LAYING THE FOUNDATION
Building Academic Excellence

Polygons II

About this Lesson
This lesson connects algebraic and geometric skills by using the characteristics of polygons to explore limit concepts. For each characteristic, students will create a function rule, sketch a graph of the function over a discrete domain, and describe the end behavior of the graph.

Prior to the lesson, students should have experience with formulas for the characteristics of regular polygons (see "Summary of Regular Polygons" at the end of the Student Activity pages).

This lesson is included in Module 8 – Limits.

Objectives
Students will
- create functions representing characteristics of polygons.
- graph functions over discrete domains.
- describe the end behavior of graphs in terms of limits.
- verbalize the relationship between the graph's end behavior and the polygon's characteristic as the number of sides of the polygon increases.

Level
Geometry

Common Core State Standards for Mathematical Content
This lesson addresses the following Common Core State Standards for Mathematical Content. The lesson requires that students recall and apply each of these standards rather than providing the initial introduction to the specific skill. The star symbol (★) at the end of a specific standard indicates that the high school standard is connected to modeling.

Explicitly addressed in this lesson

Code	Standard	Level of Thinking	Depth of Knowledge
A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★	Apply	II
F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★	Analyze	III

T E A C H E R

Code	Standard	Level of Thinking	Depth of Knowledge
F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$ (n is greater than or equal to 1).	Apply	II
F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★	Analyze	III
F-BF.1a	Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.★	Apply	II
F-IF.7a	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.★	Apply	II
F-IF.7b	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.★	Apply	II
F-IF.7d	(+) Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.★	Apply	II
A-REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	Apply	II

Common Core State Standards for Mathematical Practice
These standards describe a variety of instructional practices based on processes and proficiencies that are critical for mathematics instruction. LTF incorporates these important processes and proficiencies to help students develop knowledge and understanding and to assist them in making important connections across grade levels. This lesson allows teachers to address the following Common Core State Standards for Mathematical Practice.

Implicitly addressed in this lesson

Code	Standard
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
4	Model with mathematics.
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning.

LTF Content Progression Chart
In the spirit of LTF's goal to connect mathematics across grade levels, the Content Progression Chart demonstrates how specific skills build and develop from sixth grade through pre-calculus. Each column, under a grade level or course heading, lists the concepts and skills that students in that grade or course should master. Each row illustrates how a specific skill is developed as students advance through their mathematics courses.

6th Grade Skills/Objectives	7th Grade Skills/Objectives	Algebra 1 Skills/Objectives	Geometry Skills/Objectives	Algebra 2 Skills/Objectives	Pre-Calculus Skills/Objectives
			Develop the formulas for the measures of angles, perimeter, and/or area of a regular polygon inscribed in a circle by recognizing the patterns. (200_GE.LI_H.04)	Develop the formulas for the measures of angles, perimeter, and/or area of a regular polygon inscribed in a circle by recognizing the patterns. (200_A2.LI_H.04)	Develop the formulas for the measures of angles, perimeter, and/or area of a regular polygon inscribed in a circle by recognizing the patterns. (200_PC.LI_H.04)
			Graph the formulas for the measures of angles, perimeter, and/or area of a regular polygon inscribed in a circle and evaluate their limits. (200_GE.LI_H.04)	Graph the formulas for the measures of angles, perimeter, and/or area of a regular polygon inscribed in a circle and evaluate their limits. (200_A2.LI_H.04)	Graph the formulas for the measures of angles, perimeter, and/or area of a regular polygon inscribed in a circle and evaluate their limits. (200_PC.LI_H.04)

Connection to AP*
AP Calculus Topic: Limits
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TEACHER

Materials and Resources

- Student Activity pages
- Graphing calculators

Assessments

The following formative assessment is embedded in this lesson:

- Students engage in independent practice.

The following additional assessments are located on the LTF website:

- Limits – Geometry Free Response Questions
- Limits – Geometry Multiple Choice Questions

Teaching Suggestions

Prior to this lesson, students should have completed the Polygons I activity, including the polygons chart. The formulas developed on the chart will be used in this lesson. If this activity does not follow the Polygon I lesson, provide the students with the summary of the formulas on the last page of the Student Activity. If the lesson follows Polygons I, have the students use their own chart from Polygons I.

If the notation for a limit has not previously been discussed, introduce students to $\lim_{x \to \infty} y = \#$, meaning that, as *x* becomes large without bound, *y* approaches the number #. Connect this idea to horizontal asymptotes and the end behavior of a function. Students will also encounter $\lim_{x \to \infty} y = \infty$, meaning that the function values increase without bound or that the function does not actually have a limit. Point out to students that using the infinity notation is more descriptive of the end behavior of the function than merely saying that the limit does not exist.

Since the functions that define the characteristics of regular polygons are discrete rather than continuous, they should be graphed as a set of points rather than as a connected line or curve. For simple functions, students should certainly graph the points by hand. For more complicated functions, two calculator techniques work well.

Method 1:

Enter the formulas into the lists and graph as a scatterplot.

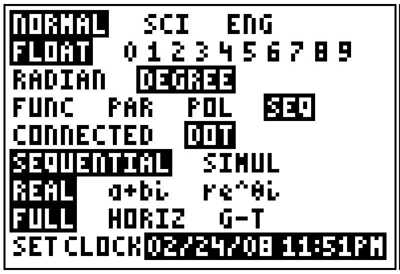
- Clear lists L1 and L2 then enter the numbers of sides for the polygons into L1. Highlight the name L2 at the top of list 2 and type in the formula using L1 in place of *n*. Press enter to fill the list.
- To put the formula into L2 so that data in L1 can be changed without having to enter the formula again, type "180(L1–2)" and press enter. L2 values will now change as different values are entered into L1. Note: The quotation marks lock the formula into the list. To clear the formula from a list, highlight the list name, press enter, then press the clear key. This will clear the formula but the values will remain in the list. The values in the list may be cleared in the normal way at this point.
- The seq command on the list key provides a nice technique for storing numbers into a list. The format for the sequence command is: 2nd List: seq(formula, x, start value, end value) From the home screen, enter seq(180(*x*-2), *x*, 3, 12) Sto L2. Since the formula may be recalled on the home screen, changes to the formula are easy to make.

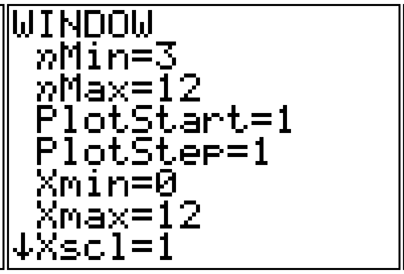
T E A C H E R

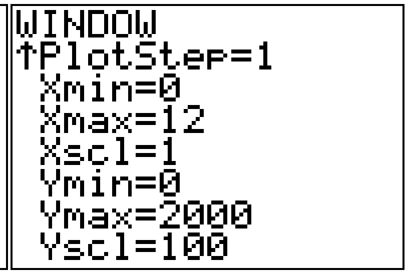
Method 2:

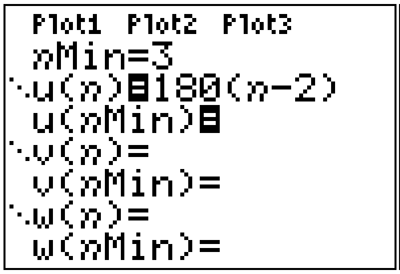
Use the sequence mode on the TI calculators. The example below graphs the sum of the interior angles for $3 \le n \le 12$.

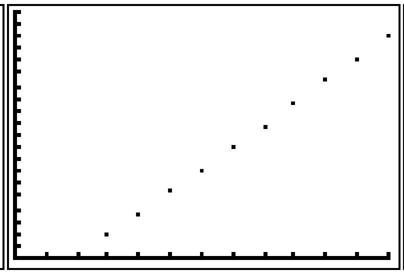
Press MODE and change the calculator from FUNC to SEQ.

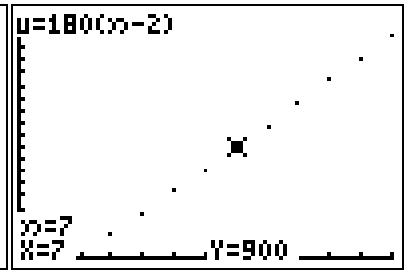






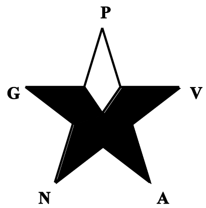






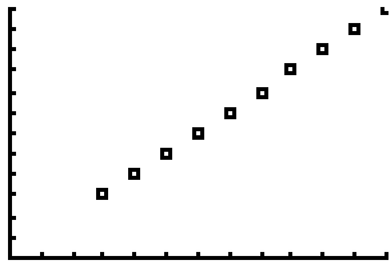
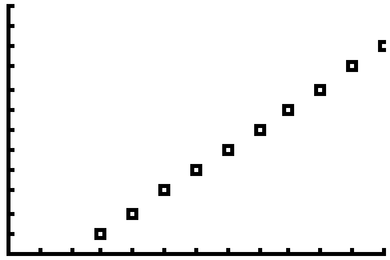
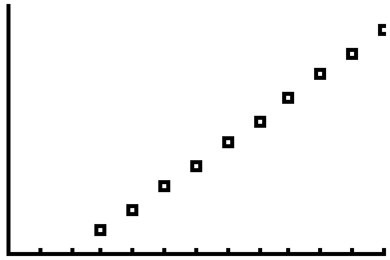
Modality

LTF emphasizes using multiple representations to connect various approaches to a situation in order to increase student understanding. The lesson provides multiple strategies and models for using these representations to introduce, explore, and reinforce mathematical concepts and to enhance conceptual understanding.



P – Physical
V – Verbal
A – Analytical
N – Numerical
G – Graphical

Answers

1. Number of vertices
   a. $y = x$, $x \in$ whole numbers, $x \geq 3$
   b.

   c. $\lim\limits_{x \to \infty} y = \infty$
   d. As the number of sides increases, the number of vertices increases at the same rate. The number of sides and vertices can become arbitrarily large. As the number of sides and vertices increases, the polygon becomes more circular.
2. Number of triangles formed when one vertex is connected to the others
   a. $y = x - 2$, $x \in$ whole numbers, $x \geq 3$
   b.

   c. $\lim\limits_{x \to \infty} y = \infty$
   d. The number of triangles formed is always two less than the number of sides. It can become arbitrarily large.
3. Sum of the angle measures of the triangles formed when one vertex is connected to the others
   a. $y = 180(x - 2)$, $x \in$ whole numbers, $x \geq 3$
   b.

   c. $\lim\limits_{x \to \infty} y = \infty$
   d. The sum of the angle measures is always 180 times the number of triangles because there are 180 degrees in each triangle formed by connecting one vertex to all the others.

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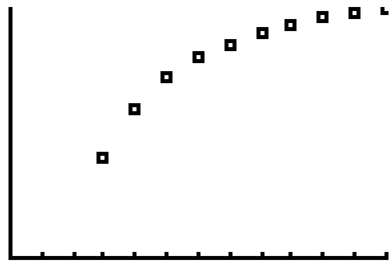
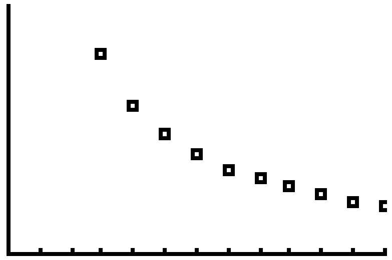
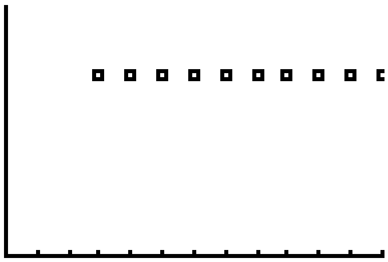
4. If the polygon is regular, the measure of each interior angle
   a. $y = \dfrac{180(x-2)}{x}$, $x \in$ whole numbers, $x \geq 3$
   b.
   
   c. $\lim\limits_{x \to \infty} y = 180$
   d. The measure of each interior angle must be less than 180 degrees because if it were 180 degrees, it would be a straight line and there would be no angle at the vertex. The graph has a horizontal asymptote of $y = 180$ that it will approach but will not reach.

5. If the polygon is regular, the measure of each exterior angle
   a. $y = \dfrac{360}{x}$, $x \in$ whole numbers, $x \geq 3$
   b.
   
   c. $\lim\limits_{x \to \infty} y = 0$
   d. The measure of each exterior angle must be 120 degrees or less because the smallest interior angle possible in a regular polygon is 60 degrees in the equilateral triangle, and the two angles' sum is 180 degrees because they form a straight line. The measure of each exterior angle decreases from a maximum of 120 degrees as the number of sides of the polygon increases. The angle measure approaches 0.

6. Sum of the measures of the exterior angles of a polygon.

   a. $y = 360$, $x \in$ whole numbers, $x \geq 3$

   b.



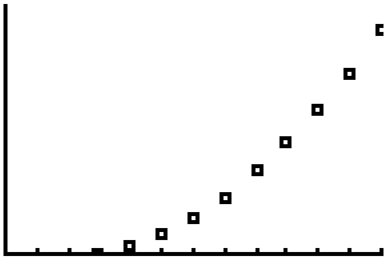
   c. $\lim\limits_{x \to \infty} y = 360$

   d. The sum of the measures of the exterior angles is always 360 degrees.

7. Number of diagonals of a polygon

   a. $y = \dfrac{x(x-3)}{2}$, $x \in$ whole numbers, $x \geq 3$

   b.



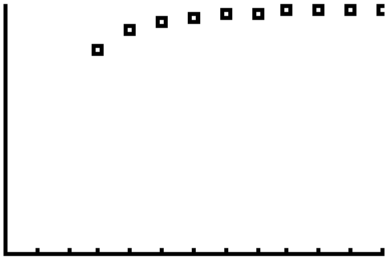
   c. $\lim\limits_{x \to \infty} y = \infty$

   d. A triangle does not have any diagonals. As the number of sides increases, the number of diagonals increases to an infinitely large number.

8. Perimeter of the polygon if the radius is 1 unit

   a. $y = x\sqrt{2 - 2\cos\left(\dfrac{360}{x}\right)}$, $x \in$ whole numbers, $x \geq 3$

   b.



   c. $\lim\limits_{x \to \infty} y = 2\pi$

   d. As the number of sides increases, the polygon becomes more circular, so its perimeter approaches the perimeter of a circle with radius 1.

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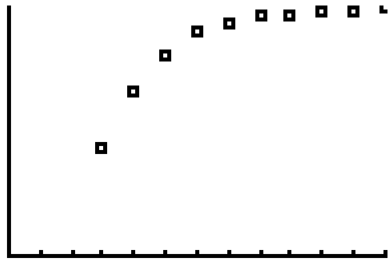
9. Area of the polygon if the radius is 1 unit

   a. $y = \frac{x}{2}\sin\left(\frac{360}{x}\right), x \in$ whole numbers, $x \geq 3$

   b. 

   c. $\lim_{x \to \infty} y = \pi$

   d. As the number of sides increases, the polygon becomes more circular, so its area approaches the area of a circle with radius 1.

TEACHER

Polygons II

Answer the following for questions 1 – 9:

a. Re-write the rule developed for the *n*-gon in the chart in the Polygons I activity as an equation ($y =$) and state the domain.

b. Sketch a graph of the equation. Be sure to graph only points in the domain. Label the axes.

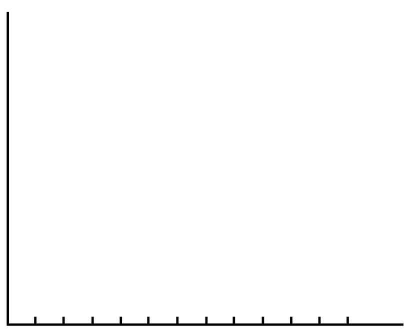
c. Describe the end behavior of the graph. End behavior can be described in terms of limits. For example, $\lim_{x \to \infty} y = \infty$ means as x becomes large without bound, y becomes large without bound.

d. Discuss the relationship of the answers in parts (a), (b), and (c) to what is occurring in the polygon as x increases.

1. Number of vertices

a. $y =$ ________________

b.



c. $\lim_{x \to \infty} y =$ ______________

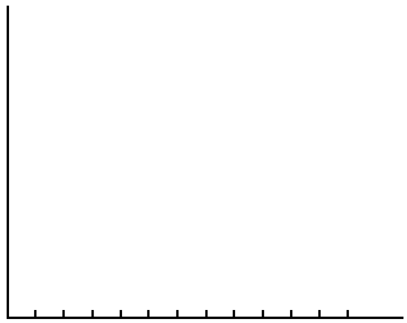
d. ____

____

2. Number of triangles formed when one vertex is connected to each of the other vertices

   a. $y =$ ____________________

   b.



   c. $\lim_{x \to \infty} y =$ ________________

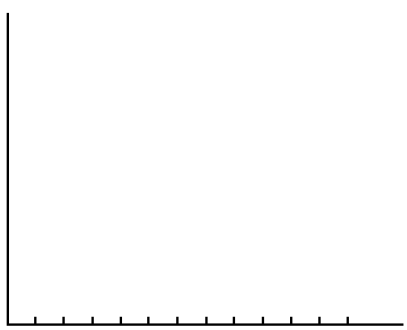
   d. ____

   ____

3. Sum of the angle measures of the triangles formed when one vertex is connected to the other vertices

   a. $y =$ ____________________

   b.



   c. $\lim_{x \to \infty} y =$ ________________

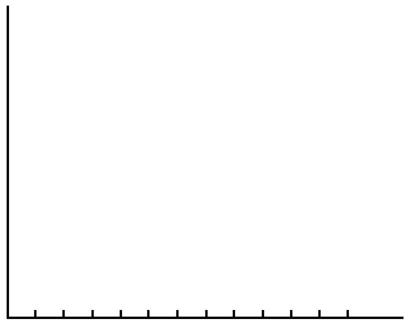
   d. ____

   ____

4. If the polygon is regular, the measure of each interior angle

   a. $y =$ ____________________

   b.



   c. $\lim_{x \to \infty} y =$ ________________

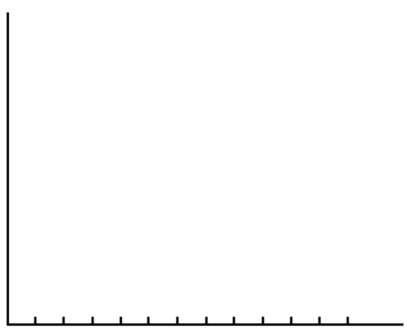
   d. ____

   ____

5. If the polygon is regular, the measure of each exterior angle

   a. $y =$ ____________________

   b.



   c. $\lim_{x \to \infty} y =$ ________________

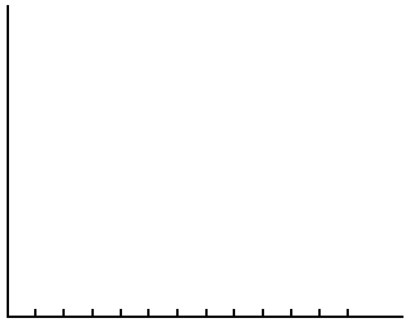
   d. ____

   ____

6. Sum of the measures of the exterior angles of a polygon, one at each vertex

   a. $y =$ ____________________

   b.



   c. $\lim_{x \to \infty} y =$ ________________

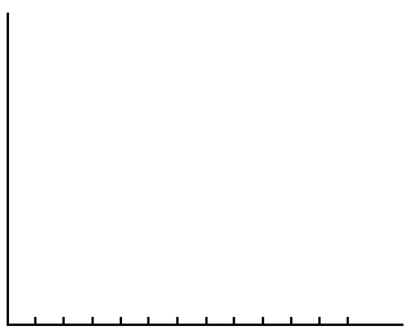
   d. ____

      ____

7. Number of diagonals of a polygon

   a. $y =$ ____________________

   b.



   c. $\lim_{x \to \infty} y =$ ________________

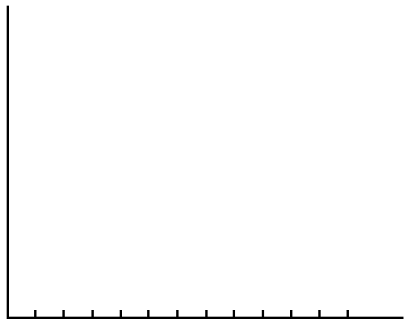
   d. ____

      ____

8. Perimeter of the polygon if the radius is 1 unit

   a. $y =$ ____________________

   b.



   c. $\lim_{x \to \infty} y =$ ________________

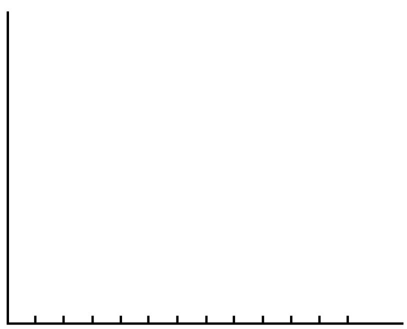
   d. ____

   ____

9. Area of the polygon if the radius is 1 unit

   a. $y =$ ____________________

   b.



   c. $\lim_{x \to \infty} y =$ ________________

   d. ____

   ____

Summary of Regular Polygons

Number of Sides:	n
Number of Vertices:	n
Number of Triangles connected from one vertex;	$n-2$
Sum of Interior Angles:	$180°(n-2)$
Interior Angle Measure:	$\frac{180°(n-2)}{n}$
Measure of Exterior Angle:	$\frac{360°}{n}$
Sum of Measures of Exterior Angles:	$360°$
Number of Diagonals:	$\frac{n(n-3)}{2}$
Central Angle Measure:	$\frac{360°}{n}$
Perimeter if Radius is 1 unit:	$2n\sin\left(\frac{180°}{n}\right)$ or $n\sqrt{2-2\cos\left(\frac{360°}{n}\right)}$
Area if Radius is 1 unit:	$\frac{n}{2}\sin\left(\frac{360°}{n}\right)$ or $n\sin\left(\frac{180°}{n}\right)\cos\left(\frac{180°}{n}\right)$