

We can construct a right triangle with $R-r$ as the hypotenuse, and $R/2$ and $r+h$ as the sides. Using the Pythagorean Theorem,

$$(R-r)^2 = \left(\frac{R}{2}\right)^2 + (r+h)^2$$

When we foil we get,

$$R^2 - 2Rr + r^2 = \frac{R^2}{4} + r^2 + 2rh + h^2$$

When we move some things around and simplify we get,

$$-2Rr - 2rh = -R^2 + \frac{R^2}{4} + h^2$$

By factoring and combining like terms we arrive at,

$$r(-2R-2h) = \frac{-R^2}{4} + h^2 - R^2$$

Then on the right side put everything over 4

$$r(-2R-2h) = \frac{-3R^2+4h^2}{4}$$

To solve for r , I divide by $(-2R-2h)$ on both sides and get,

$$\frac{-3R^2+4h^2}{-8(R+h)} \text{ which equals } \frac{3R^2-4h^2}{8(R+h)}. \text{ Thus } r = \frac{3R^2-4h^2}{8(R+h)}.$$