Warm up: 3D Graphics Tools

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Movement Tools
- Move Tool

Point Tools
- Point Tool
- Point on Object Tool
- Intersect Tool
- Midpoint or Center Tool
- Attach / Detach Point Tool

Line Tools
- Line Tool
- Segment Tool
- Segment with Given Length Tool
- Ray Tool
- Vector Tool
- Vector from Point Tool

Special Line Tools
- Perpendicular Line Tool
- Parallel Line Tool
- Angle Bisector Tool
- Tangents Tool
- Polar or Diameter Line Tool
- Locus Tool

Polygon Tools
- Polygon Tool

Circle, Arc, and Conics Tools
- Circle with Axis through Point Tool
- Circle with Center, Radius and Direction Tool
- Circle through 3 Points Tool
- Circumcircular Arc Tool
- Circumcircular Sector Tool
- Ellipse Tool
- Hyperbola Tool
- Parabola Tool
- Conic through 5 Points Tool

Instersection Tools
- Intersect Two Surfaces Tool

Plane Tools
- Plane through 3 Points Tool
- Plane Tool
- Perpendicular Plane Tool
- Parallel Plane Tool

Geometric Solids Tools
- Pyramid Tool
- Prism Tool
- Extrude to Pyramid or Cone Tool
- Extrude to Prism or Cylinder Tool
- Cone Tool
- Cylinder Tool
- Regular Tetrahedron Tool
- Cube Tool
- Net Tool

Sphere Tools
- Sphere with Center through Point Tool
- Sphere with Center and Radius Tool

Measurement Tools
- Angle Tool
- Distance or Length Tool
- Area Tool
- Volume Tool

Transformation Tools
- Reflect about Plane Tool
- Reflect about Line Tool
- Reflect about Point Tool
- Rotate around Line Tool
- Translate by Vector Tool
- Dilate from Point Tool

Special Objects Tools
- Text Tool

General Tools
- Rotate 3D Graphics View Tool
- Move Graphics View Tool
- Zoom In Tool
- Zoom Out Tool
- Show / Hide Object Tool
- Show / Hide Label Tool
- Copy Visual Style Tool
- Delete Tool
- View in front of Tool

Task A: <u>Construction of a Pyramid</u>

To construct the 3D representation of a pyramid according to a question in Paper 1 of Compulsory Part, HKDSEE 2014.

17. Figure 6(a) shows a solid pyramid $VABCD$ with a rectangular base, where $AB = 18$ cm, $BC = 10$ cm, $VB = VC = 30$ cm and $\angle VAB = \angle VDC = 110°$.

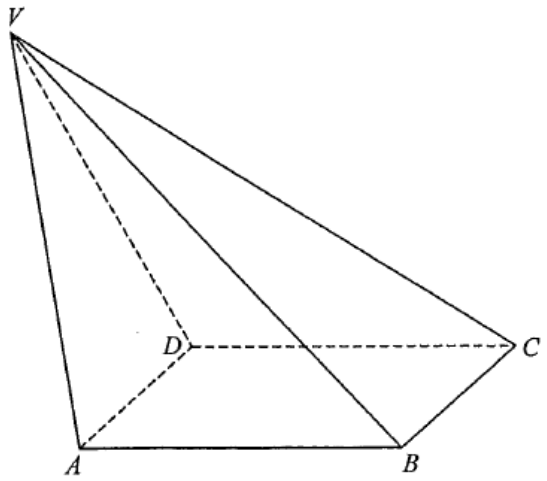


Figure 6(a)

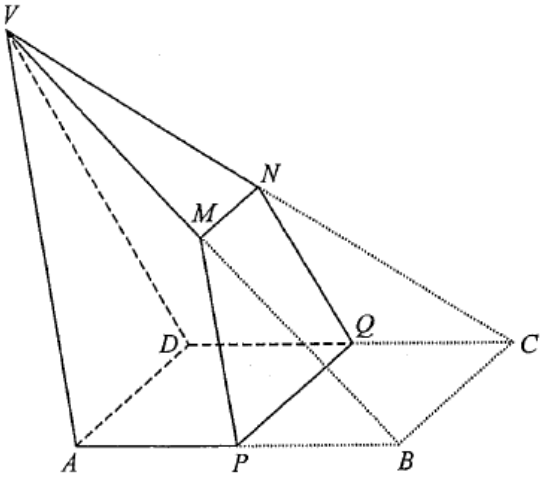


Figure 6(b)

(a) Find $\angle VBA$. (2 marks)

(b) P, Q, M and N are the mid-points of AB, CD, VB and VC respectively. A geometric model is made by cutting off $PBCQNM$ from $VABCD$ as shown in Figure 6(b). A craftsman claims that the area of the trapezium $PQNM$ is less than $70\ cm^2$. Do you agree? Explain your answer. (5 marks)

(Q. 17, Paper 1, Compulsory Part, Mathematics, HKDSEE 2014)

Create objects on the Graphics window as follows:

Steps	Action / Command	Remarks
1.	Show Graphics 3D view.	Hide axes and clipping box.
2.	A=(0,0), B=(18,0), C=(18,10), D=(0,10)	Fix. <u>Or</u> use Execute[{"..","..", …}][1]
	Base=Polygon[A, B, C, D]	
3.	SpB=Sphere[B, 30], SpC=Sphere[C, 30]	Hide
	IntCircle =IntersectConic[SpB, SpC]	Hide
	V=Point[IntCircle]	Fix V(-6.37, 5, 16.77)
4.	aVAB=Angle[V, A, B], aVDC=Angle[V, D, C]	Show Label: Value
	PyramidVABCD= Pyramid[Base, V]	
	aVBA= Angle[V, B, A]	17a) ∠VBA=35.68°
5.	M=Midpoint[V, B], N=Midpoint[V, C], P=Midpoint[A, B], Q=Midpoint[C, D]	Midpoint[edgeBV], …
	TrapeziumPQNM=Polygon[P, Q, N, M]	17b) Area=67.26 < 70
	BasePlane=Plane[Base]	Plane[A, B, C]
	HeightLine=PerpendicularLine[V, BasePlane]	

[1] Execute[{"A=(0,0)", "B=(18,0)", "C=(18,10)","D=(0,10)"}]

Task B: <u>Construction of a Model of Paper-folding</u>

To construct the 3D representation of a model of paper-folding according to a question in Paper 1 of Compulsory Part, HKDSEE 2015.

19. In Figure 3(a), $ABCDB'$ is a pentagonal paper card. It is given that $AB = AB' = 40$ cm, $BC = B'D = 24$ cm and $\angle ABC = \angle AB'D = 80°$.

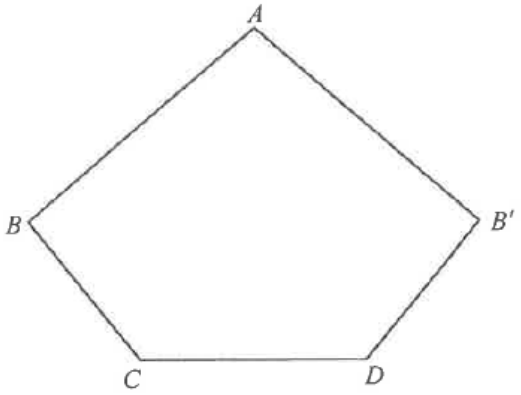


Figure 3(a)

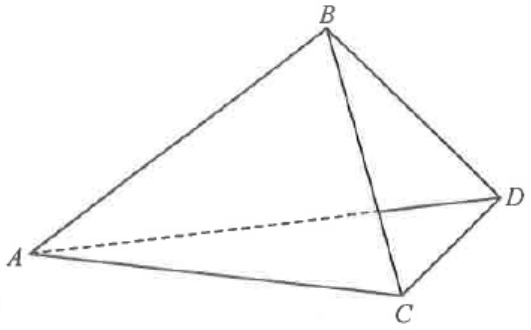


Figure 3(b)

(a) Suppose that $105° \le \angle BCD \le 145°$.

(i) Find the distance between A and C.

(ii) Find $\angle ACB$.

(iii) Describe how the area of the paper card varies when $\angle BCD$ increases from $105°$ to $145°$. Explain your answer.

(7 marks)

(b) Suppose that $\angle BCD = 132°$. The paper card in Figure 3(a) is folded along AC and AD such that AB and AB' join together to form a pyramid $ABCD$ as shown in Figure 3(b). Find the volume of the pyramid $ABCD$. (6 marks)

(Q. 19, Paper 1, Compulsory Part, Mathematics, HKDSEE 2015)

Create objects on the Graphics window as follows:

Steps	**Action / Command**	**Remarks**
1.	Show Graphics 3D view.	Hide axes and clipping box.
2.	A=(0,0), cA=Circle[A, 40]	Free **Point A**. Hide cA.
	B=Point[cA], cB=Circle[B, 24]	**Point B**. Hide cB.
	A'=Rotate[A, -80°, B], rayBA'=Ray[B, A']	Hide. Show Angle[A', B, A]=80°
	C=Intersect[cB, rayBA']	**Point C**
	R=Rotate[B, -132°, C], rayCR=Ray[C, R]	Hide. Show Angle
	cAC= Circle[A, C]	Hide
	D=Intersect[cAC, rayCR, 2], cD=Circle[D, 24]	**Point D**. Hide cD.
	B'=Intersect[cA, cD, 1]	**Point B'**
	Pentagon=Polygon[A, B, C, D, B']	
3.	TriangleABC=Polygon[A, B, C]	
	TriangleAB'D=Polygon[A, D, B']	
	theta=Slider[0°, 180°, 0.1°]	Fixed. Width=540px

Steps	Action / Command	Remarks
	B_r=Rotate[B, -theta, Line[A,C]]	
	rABC=Polygon[A, B_r, C]	
	B'_r=Rotate[B', theta, Line[A,D]]	
	rAB'D=Polygon[A, B'_r, D]	

Task B': <u>Construction of a Model of Paper-folding (Modified)</u>

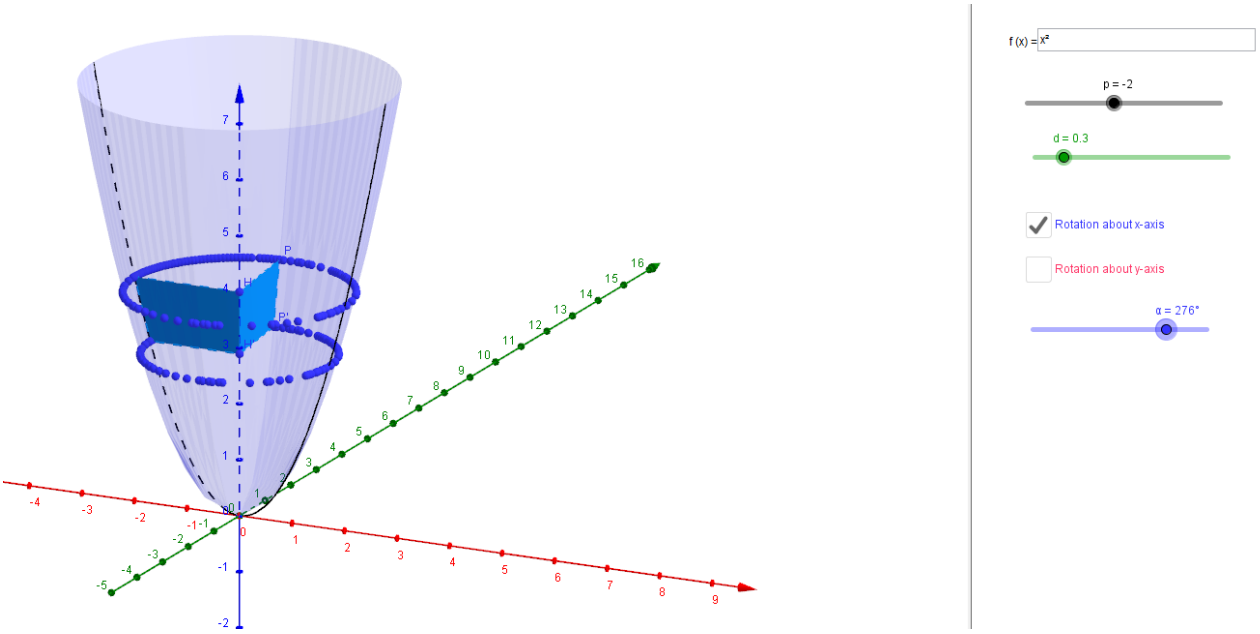
Create objects on the Graphics window as follows:

Steps	Action / Command	Remarks
1.	Show Graphics 3D view.	Hide axes and clipping box.
2.	A=(0,0), cA=Circle[A, 40]	Free **Point A**. Hide cA.
	B=Point[cA], cB=Circle[B, 24]	**Point B**. Hide cB.
	A'=Rotate[A, -80°, B], rayBA'=Ray[B, A']	Hide. Show Angle[A', B, A]=80°
	C=Intersect[cB, rayBA']	**Point C**
	aBCD=Slider[105°, 145°, 1°]	Show Label: Caption: ∠BCD = %v
	R=Rotate[B, -aBCD, C], rayCR=Ray[C, R]	Hide
	cAC= Circle[A, C]	Hide
	D=Intersect[cAC, rayCR, 2], cD=Circle[D, 24]	**Point D**. Hide cD.
	B'=Intersect[cA, cD, 1]	**Point B'**
	AC= Segment[A, C]	19a)i) AC=42.93
	aACB=Angle[A, C, B]	19a)ii) ∠ACB=66.59°
	Pentagon=Polygon[A, B, C, D, B']	19a)iii) Area of Pentagon. (111.59°)
3.	BP=BC sin(aACB), AP=AC-BC cos(aACB)	P is the projection of B onto AC.
	aCAD=Angle[C, A, D]	Hide
	PN=AP*tan(aCAD/2)	N is the projection of B onto ACD.
	BPN=acos(PN/BP)/°, thetaMax=(180-BPN)°	
	theta=Slider[0°, thetaMax, thetaMax/100]	Show Label: Caption: fold
	Set ∠BCD=132°. Hide Pentagon, ∠ACB, AC.	
	TriangleACD=Polygon[A, C, D]	ΔABC & ΔAB'D[2]
	B_r=Rotate[B, -theta, Line[A,C]]	Show Label: Caption: B
	rABC=Polygon[A, B_r, C]	
	B'_r=Rotate[B', theta, Line[A,D]]	Hide
	rAB'D=Polygon[A, B'_r, D]	
	Meet= B_r== B'_r	
	N=Intersect[PerpendicularLine[B_r, Plane[Pentagon]], Plane[Pentagon]]	Point N.
	H_{Pyramid}=Segment[B_r, N]	Style: Dotted line. H=15.86
	If[Meet, Pyramid[B_r,A,C,D]]	19b) Volume of Pyramid = 3686.28

[2] TriangleABC=Polygon[A, C, B], TriangleAB'D=Polygon[A, D, B']

Task C: Solid of revolution

To create a dynamic worksheet to explore how a solid of revolution is generated and hence the method of calculation the volume of the solid.



Create objects on the Graphics window as follows:

Steps	Action / Command	Remarks
1.	Show Graphics 3D view.	Hide *xOy* plane and Clipping Box.
	f(x) = x^2,	Arbitrary function. Hide
	InputBox[f]	Caption "f (x) = ". Hide the graph.
	c(t)=curve[t,0,f(t),t,-20,20]	Parametric function
2.	p=Slider[-20, 20, 0.5], d=Slider[0, 2, 0.1]	
	P=c(p), P'=c(p+d)	
	H=P-p*(1,0,0), H'=P'-(p+d)*(1,0,0), V=P-f(p)*(0,0,1), V'=P'-f(p+d)*(0,0,1)	
	PHH'P'=Polygon[P, H, H', P'], PVV'P'=Polygon[P, V, V', P']	Label with two colours and the value of opacity being 100.
	CheckX=Checkbox["Rotation about x-axis", {PVV'P', V, V'}], CheckY=Checkbox["Rotation about y-axis", {PHH'P', H, H'}]	Advanced: Condition to Show Object: Checkbox
3.	theta=Slider[0°, 360°, 1°]	
	rP=Rotate[P, theta, zAxis], rP'=Rotate[P', theta, zAxis], rPVV'P'=Rotate[PVV'P', theta, zAxis]	Trace rP & rP'.
	Surface[t*cos(θ), t*sin(θ), f(t), t, -10, 10, θ, 0, theta]	Surface of revolution. Value of opacity being 25.