(e) one die showed an odd number and the other die an even number.

Answers

1. (a) $\frac{8}{125}$ (b) $\frac{27}{125}$ (c) $\frac{12}{125}$ (d) $\frac{4}{25}$
2. (a) $\frac{7}{45}$ (b) $\frac{1}{40}$ (c) $\frac{1}{5}$ (d) $\frac{2}{9}$
3. (a) $\frac{3}{8}$ (b) $\frac{3}{20}$ (c) $\frac{3}{8}$ (d) $\frac{1}{4}$ (e) $\frac{9}{40}$ (f) $\frac{19}{40}$
4. (a) $\frac{1}{60}$ (b) $\frac{1}{5}$ (c) $\frac{3}{10}$ (d) $\frac{1}{5}$ (e) $\frac{1}{2}$

Sheet "B"

1. From a group of 5 students and 3 teachers, two people are selected at random. What is the probability that:

(a) no student is selected; (b) no teachers are selected;

(c) the first person selected is a student and the second is a teacher;

(d) one person selected is a student and the other a teacher.

2. The probability that it rains tomorrow is $\frac{1}{4}$. If it rains tomorrow then the probability that it is fine the next day is $\frac{3}{5}$. Find the probability that:

(a) it rains tomorrow and is fine the next day;

(b) it rains both days.

3. Two identical boxes X and Y contain 10 balls. Box X contains 3 red and 7 black balls while Box Y contains 1 red and 9 black balls. A box is chosen at random and from that box a ball is selected at random. What is the probability that:

(a) Box X was chosen and the ball was red;

(b) Box Y was chosen and the ball was red;

(c) the ball was black and it came from Box X.

Answers

1. (a) $\frac{3}{28}$ (b) $\frac{5}{14}$ (c) $\frac{15}{56}$ (d) $\frac{15}{28}$

2. (a) $\frac{3}{20}$ (b) $\frac{1}{10}$

3. (a) $\frac{3}{20}$ (b) $\frac{1}{20}$ (c) $\frac{7}{20}$

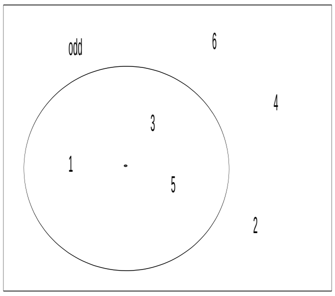
Conditional Probability

"When you are given extra information about the outcome" **(KEYWORD: GIVEN)**

- For conditional probability Pr(*A*|*B*) – the probability that *A* occurs given that *B* has occurred.
- Rule: $\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$
- Two events *A* and *B* are **independent** if the occurrence of one event has no effect on the probability of the occurrence of the other.
  - $\Pr(A \mid B) = \Pr(A)$
  - $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$
- Two types of problems:

Type 1
Example 1: A die is thrown. Find the probability that a three turns up given that the number is odd.

Solution:
Let A = a three is thrown
Let B = an odd is thrown



	3	3'	
odd	1	2	3
odd'	0	3	3
	1	5	6

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$\Pr(A \mid B) = \frac{\Pr(3 \text{ and odd})}{\Pr(odd)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

Example 2: From a pack of 52 playing cards, one is drawn. If it is a heart, what is the probability that it is the ace of hearts?

Let *A* = ace of hearts drawn
Let *B* = a heart is drawn

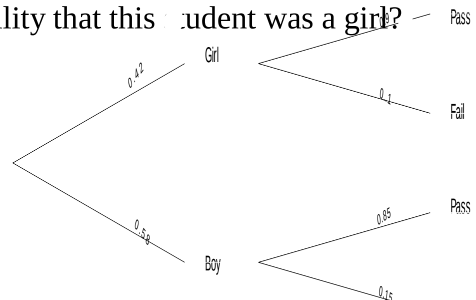
$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$\Pr(A \mid B) = \frac{\Pr(ace\ of\ hearts\ and\ heart)}{\Pr(heart)} = \frac{\frac{1}{52}}{\frac{1}{4}} = \frac{1}{13}$$

TYPE 2
Example 3: In a certain VCE mathematics examination, 42% of the candidates were girls, and 90% of these girls passed in mathematics. The rest of the candidates were boys and 85% of these passed in mathematics.

(a) What was the overall percentage of candidates who passed in mathematics?
(b) A randomly selected mathematics paper was found to be the paper of a student who has passed in mathematics. What is the probability that this student was a girl?

Solution:



(a) % passed

= 0.42 x 0.9 + 0.58 x .85

= 0.871

= 87.1 %

(b) Pr (Girl given that the student passed)

$$= \frac{\Pr(Girl \cap student\ \ passed)}{\Pr(student\ \ passed)}$$

$$= = \frac{0.42 \times 0.9}{0.871}$$

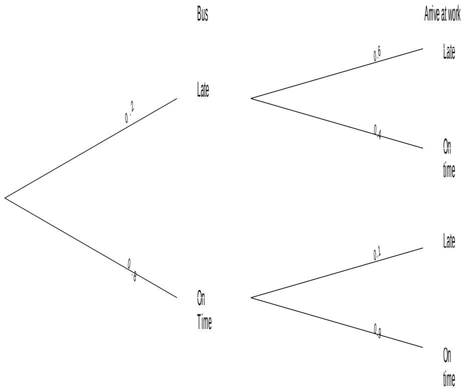
$= 0.434$

Alternative approach: Karnaugh (or Two-way) Table.

	Girl	Boy	Total
Pass	0.378	0.493	0.871
Fail	0.042	0.087	0.129
Total	0.420	0.580	1.000

Example 4: There is only one bus service passing a man’s house each morning. If the bus is on time then he arrives at work on time on average of 9 out 10 occasions. If the bus is late then he arrives at work on time on only an average of 4 out 10 times. The bus is late 20% of the time. Find the probability that the bus was late on a day he was late to work.

Solution:



Pr (Bus late *knowing* Man was late for work)

$$= \frac{\Pr(Bus\ late \cap Man\ late\ \ for\ \ work)}{\Pr(Man\ late\ \ for\ \ work)} = \frac{0.2 \times 0.6}{(0.2 \times 0.6) + (0.8 \times 0.1)} = \frac{0.12}{0.20} = 0.6$$

or

		Bus		
		On time	Late	Total
Work	On time	0.72	0.08	0.80
	Late	0.08	0.12	0.20
	Total	0.80	0.20	1.000

Example 5: The probability that Monica remembers to do her homework is 0.7, while the probability that Patrick remembers to do his homework is 0.4. If these events are independent, then what is the probability that:

a both will do their homework

b Monica will do her homework but Patrick forgets?

Solution:

Let M = Monica does her homework
Let P = Patrick does his homework

a

$$\begin{aligned}\Pr(M \cap P) &= \Pr(M) \times \Pr(P) \quad \textit{Independent} \\ &= 0.7 \times 0.4 \\ &= 0.28\end{aligned}$$

b

$$\begin{aligned}\Pr(M \cap P') &= \Pr(M) \times \Pr(P') \quad \textit{Independent} \\ &= 0.7 \times 0.6 \\ &= 0.42\end{aligned}$$

- **Ex 13 B** 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 15, 17, 19

Discrete Probability

- Discrete Random Variables (DRV) and Discrete Probability Distributions (DPD)
- A DRV is a variable.
- The value of a DRV is usually an integer and countable.
- Some examples and non-examples of DRV's

    - If X = the number of boys in a family, X is a DRV
    - If Y = 2^{nd} innings score of a cricket match, Y is a DRV
    - If A = the height of a Year 12 student at RSC, A is not a DRV
    - If T = the time taken to get home, T is not a DRV.

- A DPD is a table with two columns (or rows) that shows all the possible values of a DRV with each of its respective possibilities.

Example 1:
Consider a family of three children.

(i) Use a probability tree to list all the possible families.

(ii) If the probability of a girl is $\frac{3}{5}$, find the probability of each of the possible families occurring.

(iii) Find the probability distribution of the discrete random variable, X, where X is "the number of boys in the family".

(iv) Using your answer to (iii) find:

a. $\Pr(X = 2)$

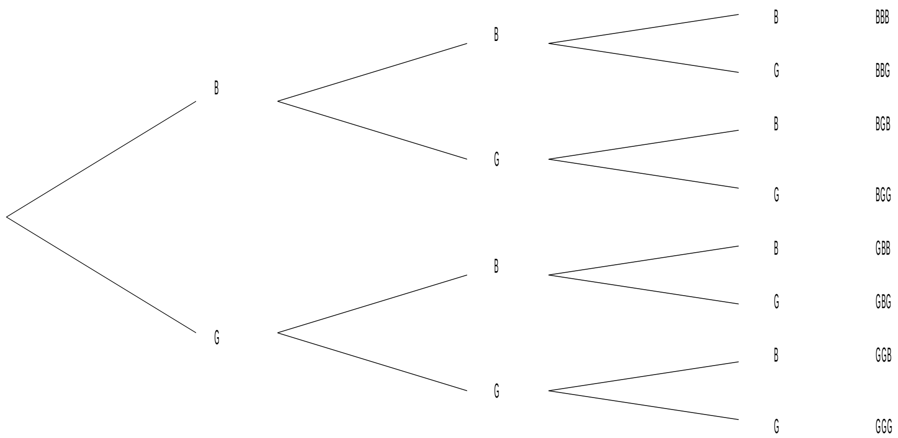
b. $\Pr(X < 3)$

c. $\Pr(X = 2 \mid X \geq 1)$

d. $\left\{x : \Pr(X = x) = \frac{36}{125}\right\}$

e. $\left\{x : \Pr(X \leq x) = \frac{81}{125}\right\}$

Solution:



(ii)

$\Pr(BBB) = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \frac{8}{125}$
$\Pr(BBG) = \frac{2}{5} \times \frac{2}{5} \times \frac{3}{5} = \frac{12}{125}$
$\Pr(BGB) = \frac{2}{5} \times \frac{3}{5} \times \frac{2}{5} = \frac{12}{125}$
$\Pr(BGG) = \frac{2}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{18}{125}$
$\Pr(GBB) = \frac{3}{5} \times \frac{2}{5} \times \frac{2}{5} = \frac{12}{125}$
$\Pr(GBG) = \frac{3}{5} \times \frac{2}{5} \times \frac{3}{5} = \frac{18}{125}$
$\Pr(GGB) = \frac{3}{5} \times \frac{3}{5} \times \frac{2}{5} = \frac{18}{125}$
$\Pr(GGG) = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{27}{125}$

(iii)

x	$\Pr(X = x)$
0	$\frac{27}{125}$
1	$\frac{54}{125}$
2	$\frac{36}{125}$
3	$\frac{8}{125}$

(iv) (a) $\Pr(X = 2) = \frac{36}{125}$

(b) $\Pr(X < 3) = \Pr(X = 0 \text{ or } 1 \text{ or } 2) = \frac{27}{125} + \frac{54}{125} + \frac{36}{125} = \frac{117}{125}$ (or $1 - \Pr(X = 3)$)

(c) $\Pr(X = 2 \mid X \geq 1) = \frac{\Pr(X = 2 \cap X \geq 1)}{\Pr(X \geq 1)}$

$$= \frac{\Pr(X = 2)}{\Pr(X \geq 1)} = \frac{\frac{36}{125}}{1 - \frac{27}{125}} = \frac{\frac{36}{125}}{\frac{98}{125}} = \frac{36}{98} = \frac{18}{49}$$

(d) $\Pr(X = x) = \frac{36}{125} \Rightarrow x = 2$

(e) $\Pr(X \leq x) = \frac{81}{125} \Rightarrow x = 1$

- **Ex 13C** 1, 2, 3,6, 7, 9
- **Ex 13C** 5, 10, 11, 12, 13, 17, 18

Expected Value, $E(X)$

- The expected value, $E(X)$, is the same as the average, mean or μ.
- The general rule: $E(X) = \sum x.\Pr(X = x)$
- *"the expected value is equal to the sum of each value of X multiplied by its probability"*
- Also: $E(f(x)) = \sum f(x) \cdot \Pr(X = x)$

Note:
Mode: Most Common
Median: Middle value
Mean: Average

Properties of $E(X)$

(i)	$E(aX) = aE(X)$
(ii)	$E(aX+b) = aE(X)+b$
(iii)	$E(a) = a$
(iv)	$E(X+Y) = E(X) + E(Y)$

Example: For the following probability distribution, find:

x	$\Pr(X = x)$
1	0.2
2	0.1
3	0.5
4	0.2

(a) $E(X)$ (d) Mode
(b) $E(X + 2)$ (e) Median
(c) $E(X^2)$

Solution:

(a) $E(X) = \sum x.\Pr(X = x)$

x	$\Pr(X = x)$	$x.\Pr(X = x)$
1	0.2	0.2
2	0.1	0.2
3	0.5	1.5
4	0.2	0.8
	Total	2.7

$= E(X)$

(b) $E(X + 2) = \sum (x+2).\Pr(X = x)$

x	$x+2$	$\Pr(X = x)$	$(x+2).\Pr(X = x)$
1	3	0.2	0.6
2	4	0.1	0.4
3	5	0.5	2.5
4	6	0.2	1.2
		Total	4.7

$= E(X+2)$

(c) $E(X^2) = \sum (x^2).\Pr(X = x)$

x	x^2	$\Pr(X = x)$	$x^2.\Pr(X = x)$
1	1	0.2	0.2
2	4	0.1	0.4
3	9	0.5	4.5
4	16	0.2	3.2
		Total	8.3

$= E(X^2)$

(d) Mode: $x = 3$ (e) Median: $x = 3$

- **Ex13D** 1, 2, 3, 4, 5, 7, 8, 9ab, 11ab(i)(ii), 12ab, 14ab, 15ab, 16a, 17a

The Variance, *Var*(*X*) of a DRV(*X*)

- The variance measures the spread of a distribution.
- $Var(X) = E(X-\mu)^2$

  $= \sum (x-\mu)^2 \cdot \Pr(X=x)$

  ....

  $= E(X^2) - (E(X))^2 \; or \; \sum x^2 \cdot \Pr(X=x) - \mu^2$
- The standard deviation of X, $SD(X) = \sqrt{Var(X)}$.
- Common Notation

Mean	*E(X)*	μ
Variance	*Var(X)*	σ^2
Standard Deviation	*SD*(*X*)	σ

Example: Consider these two distributions:

x	$\Pr(X=x)$
2	0.3
3	0.5
4	0.2

y	$Pr(Y = y)$
0	0.05
1	0.1
2	0.15
3	0.3
4	0.4

x	$\Pr(X=x)$	$x.\Pr(X=x)$	x^2	$x^2.\Pr(X=x)$
2	0.3	0.6	4	1.2
3	0.5	1.5	9	4.5
4	0.2	0.8	16	3.2
	$E(X)=$	2.9	$E(X^2)=$	8.9

y	$\Pr(Y=y)$	$y.\Pr(Y=y)$	y^2	$y^2.\Pr(Y=y)$
0	0.05	0.0	0	0
1	0.1	0.1	1	0.1
2	0.15	0.30	4	0.6
3	0.3	0.9	9	2.7
4	0.4	1.6	16	6.4
	$E(Y)=$	2.9	$E(Y^2)=$	9.8

Both distributions have the same mean, $\mu_X = \mu_Y$, but the *y*-values have more spread.

$Var(X) = E(X^2) - \mu^2$

$\sigma^2 = 8.9 - (2.9)^2$

$\sigma^2 = 0.49$

$SD(X) = \sigma = \sqrt{0.49} = 0.7$

$Var(Y) = E(Y^2) - \mu^2$

$\sigma^2 = 9.8 - (2.9)^2$

$\sigma^2 = 1.39$

$SD(Y) = \sigma = \sqrt{1.39} = 1.18$

Example: A box contains three white and two red balls. The balls are taken out one at a time (and not replaced) until red ball is obtained.

(a) find the probability distribution for the number of balls chosen.
(b) How many draws do you expect until you get a red?
(c) Find the variance and standard variance.

Solution:
Let X = the number of balls chosen.

x	Order of balls	$\Pr(X = x)$	$x.\Pr(X = x)$	x^2	$x^2.\Pr(X = x)$
1	R	$\frac{2}{5}$	$\frac{2}{5}$	1	$\frac{2}{5}$
2	WR	$\frac{3}{5}\times\frac{2}{4}=\frac{3}{10}$	$\frac{3}{5}$	4	$\frac{6}{5}$
3	WWR	$\frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}=\frac{1}{5}$	$\frac{3}{5}$	9	$\frac{9}{5}$
4	WWWR	$\frac{3}{5}\times\frac{2}{4}\times\frac{1}{3}\times\frac{2}{2}=\frac{1}{10}$	$\frac{2}{5}$	16	$\frac{8}{5}$
		$E(X)=$	2	$E(X^2)=$	5

(b) $E(X) = 2$
(c) $Var(X) = 5 - (2)^2 = 1$
$SD(X)=1$

- Useful property: $VAR(aX+b) = a^2Var(X)$

- **Ex13D** 9c, 11b(iii) 12c, 13, 14c, 15c, 16b, 17b

The relationship between the mean and the standard deviation. ($\mu \pm 2\sigma$)

- For many probability distributions (but not all), about 95% of the distribution lies within two standard deviations of the mean.
- i.e. $\Pr(\mu - 2\sigma \le X \le \mu + 2\sigma) \approx 0.95$

Example: For the family of three children, used before, where the chance of the birth of a girl was $\frac{3}{5}$:

x	$\Pr(X = x)$	$x.\Pr(X = x)$	x^2	$x^2.\Pr(X = x)$
0	$\frac{27}{125}$	0	0	0
1	$\frac{54}{125}$	$\frac{54}{125}$	1	$\frac{54}{125}$
2	$\frac{36}{125}$	$\frac{72}{125}$	4	$\frac{144}{125}$
3	$\frac{8}{125}$	$\frac{24}{125}$	9	$\frac{72}{125}$
	$E(X)$=	$\frac{150}{125} = 1.2$	$E(X^2)$=	$\frac{270}{125} = 2.16$

$Var(X) = E(X^2) - \mu^2$

$\sigma^2 = 2.16 - (1.2)^2$

$\sigma^2 = 0.72$

$SD(X) = \sigma = \sqrt{0.72} = 0.8485$

$\mu - 2\sigma = 1.2 - (2 \times 0.8485) = -0.497$

$\mu + 2\sigma = 1.2 + (2 \times 0.8485) = 2.897$

$\therefore \Pr(\mu - 2\sigma \le X \le \mu + 2\sigma) = \Pr(-0.497 \le X \le 2.897)$

- the values of X (the number of boys) that lie between these two numbers are 0, 1 & 2.
- $\Pr(\mu - 2\sigma \le X \le \mu + 2\sigma) = \Pr(0 \le X \le 2) = \frac{117}{125} = 0.936 \quad or \quad 93.6\%$

- **Ex13D** 14d, 15d, 16c, 17c, 18
- Review questions chapter 13

The Binomial Distribution – an example of Discrete Probability

The Binomial Theorem

Expansions of the form $(a+b)^n$

Consider: $(x+1)^2$

$(x+1)^2 = (x+1)(x+1) = x^2 + x + x + 1 = x^2 + 2x + 1$

Now:

$(x-1)^2 = (x-1)(x-1) = x^2 - x - x + 1 = x^2 - 2x + 1$

There is a pattern: the identity, $(a \pm b)^2 = a^2 \pm 2ab + b^2$

There is an identity for $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

And there is for….

Interesting is that the co-efficients of the expansions belong to Pascal's Triangle:

PASCAL'S TRIANGLE

1

1 1

1 2 1 $\binom{2}{0}$ $\binom{2}{1}$ $\binom{2}{2}$

1 $\binom{3}{0}$ $\binom{3}{1}$ $\binom{3}{2}$ $\binom{3}{3}$

$\binom{4}{0}$ $\binom{4}{1}$ $\binom{4}{2}$ $\binom{4}{3}$ $\binom{4}{4}$

Etc..

In general the Binomial Expansion is:

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots\ldots + \binom{n}{n}a^0 b^n$$

Here we have the:

1st term $\binom{n}{0}a^n b^0$, 2nd term $\binom{n}{1}a^{n-1}b^1$, 3rd term $\binom{n}{2}a^{n-2}b^2$, the general term $\binom{n}{r}a^{n-r}b^r$ and the last term. $\binom{n}{n}a^0 b^n$.

Properties of combinations:

$\binom{n}{0}$ *and* $\binom{n}{n} = 1$

$\binom{7}{1} = \binom{7}{6}$ and in general $\binom{n}{r} = \binom{n}{n-r}$

Example: Using the binomial expansion expand $(2x-7)^5$.

Solution: $a = 2x, b = -7, n = 5$

$$(2x-7)^5 = \binom{5}{0}(2x)^5(-7)^0 + \binom{5}{1}(2x)^4(-7)^1 + \binom{5}{2}(2x)^3(-7)^2 + \binom{5}{3}(2x)^2(-7)^3 + \binom{5}{4}(2x)^1(-7)^4 + \binom{5}{5}(2x)^0(-7)^5$$
$$= \left(1\times(2x)^5\times 1\right) + \left(5\times(2x)^4\times(-7)^1\right) + \left(10\times(2x)^3\times(-7)^2\right) + \left(10\times(2x)^2\times(-7)^3\right) + \left(5\times(2x)^1\times(-7)^4\right) + \left(1\times(2x)^0\times(-7)^5\right)$$
$$= 32x^5 - 560x^4 + 3920x^3 - 13720x^2 + 24010x - 16807$$

**There is always one more term than the power.

** For each term the sum of the indices add up to "n".

The Binomial Probability Distribution

CHARACTERISTICS

In a binomial experiment,
1. there are *two* possible outcomes for each trial.
 ‘success’ $\rightarrow$ this is the ‘desired’ outcome
 ‘failure’ $\rightarrow$ this outcome is ‘not desired’.
2. the probability of a ‘success’ is the *same* for each trial.
 i.e. the trials are *independent.*

Note: Trials of this type are called BERNOULLI trials. (pronounced Burnooey)

The FORMULA for the PROBABILITY of a BINOMIAL r.v.

$$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

where $\Pr(X = x)$ = Pr (getting $\underline{x}$ successes in $\underline{n}$ trials)
and p = *Pr(Success)*

NB. This formula takes into account all possible orders.

When do you use the Binomial Distribution?

When the situation has BOTH of the characteristics: 1 & 2
This usually involves:
* sampling with replacement OR
* sampling without replacement from a ‘large’ population OR
* no sampling at all: just observing

Notation:

The random variable X has a binomial distribution with n independent trials and p = probability of a success, is written as:

$$X \sim Bi(n, p) \quad \text{or} \quad X \underset{=}{d}\ Bi(n, p)$$

e.g. $X \sim Bi(20, 0.3)$

Example: A machine manufacturing calculators is known to have a defective rate of 1 in 10. Find the probability that in a sample of 6 calculators taken at random:
(a) exactly two are defective;
(b) no more than 2 are defective.

- 2 possible outcomes – defective or not defective.
- Let X = the number of defective calculators.
- $X \sim Bi\left(6, \frac{1}{10}\right)$

(a) $\Pr(X = 2) = \binom{6}{2}\left(\frac{1}{10}\right)^2\left(\frac{9}{10}\right)^4 = 0.098415$

(b)

$$\Pr(X \le 2) = \Pr(X = 0) + \Pr(X = 1) + \Pr(X = 2) = \binom{6}{0}\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^6 + \binom{6}{1}\left(\frac{1}{10}\right)^1\left(\frac{9}{10}\right)^5 + \binom{6}{2}\left(\frac{1}{10}\right)^2\left(\frac{9}{10}\right)^4 = 0.984$$

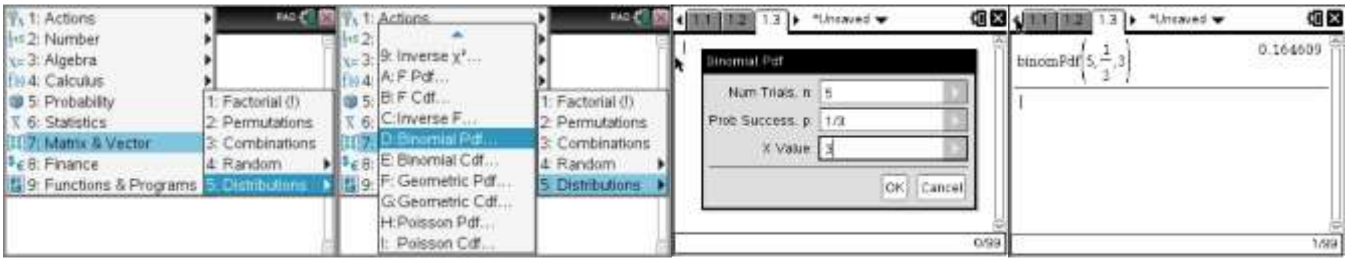
- **Ex 14A** 1, 3, 4, 6, 7, 10, 11, 12, 13, 14 , 16, 18, 19, 20, 21

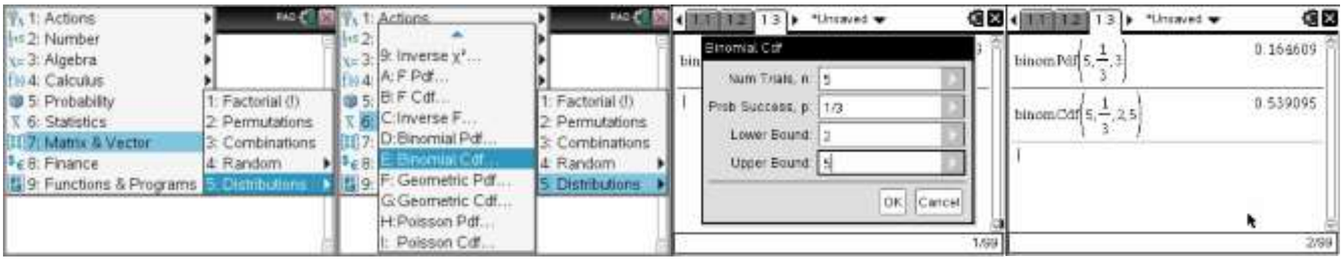
Using The Graphics Calculator

Example: A hitter has a probability of $\frac{1}{3}$ of getting a hit each time at bat, with each at-bat independent of other at-bat. In the next 5 times at-bat,

(a) What is the probability of getting exactly three hits?

(b) What is the probability of getting at least two hits?





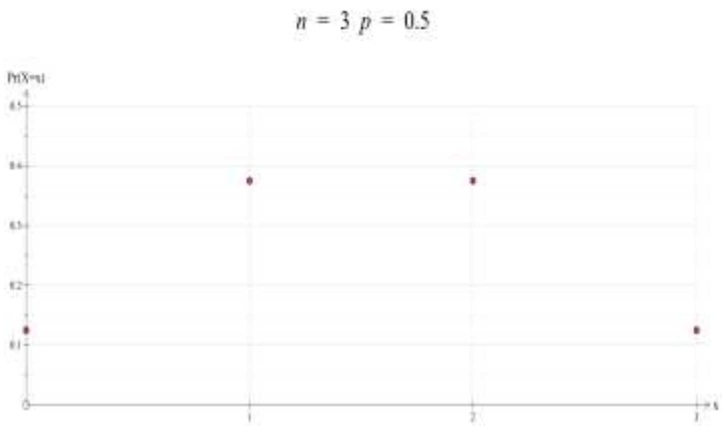
The graph of the binomial probability distribution

Example: Find the probability distribution of the number of girls in a family of three children. Assume that the probability of a girl being born is 0.5. Hence graph $\Pr(X = x)$ versus x, where X = the number of girls in the family.

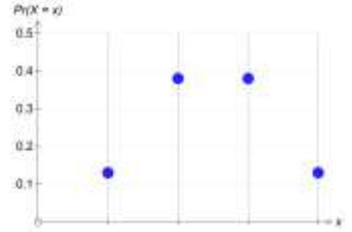
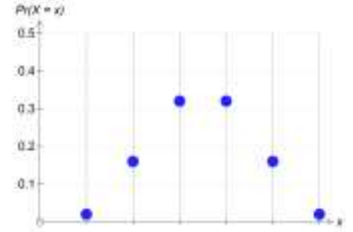
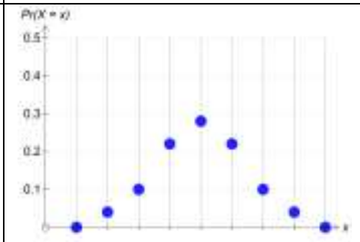
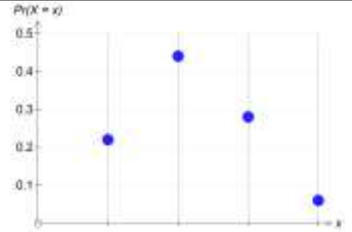
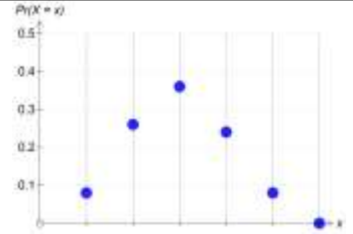
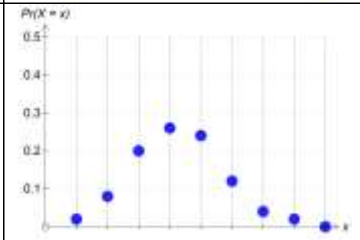
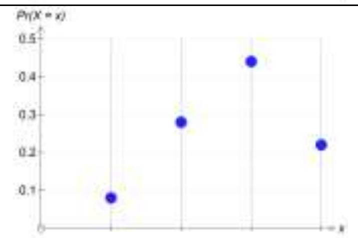
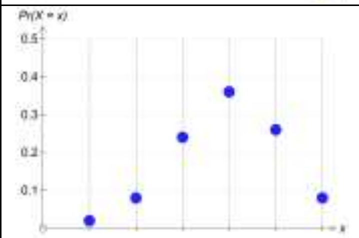
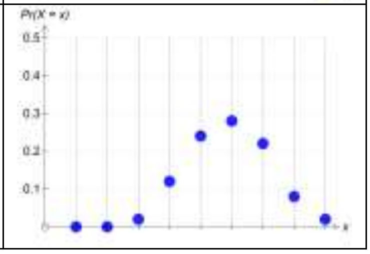
Solution:
$X \sim Bi(3, 0.5)$

$$\Pr(X = x) = \binom{n}{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{n-x}$$

x	$\Pr(X = x)$
0	0.125
1	0.375
2	0.375
3	0.125



Repeat for: all combinations of n = 3, 5, 8 and p = 0.4, 0.5, 0.6

	$n = 3$	$n = 5$	$n = 8$
$p = 0.5$			
$p = 0.4$			
$p = 0.6$			

- The effect of the parameters (variables) n and p on the shape of the graph:
  - As the value of n increases, the peak of the graph shifts to the right. (i.e. the expected value (the mean) increases)
  - When $p = 0.5$, the curve is perfectly symmetrical.
  - When $p < 0.5$ the distribution is skewed to the right (or positively skewed) NOTE: Skewness refers to the tail.
  - When $p > 0.5$ the distribution is skewed to the left (negatively skewed).

- **Ex 14B** Q 1, 2, 3

Expectation and Variance of the Binomial Distribution

- The mean of a binomial distribution DRV can be found by:
  $\mu = E(X) = \sum x.\Pr(X = x) = n.p$
- The variance of a binomial distribution DRV can be found by:
  $\sigma^2 = E(X^2) - \mu^2 = np(1-p)$
- The standard deviation : $SD(X) = \sigma = \sqrt{np(1-p)}$.

Example: A binomial random variable has a mean of 3 and a variance of 2. find the parameters *n* and *p*.

Solution:

$X \sim Bi(n, p)$

$\mu = np \qquad \Rightarrow np = 3$

$\sigma^2 = np(1-p) \quad \Rightarrow np(1-p) = 2$

$\therefore 3(1-p) = 2$

$1 - p = \frac{2}{3}$

$p = \frac{1}{3} \quad \Rightarrow \quad n = 9$

Example: Give the 95% confidence limits for the number of girls in a family of eight children. Assume Pr(girl)=0.55.

Solution:

Let X = the number of girls in the family

$X \sim Bi(8, 0.55)$

We know: $\Pr(\mu - 2\sigma \le X \le \mu + 2\sigma) \approx 0.95$

$\mu = 8 \times 0.55 = 4.4$

$\sigma = \sqrt{8 \times 0.55 \times 0.44} = 1.4071$

$\mu - 2\sigma = 4.4 - 2(1.4071) = 1.5858$

$\mu + 2\sigma = 4.4 + 2(1.4071) = 7.2142$

$\therefore \Pr(1.5858 \le x \le 7.2142) = \Pr(2 \le x \le 7)$

- **Ex 14B** 4, 5, 6, 7, 8, 9, 10

Binomial Distribution: Solving for '*n*'

Example:

A group of people meet for a fancy dress party. Each person comes dressed in something related to his or her zodiac sign. Assume that the probability of a person at the party having a particular zodiac sign is $\frac{1}{12}$.

(a) What is the least number of people who need to attend the party so that the probability that there will be at least one Scorpio is greater than 0.8?

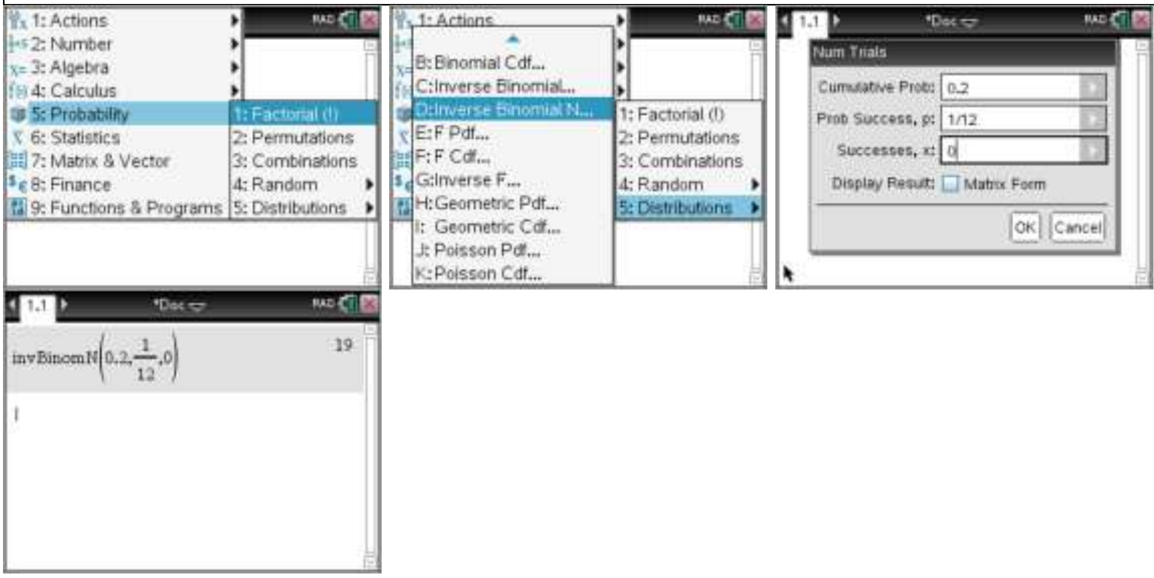
Solution: Let X = the number of Scorpios at the party

$\Pr(X \geq 1) > 0.8$

$\therefore \Pr(X < 1) < 0.2$

$\Pr(X = 0) < 0.2$

$\binom{n}{0}\left(\frac{1}{12}\right)^0\left(\frac{11}{12}\right)^n < 0.2$ $\quad$ $solve\left(\left(\frac{11}{12}\right)^\wedge n < 0.2, n\right)$ $\quad$ $n > 18.496\ldots \Rightarrow \textit{at least } 19 \textit{ people}$

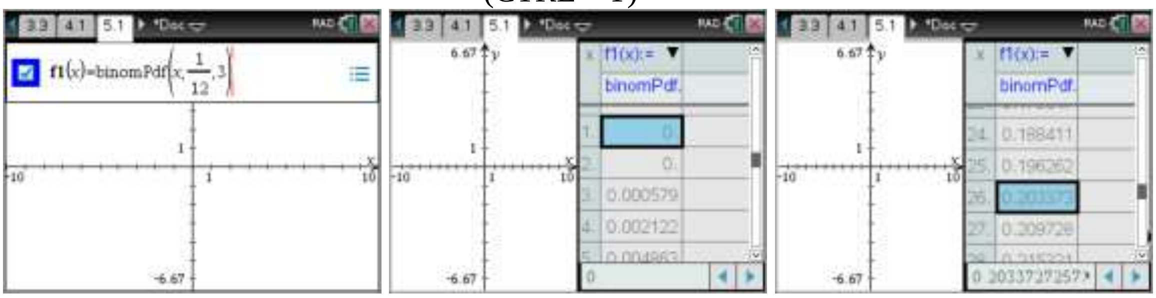


(b) What is the least number of people who need to be at the party so that the probability that there will be exactly 3 Scorpios is greater than 20%?

Solution:

$n = 26$

(CTRL – T)



Can't use invBinomN( ) command as it is not a Cumulative Probability.

(c) What is the least number of people who need to attend the party so that the probability of fewer than two Scorpios is closest to 0.8?

Solution:

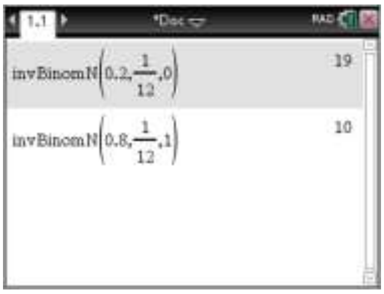
$$\Pr(X < 2) = 0.8$$

$$\Pr(X \le 1) = 0.8$$

$$\textit{answer}\ 10\ \textit{people}$$

Go to tableset put in number 0, 1 then go to table and scroll down to the value that is closest to 0.8 .

OR



- **Ex 14C** Q 1, 2, 3, 4, 5, 6, 7 **Chapter 14 Review**

Past Exam Questions

2008 Exam 1

Question 7

Jane drives to work each morning and passes through three intersections with traffic lights. The number X of traffic lights that are red when Jane is driving to work is a random variable with probability distribution given by

x	0	1	2	3
$\Pr(X = x)$	0.1	0.2	0.3	0.4

a. What is the mode of X?

b. Jane drives to work on two consecutive days. What is the probability that the number of traffic lights that are red is the same on both days?

1 + 2 = 3 marks

Question 8

Every Friday Jean-Paul goes to see a movie. He always goes to one of two local cinemas – the Dandy or the Cino.

If he goes to the Dandy one Friday, the probability that he goes to the Cino the next Friday is 0.5. If he goes to the Cino one Friday, then the probability that he goes to the Dandy the next Friday is 0.6.

On any given Friday the cinema he goes to depends only on the cinema he went to on the previous Friday.

If he goes to the Cino one Friday, what is the probability that he goes to the Cino on exactly two of the next three Fridays?

3 marks

2008 Exam 2

Question 5

Let X be a discrete random variable with a binomial distribution. The mean of X is 1.2 and the variance of X is 0.72.

The values of n (the number of independent trials) and p (the probability of success in each trial) are

A. $n = 4$, $p = 0.3$
B. $n = 3$, $p = 0.6$
C. $n = 2$, $p = 0.6$
D. $n = 2$, $p = 0.4$
E. $n = 3$, $p = 0.4$

Question 13

According to a survey, 30% of employed women have never been married.

If 10 employed women are selected at random, the probability (correct to four decimal places) that at least 7 have never been married is

A. 0.0016
B. 0.0090
C. 0.0106
D. 0.9894
E. 0.9984

Question 14

The minimum number of times that a fair coin can be tossed so that the probability of obtaining a head on each trial is less than 0.0005 is

A. 8
B. 9
C. 10
D. 11
E. 12

Question 15

The sample space when a fair die is rolled is $\{1, 2, 3, 4, 5, 6\}$, with each outcome being equally likely.

For which of the following pairs of events are the events independent?

A. $\{1, 2, 3\}$ and $\{1, 2\}$
B. $\{1, 2\}$ and $\{3, 4\}$
C. $\{1, 3, 5\}$ and $\{1, 4, 6\}$
D. $\{1, 2\}$ and $\{1, 3, 4, 6\}$
E. $\{1, 2\}$ and $\{2, 4, 6\}$

Question 1

Sharelle is the goal shooter for her netball team. During her matches, she has many attempts at scoring a goal.

Assume that each attempt at scoring a goal is independent of any other attempt. In the long term, her scoring rate has been shown to be 80% (that is, 8 out of 10 attempts to score a goal are successful).

a. i. What is the probability, correct to four decimal places, that her first 8 attempts at scoring a goal in a match are successful?

ii. What is the probability, correct to four decimal places, that exactly 6 of her first 8 attempts at scoring a goal in a match are successful?

1 + 2 = 3 marks

Assume instead that the success of an attempt to score a goal depends only on the success or otherwise of her previous attempt at scoring a goal.

If an attempt at scoring a goal in a match is successful, then the probability that her next attempt at scoring a goal in the match is successful is 0.84. However, if an attempt at scoring a goal in a match is unsuccessful, then the probability that her next attempt at scoring a goal in the match is successful is 0.64.

Her first attempt at scoring a goal in a match is successful.

b. i. What is the probability, correct to four decimal places, that her next 7 attempts at scoring a goal in the match will be successful?

ii. What is the probability, correct to four decimal places, that exactly 2 of her next 3 attempts at scoring a goal in the match will be successful?

iii. What is the probability, correct to four decimal places, that her 8th attempt at scoring a goal in the match will be successful?

iv. In the long term, what percentage of her attempts at scoring a goal are successful?

1 + 3 + 3 + 1 = 8 marks

2009 Exam 1

Question 8

Four identical balls are numbered 1, 2, 3 and 4 and put into a box. A ball is randomly drawn from the box, and not returned to the box. A second ball is then randomly drawn from the box.

a. What is the probability that the first ball drawn is numbered 4 and the second ball drawn is numbered 1?

1 mark

b. What is the probability that the sum of the numbers on the two balls is 5?

1 mark

c. Given that the sum of the numbers on the two balls is 5, what is the probability that the second ball drawn is numbered 1?

2 marks

Question 7

The random variable X has this probability distribution.

X	0	1	2	3	4
$\Pr(X = x)$	0.1	0.2	0.4	0.2	0.1

Find

a. $\Pr(X > 1 \mid X \leq 3)$

2 marks

b. $\text{Var}(X)$, the variance of X.

3 marks

2009 Exam 2

Question 10

The discrete random variable X has a probability distribution as shown.

x	0	1	2	3
$\Pr(X = x)$	0.4	0.2	0.3	0.1

The median of X is

A. 0

B. 1

C. 1.1

D. 1.2

E. 2

Question 13

A fair coin is tossed twelve times.

The probability (correct to four decimal places) that at most 4 heads are obtained is

A. 0.0730

B. 0.1209

C. 0.1938

D. 0.8062

E. 0.9270

Question 17

The sample space when a fair twelve-sided die is rolled is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. Each outcome is equally likely.

For which one of the following pairs of events are the events independent?

A. $\{1, 3, 5, 7, 9, 11\}$ and $\{1, 4, 7, 10\}$

B. $\{1, 3, 5, 7, 9, 11\}$ and $\{2, 4, 6, 8, 10, 12\}$

C. $\{4, 8, 12\}$ and $\{6, 12\}$

D. $\{6, 12\}$ and $\{1, 12\}$

E. $\{2, 4, 6, 8, 10, 12\}$ and $\{1, 2, 3\}$

2010 Exam 1

Question 8

The discrete random variable X has the probability distribution

x	−1	0	1	2
$\Pr(X = x)$	p^2	p^2	$\frac{p}{4}$	$\frac{4p+1}{8}$

Find the value of p.

3 marks

2010 Exam 2

Question 12

A soccer player is practising her goal kicking. She has a probability of $\frac{3}{5}$ of scoring a goal with each attempt. She has 15 attempts.

The probability that the number of goals she scores is less than 7 is closest to

A. 0.0612

B. 0.0950

C. 0.1181

D. 0.2131

E. 0.7869

Question 14

A bag contains four white balls and six black balls. Three balls are drawn from the bag without replacement.

The probability that they are all black is

A. $\frac{1}{6}$

B. $\frac{27}{125}$

C. $\frac{24}{29}$

D. $\frac{3}{500}$

E. $\frac{8}{125}$

Question 15

The discrete random variable X has the following probability distribution.

X	0	1	2
$\Pr(X = x)$	a	b	0.4

If the mean of X is 1 then

A. $a = 0.3$ and $b = 0.1$

B. $a = 0.2$ and $b = 0.2$

C. $a = 0.4$ and $b = 0.2$

D. $a = 0.1$ and $b = 0.5$

E. $a = 0.1$ and $b = 0.3$

Question 21

Events A and B are mutually exclusive events of a sample space with

$$\Pr(A) = p \text{ and } \Pr(B) = q \text{ where } 0 < p < 1 \text{ and } 0 < q < 1.$$

$\Pr(A' \cap B')$ is equal to

A. $(1 - p)(1 - q)$

B. $1 - pq$

C. $1 - (p + q)$

D. $2 - p - q$

E. $1 - (p + q - pq)$

2010 Exam 2

Question 2

Victoria Jones runs a small business making and selling statues of her cousin the adventurer Tasmania Jones.

The statues are made in a mould, then finished (smoothed and then hand-painted using a special gold paint) by Victoria herself. Victoria sends the statues **in order of completion** to an inspector, who classifies them as either 'Superior' or 'Regular', depending on the quality of their finish.

If a statue is Superior then the probability that the next statue completed is Superior is p.

If a statue is Regular then the probability that the next statue completed is Superior is $p - 0.2$.

On a particular day, Victoria knows that $p = 0.9$.

On that day

a. if **the first statue inspected is Superior**, find the probability that the third statue is Regular.

2 marks

b. if **the first statue inspected is Superior**, find the probability that the next three statues are Superior.

1 mark

c. find the steady state probability that any one of Victoria's statues is Superior.

1 mark

On another day, Victoria finds that if **the first statue inspected is Superior** then the probability that the third statue is Superior is 0.7.

d. i. Show that the value of p on this day is 0.75.

On this day, a group of 3 consecutive statues is inspected. Victoria knows that the first statue of the 3 statues is **Regular**.

ii. Find the expected number of these 3 statues that will be Superior.

3 + 4 = 7 marks

Victoria hears that another company, Shoddy Ltd, is producing similar statues (also classified as Superior or Regular), but its statues are entirely made by machines, on a construction line. The quality of any one of Shoddy's statues is independent of the quality of any of the others on its construction line. The probability that any one of Shoddy's statues is Regular is 0.8.

Shoddy Ltd wants to ensure that the probability that it produces at least two Superior statues in a day's production run is at least 0.9.

e. Calculate the minimum number of statues that Shoddy would need to produce in a day to achieve this aim.

3 marks

Total 14 marks

2011 Exam 1

Question 7

A biased coin is tossed three times. The probability of a head from a toss of this coin is p.

a. Find, in terms of p, the probability of obtaining

i. three heads from the three tosses

ii. two heads and a tail from the three tosses.

1 + 1 = 2 marks

b. If the probability of obtaining three heads equals the probability of obtaining two heads and a tail, find p.

2 marks

Question 8

Two events, A and B, are such that $\Pr(A) = \frac{3}{5}$ and $\Pr(B) = \frac{1}{4}$.

If A' denotes the complement of A, calculate $\Pr(A' \cap B)$ when

a. $\Pr(A \cup B) = \frac{3}{4}$

2 marks

b. A and B are mutually exclusive.

1 mark

2011 Exam 2

Question 21

For two events, P and Q, $\Pr(P \cap Q) = \Pr(P' \cap Q)$.

P and Q will be independent events exactly when

A. $\Pr(P') = \Pr(Q)$

B. $\Pr(P \cap Q') = \Pr(P' \cap Q)$

C. $\Pr(P \cap Q) = \Pr(P) + \Pr(Q)$

D. $\Pr(P \cap Q') = \Pr(P \cap Q)$

E. $\Pr(P) = \frac{1}{2}$

2012 Exam 1

Question 4

On any given day, the number X of telephone calls that Daniel receives is a random variable with probability distribution given by

x	0	1	2	3
$\Pr(X = x)$	0.2	0.2	0.5	0.1

a. Find the mean of X.

2 marks

b. What is the probability that Daniel receives only one telephone call on each of three consecutive days?

1 mark

c. Daniel receives telephone calls on both Monday and Tuesday.
What is the probability that Daniel receives a total of four calls over these two days?

3 marks

2012 Exam 2

Question 12

Demelza is a badminton player. If she wins a game, the probability that she will win the next game is 0.7. If she loses a game, the probability that she will lose the next game is 0.6. Demelza has just won a game.
The probability that she will win exactly one of her next two games is

A. 0.33
B. 0.35
C. 0.42
D. 0.49
E. 0.82

Question 13

A and B are events of a sample space S.

$$\Pr(A \cap B) = \frac{2}{5} \text{ and } \Pr(A \cap B') = \frac{3}{7}$$

$\Pr(B'|A)$ is equal to

A. $\frac{6}{35}$
B. $\frac{15}{29}$
C. $\frac{14}{35}$
D. $\frac{29}{35}$
E. $\frac{2}{3}$

Question 20

A discrete random variable X has the probability function $\Pr(X = k) = (1 - p)^k p$, where k is a non-negative integer.
$\Pr(X > 1)$ is equal to

A. $1 - p + p^2$
B. $1 - p^2$
C. $p - p^2$
D. $2p - p^2$
E. $(1 - p)^2$

Question 3

Steve, Katerina and Jess are three students who have agreed to take part in a psychology experiment. Each student is to answer several sets of multiple-choice questions. Each set has the same number of questions, n, where n is a number greater than 20. For each question there are four possible options (A, B, C or D), of which only one is correct.

a. Steve decides to guess the answer to every question, so that for each question he chooses A, B, C or D at random.
Let the random variable X be the number of questions that Steve answers correctly in a particular set.

i. What is the probability that Steve will answer the first three questions of this set correctly?

ii. Find, to four decimal places, the probability that Steve will answer at least 10 of the first 20 questions of this set correctly.

iii. Use the fact that the variance of X is $\frac{75}{16}$ to show that the value of n is 25.

1 + 2 + 1 = 4 marks

If Katerina answers a question correctly, the probability that she will answer the next question correctly is $\frac{3}{4}$. If she answers a question incorrectly, the probability that she will answer the next question incorrectly is $\frac{2}{3}$.

In a particular set, Katerina answers Question 1 incorrectly.

b. i. Calculate the probability that Katerina will answer Questions 3, 4 and 5 correctly.

ii. Find the probability that Katerina will answer Question 25 correctly. Give your answer correct to four decimal places.

3 + 2 = 5 marks

c. The probability that Jess will answer any question correctly, independently of her answer to any other question, is p ($p > 0$). Let the random variable Y be the number of questions that Jess answers correctly in any set of 25.

If $\Pr(Y > 23) = 6\Pr(Y = 25)$, show that the value of p is $\frac{5}{6}$.

2 marks

2013 Exam 1

Question 7 (6 marks)

The probability distribution of a discrete random variable, X, is given by the table below.

x	0	1	2	3	4
$\Pr(X = x)$	0.2	$0.6p^2$	0.1	$1 - p$	0.1

a. Show that $p = \frac{2}{3}$ or $p = 1$. 3 marks

b. Let $p = \frac{2}{3}$.

i. Calculate $E(X)$. 2 marks

ii. Find $\Pr(X \geq E(X))$. 1 mark

2013 Exam 2

Question 8

When Xena travels to work, she either drives or takes the bus.

If she takes the bus to work one day, the probability that she takes the bus to work the next day is $\frac{7}{10}$.

If she drives to work one day, the probability that she drives to work the next day is $\frac{3}{5}$.

(Assume that Xena will always travel to work according to these conditions only.)

What is the long-term probability that Xena will take the bus to work?

A. $\frac{3}{4}$

B. $\frac{7}{10}$

C. $\frac{4}{7}$

D. $\frac{6}{13}$

E. $\frac{3}{7}$

Question 9

Harry is a soccer player who practises penalty kicks many times each day.

Each time Harry takes a penalty kick, the probability that he scores a goal is 0.7, independent of any other penalty kick.

One day Harry took 20 penalty kicks.

Given that he scored at least 12 goals, the probability that Harry scored exactly 15 goals is closest to

A. 0.1789

B. 0.8867

C. 0.8

D. 0.6396

E. 0.2017

Question 10

For events A and B, $\Pr(A \cap B) = p$, $\Pr(A' \cap B) = p - \frac{1}{8}$ and $\Pr(A \cap B') = \frac{3p}{5}$.

If A and B are independent, then the value of p is

A. 0

B. $\frac{1}{4}$

C. $\frac{3}{8}$

D. $\frac{1}{2}$

E. $\frac{3}{5}$

Question 17

A and B are events of a sample space.

Given that $\Pr(A|B) = p$, $\Pr(B) = p^2$ and $\Pr(A) = p^{\frac{1}{2}}$, $\Pr(B|A)$ is equal to

A. p

B. $p^{\frac{4}{3}}$

C. $p^{\frac{5}{2}}$

D. $p^{\frac{3}{2}}$

E. p^3

Question 2 (11 marks)

FullyFit is an international company that owns and operates many fitness centres (gyms) in several countries. At every one of FullyFit's gyms, each member agrees to have his or her fitness assessed every month by undertaking a set of exercises called S. There is a five-minute time limit on any attempt to complete S and if someone completes S in less than three minutes, they are considered fit.

a. At FullyFit's Melbourne gym, it has been found that the probability that any member will complete S in less than three minutes is $\frac{5}{8}$. This is independent of any other member.

In a particular week, 20 members of this gym attempt S.

i. Find the probability, correct to four decimal places, that at least 10 of these 20 members will complete S in less than three minutes. 2 marks

ii. Given that at least 10 of these 20 members complete S in less than three minutes, what is the probability, correct to three decimal places, that more than 15 of them complete S in less than three minutes? 3 marks

b. Paula is a member of FullyFit's gym in San Francisco. She completes S every month as required, but otherwise does not attend regularly and so her fitness level varies over many months. Paula finds that if she is fit one month, the probability that she is fit the next month is $\frac{3}{4}$, and if she is not fit one month, the probability that she is not fit the next month is $\frac{1}{2}$.

If Paula is not fit in one particular month, what is the probability that she is fit in exactly two of the next three months? 2 marks

2014 Exam 1

Question 9 (6 marks)

Sally aims to walk her dog, Mack, most mornings. If the weather is pleasant, the probability that she will walk Mack is $\frac{3}{4}$, and if the weather is unpleasant, the probability that she will walk Mack is $\frac{1}{3}$.

Assume that pleasant weather on any morning is independent of pleasant weather on any other morning.

a. In a particular week, the weather was pleasant on Monday morning and unpleasant on Tuesday morning.

Find the probability that Sally walked Mack on at least one of these two mornings. 2 marks

b. In the month of April, the probability of pleasant weather in the morning was $\frac{5}{8}$.

i. Find the probability that on a particular morning in April, Sally walked Mack. 2 marks

ii. Using your answer from part b.i., or otherwise, find the probability that on a particular morning in April, the weather was pleasant, given that Sally walked Mack that morning. 2 marks

2014 Exam 2

Question 11

A bag contains five red marbles and four blue marbles. Two marbles are drawn from the bag, without replacement, and the results are recorded.

The probability that the marbles are different colours is

A. $\frac{20}{81}$

B. $\frac{5}{18}$

C. $\frac{4}{9}$

D. $\frac{40}{81}$

E. $\frac{5}{9}$

Question 14

If X is a random variable such that $\Pr(X > 5) = a$ and $\Pr(X > 8) = b$, then $\Pr(X < 5 \mid X < 8)$ is

A. $\frac{a}{b}$

B. $\frac{a-b}{1-b}$

C. $\frac{1-b}{1-a}$

D. $\frac{ab}{1-b}$

E. $\frac{a-1}{b-1}$

Question 22

John and Rebecca are playing darts. The result of each of their throws is independent of the result of any other throw. The probability that John hits the bullseye with a single throw is $\frac{1}{4}$. The probability that Rebecca hits the bullseye with a single throw is $\frac{1}{2}$. John has four throws and Rebecca has two throws.

The ratio of the probability of Rebecca hitting the bullseye at least once to the probability of John hitting the bullseye at least once is

A. 1:1

B. 32:27

C. 64:85

D. 2:1

E. 192:175

2015 Exam 1

Question 8 (3 marks)

For events A and B from a sample space, $\Pr(A \mid B) = \frac{3}{4}$ and $\Pr(B) = \frac{1}{3}$.

a. Calculate $\Pr(A \cap B)$. 1 mark

b. Calculate $\Pr(A' \cap B)$, where A' denotes the complement of A. 1 mark

c. If events A and B are independent, calculate $\Pr(A \cup B)$. 1 mark

2015 Exam 2

Question 10

The binomial random variable, X, has $E(X) = 2$ and $\text{Var}(X) = \frac{4}{3}$.

$\Pr(X = 1)$ is equal to

A. $\left(\frac{1}{3}\right)^6$

B. $\left(\frac{2}{3}\right)^6$

C. $\frac{1}{3} \times \left(\frac{2}{3}\right)^5$

D. $6 \times \frac{1}{3} \times \left(\frac{2}{3}\right)^5$

E. $6 \times \frac{2}{3} \times \left(\frac{1}{3}\right)^5$

Question 12

A box contains five red balls and three blue balls. John selects three balls from the box, without replacing them.

The probability that at least one of the balls that John selected is red is

A. $\frac{5}{7}$

B. $\frac{5}{14}$

C. $\frac{7}{28}$

D. $\frac{15}{56}$

E. $\frac{55}{56}$

Question 14

Consider the following discrete probability distribution for the random variable X.

x	1	2	3	4	5
$\Pr(X = x)$	p	$2p$	$3p$	$4p$	$5p$

The mean of this distribution is

A. 2

B. 3

C. $\frac{7}{2}$

D. $\frac{11}{3}$

E. 4