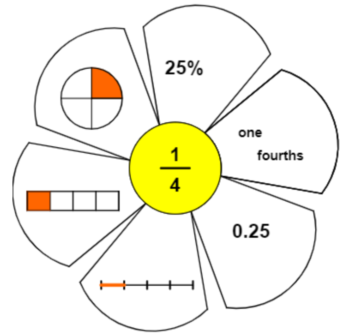
Fraction Operations (+, −, ×, ÷)

"All **ratios** are fractions, but not all **fractions** are ratios."

Fractions have been part of your vocabulary from before you went to school, what is a fraction? Why do I need them? Why is fraction arithmetic hard? Here are the answers! A fraction is a part of a whole. "**I ate half of a pizza**," means that the speaker ate one part of a whole pizza cut into two equal parts (or, the speaker ate three parts of a pizza cut into six equal parts, …). The fractions can be represented using words, pictures, or numerical formats such as: decimals, division, fractions, and percentages. Ratios are similar to fractions, but they have differences. Although ratios follow many of the rules of fractions, they do NOT follow all the rules! This online book Number Town is worth a visit. http://dmcpress.org/cm/number_town/page1/

$$\textbf{Fraction} = \frac{\textbf{Numerator}}{\textbf{Denominator}} = \frac{\text{Number of } \textbf{Parts} \text{ Used}}{\text{Total Number of } \textbf{Parts} \text{ in Whole}}$$



Words: half, third, fourth, three-fourths, nine-tenths, …

Numerical Formats:

Decimals: 0.5, 0.3333…, 0.25, 0.75, 0.9, … (5 tenths, …)

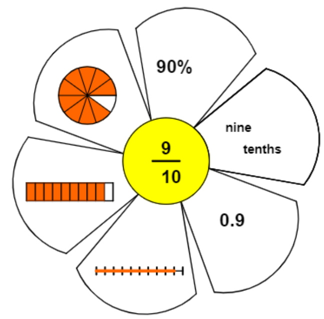
Division: 1 ÷ 2, 1 ÷ 3, 1 ÷ 4, 3 ÷ 4, 9 ÷ 10, … (1 divided by 2, …)

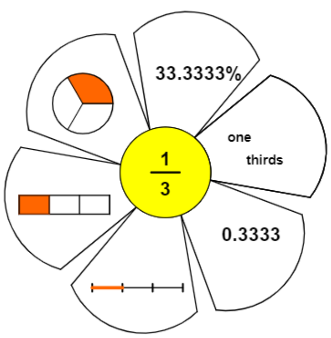
Fractions: $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{3}{4}, \frac{9}{10}, \ldots$ (1 over 2, …), $\frac{5}{\frac{2}{3}} = \frac{15}{2}, \frac{\frac{5}{6}}{7} = \frac{5}{42}, \frac{\frac{2}{3}}{\frac{3}{4}} = \frac{8}{9}$

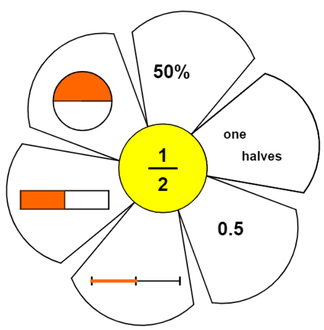
$$\frac{\frac{2}{3}}{\frac{3}{4}} = \frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$$

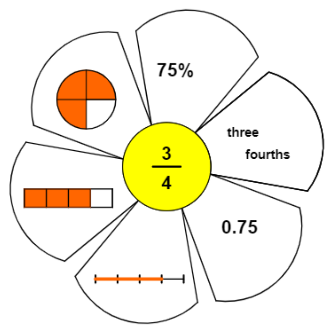
Percentage: 50%, $33.33\frac{1}{3}$%, 25%, 75%, 90%, … (50 percent, …)

Pictorially: https://www.geogebra.org/m/j4UyPdKW#material/djAAFVQC









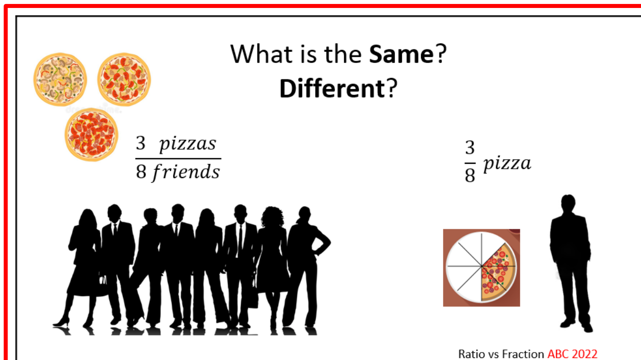
Ratios: 1:2, 1:3, 1:4, 3:4, 9:10, …
(1 is to 2, 1 is to 3, …, 3 is to 4, …) ¥

Remember: "***Fractions*** are your ***friends***."

% sign means per one hundred: $\frac{1}{100} = \%$

¥ All **ratios** are fractions, but not all **fractions** are ratios.

$$\text{ratio} = \frac{\text{collection } \boldsymbol{A}}{\text{collection } \boldsymbol{B}}$$



Knowledge of many skills are needed for operations with fractions:

1. Basic Arithmetic Skills
2. Proper fractions
3. Improper fractions
4. Multiples of a number
5. Mixed numbers
6. Common denominators
7. Equivalent fractions (simplify)
8. Equivalent decimal, percent
9. Factors of a number
10. Prime factorization: ladder/tree
11. Relatively prime
12. Vertical reducing: +, –, ×
13. Crosswise reducing: ×
14.
15. These are the start…

Definitions Important When Working with Fractions

Fraction: a number that represents a part of the whole of something; *Latin: fractus—to break*

Numerator: the number of parts used by the fraction; *Latin 'enumerate'* – to count

Denominator: the total number of parts which a fraction represents; *Latin—'that which names'* or *'indicates'*, the type of fraction that is counted by the numerator.

$$\frac{\textbf{Numerator}}{\textbf{Denominator}} \rightarrow \frac{\text{number of parts used}}{\text{total number of parts}}$$

Common Denominators: fractions with the same denominators or total parts; common denominators are required to add or subtraction fractions

Common Factors: a whole number which is a factor two or more numbers; the common factors shared factors by 12 and 15 are as follows (12:{1,2,3,4,6,12} and 15:{1,3,5,15}; the common factors are 1 and 3; 3 is GCF(12,15).)

Related Definitions

Quotient
Divisor) Dividend
– ______
Remainder

Divisor: the number we divide by (denominator)

Dividend: the number we are dividing into parts

Quotient: the answer to a division problem

Remainder: the difference between the quotient and the product of Divisor and Quotient

Improper Fraction: a fraction where the numerator is greater than or equal to the denominator; numerator $\geq$ denominator (i.e., $\frac{15}{12} = 1\frac{3}{12} = 1\frac{1}{4}$; also, $\frac{15}{12} = \frac{15 \div 3}{12 \div 3} = \frac{5}{4} = 1\frac{1}{4}$).

Mixed Number: a number having an integer number part and a simplified fractional part; many times you may want to change a mixed number to an improper fraction; a whole number plus a reduced (proper) fraction (i.e., $3\frac{4}{15} = 1 + 1 + 1 + \frac{4}{15} = \frac{15}{15} + \frac{15}{15} + \frac{15}{15} + \frac{4}{15} = \frac{49}{15}$.)

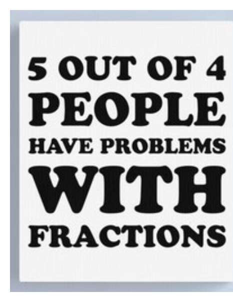
{Many people have noticed that since $3 \times 15 + 4 = 49$, so they write $\frac{49}{15}$. **Or *whole × denominator + numerator***}

Proper Fraction: a fraction where the numerator is less than the denominator; $\frac{3}{4}$. numerator < denominator

The operations of addition/subtraction need equivalent fractions that **require** a common denominator, to complete the operations. Finding and using the LCD, the lowest common denominator reduces the work in performing these operations. Here we are looking at different sets of multiples from the multiplication tables.

1s: 1, 2, 3, 4, 5, 6, 7,…
2s: 2, 4, 6, 8, 10, 12, 14,…
3s: 3, 6, 9, 12, 15, 18, 21,… a 3 and 4 share LCM=12
4s: 4, 8, 12, 16, 20, 24, 28,… a 4 and 5 share LCM=20
5s: 5, 10, 15, 20, 25, 30, 35,… a 5 and 15 share LCM=15
:
15s: 15, 30, 45, 60, 75, 90, 105, 120, …
20s; 20, 40, 60, 80, 100, 120, … a 15 and 20 share LCM=____?

Kapan, p. 246



Examples of the common factors on adjacent lines are highlighted, one can observe there are other common factors within different rows. If you chose a multiple that is NOT the lowest, you **WILL have to REDUCE.**

The steps for determining a common denominators: (**minimal version**, abbreviated p. 5, full p. 7)

1. Are the denominators the same (alike)? Do the work! If not, go to next step.
2. Does one of the denominators divide the other one? If so, find the factor to multiply by.
3. Do the denominators have any common factors? Find the LCM (LCD) of the denominators… (list the multiples of each to find the LCM or use prime factorizations)
4. Are either of the denominators what are known as **prime numbers**? {2,3,5,7,11,13, …} (or what is called **relatively prime**, they shared no common factors)?

If the denominators are alike (Step 1), we add/sub the numerators. If different, we find a common denominator depending on the evaluations in Steps 2-4. *Each of these steps involve different processes you need to know.*

Other Important Definitions need for Fraction Operations

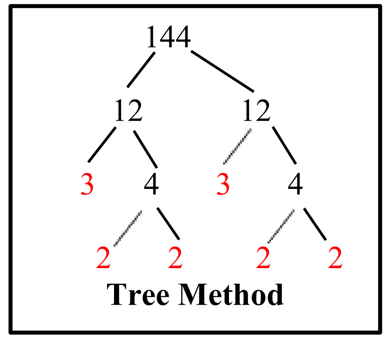
Factoring Methods: used to find the only prime factorization of a number

Factor Tree Method: illustration is only one of multiple methods

Ladder Method: using the prime factors of two or more numbers to find the LCM and GCF shared by the numbers. Note: if none of the prime numbers are shared by all sets, the GCF = 1, the common factor or every number.

2)144
2) 72
2) 36
2)18
3) 9
3) 3
1

Prime Factorization
2^4 x 3^2 or 2 x 2 x 2 x 2 x 3 x 3



{See lesson 04 LCM, GCF, Prime Numbers}

Inverse Operations: operations which undo or reverse the action of a previous operation;

addition has two inverse operations: 5 + 6 = 11 ➔ 11 – 6 = 5 or 11 – 5 = 6

Multiplication has two inverses: 5 × 6 = 30 ➔ 30 ÷ 5 = 6 or 30 ÷ 6 = 5

Greatest Common Factor: The largest value shared between two or more numbers; when the **GCF** is **1**, there are no shared factors, i.e., the values are **relatively prime**.

Lowest Common Multiple: the smallest number shared between two or more sets of numbers; a.k.a., **LCM** , you can call it the Least Common Multiple.

Lowest Common Denominator: the smallest LCM of the multiples of the denominators of fractions to be added/subtracted; a.k.a., **LCD**

LCM ≝ LCD

Prime: a number that has ***exactly two factors***, the number and one (1). The most used primes (for GED®) are 2, 3, 5, 7, 11, 13, 17, 19, ….

Ratio: a quantitative relationship between two dissimilar quantities {the parts of a ratio do not use decimal values}

Rate: a special ratio where the singular denominator is 1 and the numerator can be any value, i.e., mpg, mph, … {since the denominator is 1, decimals fractions are common}

Reduce: divide out common factors in numerator and denominator, including making improper fractions into mixed numbers, also known as, simplifying ($\frac{12}{15} = \frac{\cancel{3}\times 4}{\cancel{3}\times 5} = \frac{4}{5}$ or $\frac{12 \div 3}{15 \div 3} = \frac{4}{5}$, 3 is the ***greatest common factor*** of 12 and 15; **GCF(12,15) = 3**.)

Relatively Prime: any numbers which do not share any prime factors. Examples of relatively prime numbers: {14 & 15}, {15 & 16}, {25 & 48}. They only share 1 as a GCF.

Simplified: indicates a fraction's parts have **no** common (shared) factors; if the numerator is larger than or equal to the denominator, a mixed number is the required the result.

The solutions are on the bottom of the page.

Learning about **equivalent fractions** is an important skill to master. To add/subtract unlike fractions, there is a need to find equivalent fractions which have the same denominator. A fraction wall can assist one in finding equivalent fractions.

Find the equivalent fraction sets in this fraction wall.

Wholes: ______________________________

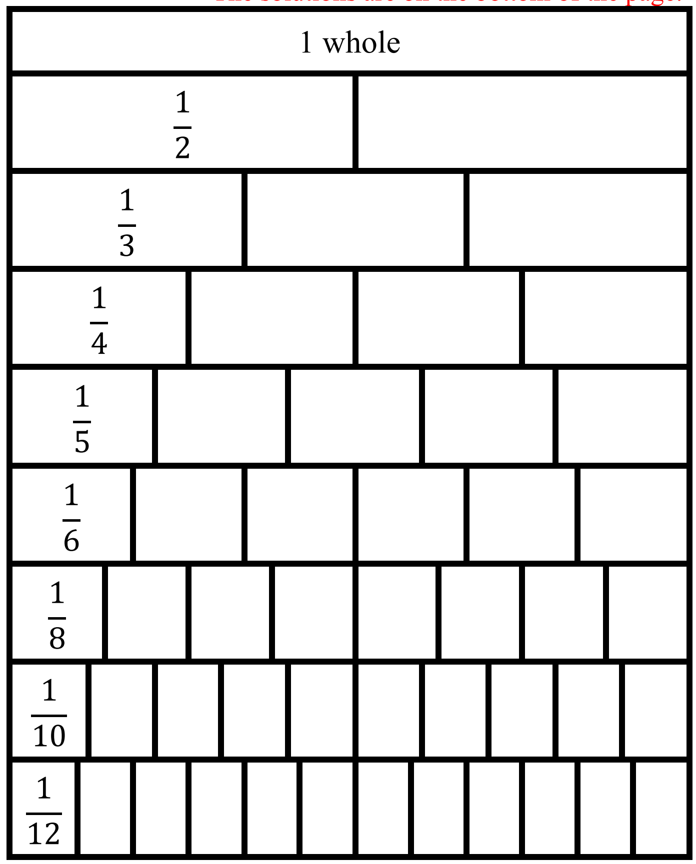
Halves: ______________________________

Thirds: ______________________________

Fourths: ______________________________

Fifths: ______________________________

Sixths: ______________________________



Fraction addition or subtraction problems require **common denominators**. This means each fraction must have the same denominator (steps 2-4). Each fraction needing to be changed must be multiplied by the same factor in both the numerator and denominator until all fractions have common denominators, only then can the numerators be added or subtracted. (see examples in **red** below)

$$\frac{\text{Numerator}}{\text{Denominator}} = \frac{\text{Parts}}{\text{Whole}} = \text{Percentage (\%)}$$

Percentages occur when a fraction/decimal is multiplied by 100 requiring the addition of the symbol (%)

Multiplication and Division operation **do not** require common denominators.

Examples:

Addition: $\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$ $\quad \frac{2}{3} + \frac{3}{4} = \frac{2\times4}{3\times4} + \frac{3\times3}{4\times3} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1\frac{5}{12}$

Subtraction: $\frac{5}{8} - \frac{3}{8} = \frac{2}{8}$ $\quad \frac{5}{6} - \frac{3}{8} = \frac{5\times4}{6\times4} - \frac{3\times3}{8\times3} = \frac{20}{24} - \frac{9}{24} = \frac{11}{24}$

Notice no common denominators are utilized below.

Multiplication: $\frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$ $\quad \frac{15}{16} \times \frac{24}{25} = \frac{3}{2} \times \frac{3}{5} = \frac{9}{10}$

Division: $\frac{2}{9} \div \frac{5}{8} = \frac{2}{9} \times \frac{8}{5} = \frac{16}{45}$

Wholes: $1 = \frac{2}{2} = \frac{3}{3} = \ldots = \frac{12}{12}$ Halves: $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$ Thirds: $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$

Fourths: $\frac{1}{4} = \frac{2}{8} = \frac{3}{12}$ Fifths: $\frac{1}{5} = \frac{2}{10}$ Sixths: $\frac{1}{6} = \frac{2}{12}$

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Steps for finding common denominators: (**abbreviated version, see page 7 for full version**)

1. Are the denominators the same (alike)?
   a. Yes, go on add/sub numerators and reduce to lowest terms.
   b. No, go to step 2.
2. Does one of the denominators divide the other one?
   a. Yes, multiply the smaller value's fraction parts by the quotient.
   b. No, go to step 3.
3. Do the denominators have any common factors? Find the LCM (LCD) of the denominators… (list the multiples of each to find the LCM or use prime factorizations) determine any common factors, or find prime factorization of each one denominator, determine the lowest common multiple (denominator).
   a. **List the multiples** of each denominator until find a common multiple
      LCM(4, 9) {This example is with relatively prime values. See Part b for prime factors of 4 and 9.}

      1 2 3 4 5 6 7 8 9 10 are the multipliers for each fraction.

      Multiples of 4: 4, 8,12,16,20,24,28,32,36,40,…
      Multiples of 9: 9,18,27,36,45,…
   b. If your values are larger values, use prime numbers of those values:
      LCM(4,9) by the **prime factorization**
      4: 2×2
      9: 3×3
      $2 \times 2 \times 3 \times 3 = 36$
   c. Go to Step 4
4. Are either of the denominators what are known as **prime numbers** (2,3,5,7,11,13, …)? (or what is called **relatively prime**, they shared no common factors, 14 {2×7} and 15{3×5})?

   If so, multiply fraction parts (in numerator & denominator) by the unshared factors. {The example in 4.a. illustrates how relatively prime denominators can be worked.}

Note: Fraction addition or subtraction problems require **common denominators.** This means each fraction must have the same denominator. Each fraction needing to be changed must be multiplied by the same factor in both the numerator and denominator until all fractions have common denominators, only then can the numerators be added or subtracted. **(HSE test requirement.)**

Since Addition and Subtraction are *inverse operations*, any methods used for addition can be used for subtraction. ***Common denominators are required for all addition or subtraction operations.*** **{The processes for addition and subtraction operations are more complex than those used for multiplication and division.}**

There will be more on multiplication and division later, but for now know the following: Multiplication and Division are *inverse operations*, too. **NO** common denominators are required. Any rule for multiplication is a rule for division. Division simply changes into multiplication by multiplying dividend by the reciprocal of the divisor.

Dividend ÷ Divisor = Quotient

Dividend $\times \frac{1}{Divisor}$ **= Quotient**

Keep the first fraction
Change the ÷ operation to ×
Reciprocal the second fraction ==> use the reciprocal of divisor

Product of Reciprocals is 1.

$\frac{a}{b} \times \frac{b}{a} = 1$

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Common Denominator Operations
+ or – (No Mixed Number Operations)

When two or more fractions with common denominators are added or subtracted, only the numerators are involved in the operations. These fractions have common denominators: $\frac{1}{12}$, $\frac{2}{12}$, $\frac{3}{12}$, $\frac{4}{12}$, $\frac{5}{12}$, $\frac{6}{12}$, $\frac{7}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$, and $\frac{11}{12}$. Notice all the 12^{ths} in this list if examples; all are proper fractions. (Some fractions could be simplified.)

$$\frac{a}{b}+\frac{c}{b}=\frac{a+b}{b};\ \frac{2}{12}+\frac{5}{12}=\frac{7}{12}$$

or

$$\frac{d}{e}-\frac{f}{e}=\frac{d-f}{e};\ \frac{7}{15}-\frac{3}{15}=\frac{4}{15}$$

$$\frac{\text{Numerator}}{\text{Denominator}}=\frac{\text{Parts}}{\text{Whole}}=\text{Percentage }(\%)$$

When working with fractions, **Addition** and **Subtraction** require common denominators (the whole for all parts; bottom number). Multiplication and Division do not need common denominators.

Recall: The **Prime Numbers** are any number with exactly two factors. These factors include the *value 1* and the *number itself*. GED®/HSE student work will usually use no more than the first eight primes: 2, 3, 5, 7, 11, 13, 17, and 19. And most of the time it will be the first four primes. These four primes are factors of most of the values used on tests.

You are expected to know the **divisibility tests** for the first three primes.

Divisible by 2, the number ends in an **even number**: 0, 2, 4, 6, 8.
Divisible by 5, the number ends in either 0 or 5.
Divisible by 3, you add the digits of the number and if the sum divides by the sum, the number divides by 3.
{2316, 2+3+1+6 = 12 and 12 divides by 3, 745, 7+4+5=16 but 16 does not divide by 3 nor does 746}

Note: The writers of standardized test use prime numbers as factors and knowledge the multiplication tables to the 16s, the squares through 25, and the cubes through 10 while developing standardized mathematics tests like HSE exams. Also, knowledge of the factor sets of all products found in the multiplication tables through the twenties. This knowledge is not something one can memorize, rather they must practice being familiar with primes, factors, and products.

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Methods to find a **Common Denominator**

Keep this Page for Review

Steps for finding common denominators need for **addition** or **subtraction**:

1. Are the denominators the same (like denominators)?
   - A. Yes, go on add/subtract numerators and **reduce** to lowest terms.
   - B. No, go to step 2.

$$\frac{8}{35}-\frac{3}{35}=\frac{5}{35}=\frac{1}{7}$$

2. Does one of the denominators divide the other one?
   - A. Yes, go on multiply the smaller values fraction parts by the quotient.
     - 12 divides by 3 four times, the result **4** is multiplied by both parts of the fraction with the smaller denominator:
     - Subtract or add, then **reduce if possible**.
   - B. No, go to step 3.

$$\frac{7}{12}-\frac{1}{3};12\div 3=4$$

$$\frac{7}{12}-\frac{1\times 4}{3\times 4}=\frac{7}{12}-\frac{4}{12}=\frac{3}{12}=\frac{1}{4}$$

3. Do the denominators have any common factors? LCM (LCD) of the denominators by one of two methods.
   - A. Write the multiples of each denominator until you find the lowest common multiple.

     1 2 3 4 5 6 7 8 9 are the multipliers for each action.

     Multiples of 6: 6, 12, 18, (24), 30, 36, 42, (48), 54, … (24 is the 4th number)

     Multiples of 8: 8, 16, (24), 32, 40, (48), 56, … (24 is the 3rd number)

     While there are many common multiples, the lowest one is **24**. So, using the 24 as a denominator, multiply the 5 by 4 and multiply the 7 by 3 to make the fractions with common denominators.

     Add or Subtract numerators, then **reduce if possible**.

$$\frac{5}{6}+\frac{7}{8}=$$

$$\frac{5\cdot 4}{6\cdot 4}+\frac{7\cdot 3}{8\cdot 3}=$$

$$\frac{20}{24}+\frac{21}{24}=$$

$$\frac{41}{24}=1\frac{17}{24}$$

   - B. Find prime factorization of each one denominator, determine any common factors (Greatest Common Factor, GCF), determine the lowest common multiple (denominator); $\frac{5}{8}+\frac{7}{18}$ **reduce if possible**. {Ladder Method or Factor Tree}

```
2 |8     2 |18
2 |4     3 |9
   2        3
```

8: {1, 2, 4, 8} **prime numbers**

18: {1, 2, 3, 6, 9, 18}

The prime factorization of 8: 2 × 2 × 2

The prime factorization of 18: 2 × 3 × 3

Lowest Common Multiple is: 2 × 2 × 2 × 3 × 3 = 72, the highlighted part is 8, the underscored is 18, and the GCF is 2 as the shared factor which we do not use in new fraction forms.

LCM(LCD) method

Multiply the 5 & 6 by 2 x 2; or 5 × 4 & 6 × 4

Multiply the 7 & 18 by 3; or 7 × 3 & 18 × 3

Write the factors of the denominators

1 2 3 4 5 6 7 8 9 10 multiply by

8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80 …

18: 18, 36, 54, 72, 90, …

https://www.geogebra.org/m/j4UyPdKW#material/xSatv2V9

Select the multiplier in light blue above LCM.

$$\frac{5\cdot 9}{8\cdot 9}+\frac{7\cdot 4}{18\cdot 4}=\frac{45}{72}+\frac{28}{72}=\frac{73}{72}=1\frac{1}{72}$$

   - C. No, go to step 4.

4. If either of the denominators is a **prime** (or is relatively prime, meaning they share no common factors), multiply fraction parts (numerator & denominator) by its unshared factor.
   - A. If any denominator is a **prime number**, multiply each denominator by the numerator and denominator of the other fraction.
   - B. If denominators are not prime number and do not share any factors other than 1, the denominators are **relatively prime** to each other. Hence, multiply each denominator by the numerator and denominator of the other fraction.
     1. The *factor pairs* of 12 are {1,12}, {2,6}, {3,4} or *factor set* = {1,2,3,4,6,12}. The **prime factorization is 2•2•3**.
     2. The factor pairs of 25 are {1,25},{5,5} or factor set = {1,5,25}. The **prime factorization is 5•5**.

     Since the factor sets share no factors other than 1, they are **relatively prime**, hence multiply each denominator by both parts of the other fraction for all of the fractions used. Add or Subtract numerators, then **reduce if possible**.

Prime Factor

$$\frac{3}{5}-\frac{4}{7}=$$

$$\frac{3\times 7}{5\times 7}-\frac{4\times 5}{7\times 5}=$$

$$\frac{21}{35}-\frac{20}{35}=\frac{1}{35}$$

Relatively Prime

$$\frac{5}{12}+\frac{12}{25}=$$

$$\frac{5\times 25}{12\times 25}+\frac{12\times 12}{25\times 12}=$$

$$\frac{125}{300}+\frac{144}{300}=\frac{269}{300}$$

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Adding and Subtracting Fractions Having a Common Denominator

When adding or subtracting fractions with a common denominator, the user just needs to add or subtract the numerators and reduce to lowest terms.

$$\frac{3}{8}+\frac{7}{8}=\frac{10}{8}=1\frac{2}{8}\text{ or }1\frac{1}{4}$$

$$\frac{7}{8}-\frac{3}{8}=\frac{4}{8}\text{ or }\frac{1}{2}$$

Adding and Subtracting Unlike Fractions Needing a Common Denominator

The addition or subtraction of unlike fractions require all fractions to be modified to equivalent fractions with common denominators. Common denominators can be found in several ways, primarily finding a common multiple of all the denominators. Once you have modified the numerators and denominators of all fractions with common denominators, the addition and subtraction problem is the operation with the numerators. Three commonly used methods:

Method 1: Use one of the three methods above to find common denominators.
Method 2: Reduce fractions if possible and use **Method 1** (use occasionally with experience).
Method 3: Use the product of the denominators to find a common denominator, then multiply the numerator by the denominator of the other fraction.

Elementary schools have used this method to assist teachers/students to help elementary students to pass the STAAR (state) exams for simple fraction addition or subtraction of two fractions. The difficult example on the right shows one of the many problems related to this method. It needs to be reduced by 6 in the final step which many student may not do. The method produces higher products than Method 1 or Method 2. It works well when the denominators are either prime or relatively prime. On HSE exams, when there are more than two fractions to add/subtract, "**Boom!**" The workload to solve expands extensively with products with 3-5 digits in the denominator. {Sometimes called the "Butterfly Method." https://teachablemath.com/butterfly-method-fractions-danger-overemphasizing-tricks/}

216 + 375

$$\frac{12}{25}+\frac{15}{18}=\frac{591}{450}$$

450

Example 1 (addition) {One denominator divides the other denominator; Step 2 of fraction operations.}

Method 1: $\frac{4}{18}+\frac{1}{6}=\frac{4}{18}+\frac{1\times3}{6\times3}=\frac{4}{18}+\frac{3}{18}=\frac{7}{18}$

Method 2: $\frac{4}{18}+\frac{1}{6}=\frac{2}{9}+\frac{1}{6}=\frac{2\times2}{9\times2}+\frac{1\times3}{6\times3}=\frac{4}{18}+\frac{3}{18}=\frac{7}{18}$

Since, one denominator divides the other exactly: 18 ÷ 6 = 3, multiply 3 times both parts of divisor's fraction.

NOTE: Since the LCM(18,6) = LCM(9,6) = 18, reducing first was not beneficial in this example; however, there times reducing can help to solve more quickly. Knowing means **PRACTICE**.

Method 3: $\frac{4}{18}+\frac{1}{6}=\frac{4\times6}{18\times6}+\frac{1\times18}{6\times18}$,

$=\frac{24}{108}+\frac{18}{108}$

$=\frac{42}{108}=\frac{42\div6}{108\div6}=\frac{7}{18}$

Butterfly: cross multiply each numerator by the other denominator; multiply denominators; add or subtract numerators… Simple with two small fractions, but with 3 or more… complicated… very hard…

*Some the parts of Method 1 or Method 2 in full **RED** can be done mentally if you have good mental arithmetic skills. It is recommended for beginning students to take all steps by whichever method you prefer to use.*

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Example 2 (addition) {One denominator relatively prime/prime the other denominator; Step 3 of Fraction Operations.}

Method 1: $\frac{2}{9}+\frac{4}{25}=\frac{2\times25}{9\times25}+\frac{4\times9}{25\times9}=\frac{50}{225}+\frac{36}{225}=\frac{86}{225}$

Method 2: same as Method 1
Method 3: same as Method 1

3 | 9 → 3　　5 | 25 → 5
9: 3 × 3
25: 5 × 5
LCD: 3 × 3 × 5 × 5 = 225
GCF = 1, relatively prime

Example 3 (addition) {Denominators share 1 or more common factors, but one does not divide the other}

Method 1: $\frac{9}{14}+\frac{7}{18}=\frac{9\times9}{14\times9}+\frac{7\times7}{18\times7}=\frac{81}{126}+\frac{49}{126}=\frac{130\div2}{126\div2}=\frac{65}{63}=1\frac{2}{63}$

Method 2: same as Method 1

Method 3: $\frac{9}{14}+\frac{7}{18}=\frac{9\times18}{14\times18}+\frac{7\times14}{18\times14}=\frac{162}{252}+\frac{98}{252}=\frac{260\div4}{252\div4}=\frac{65}{63}=1\frac{2}{63}$

2 | 14 → 7　　2 | 18, 3 | 9 → 3
14: 2 × 7
18: 2 × 3 × 3
LCD = 2 × 7 × 3 × 3 = 126
GCF = 2, common factors

When adding and subtracting fractions, one uses the same basic concepts to find common denominators. Then add or subtract, as necessary.

Example 1 (subtraction) **The only difference is the operation, subtraction versus addition. See notes from addition example above.**

Method 1: $\frac{4}{18}-\frac{1}{6}=\frac{4}{18}-\frac{1\times3}{6\times3}=\frac{4}{18}-\frac{3}{18}=\frac{1}{18}$

Method 2: $\frac{4}{18}-\frac{1}{6}=\frac{2}{9}-\frac{1}{6}=\frac{2\times2}{9\times2}-\frac{1\times3}{6\times3}=\frac{4}{18}-\frac{3}{18}=\frac{1}{18}$

Method 3: $\frac{4}{18}-\frac{1}{6}=\frac{4\times6}{18\times6}-\frac{1\times18}{6\times18}$
$=\frac{24}{108}-\frac{18}{108}$
$=\frac{6}{108}=\frac{6\div6}{108\div6}=\frac{1}{18}$

Subtraction version Example 2 (addition)
$\frac{2}{9}-\frac{4}{25}=\frac{2\times25}{9\times25}-\frac{4\times9}{25\times9}=\frac{50}{225}-\frac{36}{225}=\frac{14}{225}$
Same notes apply.
Subtraction version Example 2 (addition)
$\frac{9}{14}-\frac{7}{18}=\frac{9\times9}{14\times9}-\frac{7\times7}{18\times7}=\frac{81}{126}-\frac{49}{126}=\frac{32\div2}{126\div2}=\frac{16}{63}$
Same notes apply.

Example 2 (subtraction) {LCM (LCD) of the denominators by one of two methods, step 4 fraction operations.}

Method 1: $\frac{5}{24}-\frac{7}{30}=\frac{5\times5}{24\times5}-\frac{7\times4}{30\times4}=\frac{25}{120}-\frac{28}{120}=\frac{-3\div3}{120\div3}=-\frac{1}{40}$

24, 48, 72, 96, 120, 144, 168,… The 5th term, so multiply 24 × 5 and 5 × 5.

30, 60, 90, 120, 150, 180,… The 4th term, so multiply 30 × 4 and 7 × 4.

Using addition:
$\frac{5}{24}+\frac{7}{30}=\frac{25}{120}+\frac{28}{120}=\frac{53}{120}$
The only difference is the result of the operations.

Method 2: $\frac{5}{24}-\frac{7}{30}=\frac{5\times5}{24\times5}-\frac{7\times4}{30\times4}=\frac{25}{120}-\frac{28}{120}=\frac{-3\div3}{120\div3}=-\frac{1}{40}$

24: 2 × 2 × 2 × 3　　Since 24 has every prime except 5; so multiply 24 × 5 and 5 × 5.
30: 2 × 3 × 5　　Since 30 has every prime except 2×2=4; so multiply 30 × 4 and 7 × 4.
120: 2 × 2 × 2 × 3 × 5　　GCF = 6

Method 3: $\frac{5}{24}-\frac{7}{30}=\frac{5\times30}{24\times30}-\frac{7\times24}{30\times24}=\frac{150}{720}-\frac{168}{720}=\frac{-18\div18}{720\div18}=-\frac{1}{40}$

How well do you work with large numbers mentally? I.e., knowing that 18 is a common factor of 18 and 720?

As shown here, the only difference between add or subtracting is the operation. This remains even when fractions are chained together in the same problem.

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Chain Operations

Chain Operations are operations which contain more than one arithmetic operation within a single problem. Similarly, this is an integer sample with 4 operations:

$$2 + 13 - 4 - 8 + 7 = 10$$

4: 2×2
6: 2×3
8: $2 \times 2 \times 2$
12: $2 \times 2 \times 3$
24: $2 \times 2 \times 2 \times 3$ LCD
GCF(4,6,8,12) = 2
8, 16, 24, 32, …
4, 8, 12, 16, 20, 24, 28, 32, …
6, 12, 18, 24, 30, …
12, 24, 36, …

Example:

Method 1: $\frac{7}{8} - \frac{3}{4} + \frac{5}{6} + \frac{8}{12} = \frac{7\times3}{8\times3} - \frac{3\times6}{4\times6} + \frac{5\times4}{6\times4} + \frac{8\times2}{12\times2} =$

$$\frac{21-18+20+16}{24} = \frac{39\div3}{24\div3} = \frac{13}{8} = 1\frac{5}{8}$$

Method 2: $\frac{7}{8} - \frac{3}{4} + \frac{5}{6} + \frac{8}{12} = \frac{7}{8} - \frac{6}{8} + \frac{5}{6} + \frac{4}{6} = \frac{1}{8} + \frac{9}{6} = \frac{1}{8} + \frac{3}{2} = \frac{1}{8} + \frac{12}{8} = \frac{13}{8} = 1\frac{5}{8}$

It is okay to use convenient common denominators combinations in chain addition and subtraction of fractions, using reduction to help find simple common denominators. (If you can see it, you can do it.)

Method 3: $\frac{7}{8} - \frac{3}{4} + \frac{5}{6} + \frac{8}{12} = \frac{7\times4\times6\times12}{8\times4\times6\times12} - \frac{3\times8\times6\times12}{4\times8\times6\times12} + \frac{5\times8\times4\times12}{6\times8\times4\times12} + \frac{8\times8\times4\times6}{12\times8\times4\times6} =$

$$\frac{2016}{2304} - \frac{1728}{2304} + \frac{1920}{2304} + \frac{1536}{2304} = \frac{3744\div288}{2304\div288} = \frac{13}{8} = 1\frac{5}{8}$$

Multiplication and Division of Fractions

The steps for **multiplication** or **division** of **fractions** are:

1. Are the individual terms all fractions? If not, change all terms into their fraction form.
2. Are any terms preceded by a division (÷) indicator? $\mathbf{a \div b = a \times \frac{1}{b}}$
   If yes, change ÷ to × and use the **reciprocal** of dividing value.
3. Can you factor out values vertically or crosswise? If yes, repeat as often as necessary to reduce fully.
   $\frac{12}{15} = \frac{4}{5}$ **vertical** $\frac{12}{7} \times \frac{13}{9} = \frac{4}{7} \times \frac{13}{3}$ **crosswise**
4. Multiply the numerators and denominators.
5. Change improper fractions to mixed numbers, reducing if needed.

Multiplication and Division do **not** require common denominators, so an LCM(LCD) is **not** needed. However, the ladder method/factor tree can be used to find the Prime Factors or a factor listing of each value can be used to find the LCM/GCF of the numerators and denominators helping you solve these problem quickly with efficiency.

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Vertical reduction and Cross reduction

Method 1: $\frac{4}{18}\times\frac{1}{6}=\frac{\cancel{2}^{1}\times\cancel{2}^{1}}{\cancel{2}_{1}\times3\times3}\times\frac{1}{\cancel{2}_{1}\times3}=\frac{1\times1\times1}{1\times3\times3\times1\times3}=\frac{1}{27}$

{use prime factorization of numbers, divide common factors out}**

Method 2: $\frac{\cancel{4}^{2}}{\cancel{18}_{9}}\times\frac{1}{6}=\frac{\cancel{2}^{1}}{9}\times\frac{1}{\cancel{6}_{3}}=\frac{1}{27}$ **{Divide out common factors: reduce or cross reduce. ONLY in multiplication}**

Method 3: $\frac{4}{18}\times\frac{1}{6}=\frac{4}{108}=\frac{4\div4}{108\div4}=\frac{1}{27}$ (multiply and reduce, if you do not reduce first, you still must reduce vertically after multiplying.)

{Methods 1 and 2 are using what is sometimes called **Cross/Vertical Reduction of Factions**; it is just factoring out the common factors before multiplying problems out.}

The Prime Factorization of numerators and denominators facilitate easily dividing out **like terms to reduce answers more quickly.

Recall: By the definition of mixed numbers: $3\frac{2}{3}=3+\frac{2}{3}$ or $1+1+1+\frac{2}{3}$ and $1=\frac{1}{1}=\frac{2}{2}=\frac{3}{3}=\frac{4}{4}=\cdots$

Using substitution, $3+\frac{2}{3}=1+1+1+\frac{2}{3}$

$=\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{2}{3}$

$=\frac{11}{3}$

This process has been simplified to multiply and add→ **whole number** × **denominator** + **numerator** to produce the numerator of the improper fraction that is equivalent to mixed number. This is the inverse operation to simplify an improper fraction.

Simple mixed number times fraction:

Method 1: $1\frac{2}{3}\div\frac{5}{8}=\frac{5}{3}\div\frac{5}{8}=\frac{5}{3}\times\frac{8}{5}=\frac{8}{3}=2\frac{2}{3}$

Method 2:

Method 3:

Method 1: $- + - = - + - = -$

Method 2: $- + - = - + - = -$

Method 3: $- + - = - + - = -$

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Division is just multiplying the dividend by the **multiplicative inverse (<u>reciprocal</u>)** of the divisor yielding the quotient.

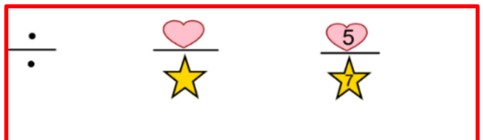
$$\text{Divisor}\overline{)\text{Dividend}}^{\ \text{Quotient}}$$

or

$$\text{Dividend} \div \text{Divisor} = \text{Quotient}$$

$$\text{Dividend} \times \frac{\mathbf{1}}{\mathbf{Divisor}} = \text{Quotient}$$

$$\text{Reciprocal} = \frac{\mathbf{1}}{\mathbf{Divisor}}$$



$$\frac{4}{2} = 4 \div 2 = 4 \cdot \frac{1}{2}$$

$$= \frac{4}{1} \div \frac{2}{1} = \frac{4}{1} \cdot \frac{1}{2}$$

Some think of this as the way the ÷ symbol replaced the ⟌.

Keep the first fraction
Change the ÷ operation to ×
Reciprocal of the second fraction ==> use the <u>reciprocal</u> of divisor

Method 1*: $\frac{4}{18} \div \frac{1}{6} = \frac{4}{18} \times \frac{6}{1} = \frac{4}{3} = 1\frac{1}{3}$ or $\frac{4}{18} \div \frac{1}{6} = \frac{4}{18} \div \frac{1}{6} = \frac{2}{9} \times \frac{6}{1} = \frac{4}{3} = 1\frac{1}{3}$

Method 2: $\frac{4}{18} \div \frac{1}{6} = \frac{2}{9} \times \frac{6}{1} = \frac{4}{3} = 1\frac{1}{3}$ {Reduce fractions mentally.}

Method 3: $\frac{4}{18} \div \frac{1}{6} = \frac{4}{18} \times \frac{6}{1} = \frac{24}{18} = \frac{4\times6}{3\times6} = \frac{4}{3} = 1\frac{1}{3}$

Method 4♦: $\frac{4}{18} \div \frac{1}{6} = \frac{4}{18} \div \frac{1\times3}{6\times3} = \frac{4}{18} \div \frac{3}{18} = \frac{4\div3}{1} = \frac{4}{3} = 1\frac{1}{3}$ (common denominator method in division)

<u>Special Situations</u>:

*Or: $\frac{4}{18} \div \frac{1}{6} = \frac{4}{3} = 1\frac{1}{3}$, however, this situation is **rare**; it only happens when both the numerators divide, and the denominators divide exactly.

♦This is another uncommon situation: $\frac{4}{18} \div \frac{5}{18} = \frac{4\div5}{18\div18} = \frac{4\div5}{1} = \frac{4}{5}$, this can be done when denominators are the same.

https://www.geogebra.org/m/cp47pguu
https://twitter.com/i/status/1478897811466817538

Knowing the factor sets of numbers helps you to work with fraction arithmetic. {02 Math Resources}

15 – {1, 3, 5, 15}
225 – {1, 3, 5, 15, 25, 75}

The product of reciprocals is 1. $\frac{8}{5} \times \frac{5}{8} = 1$

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Method 1:

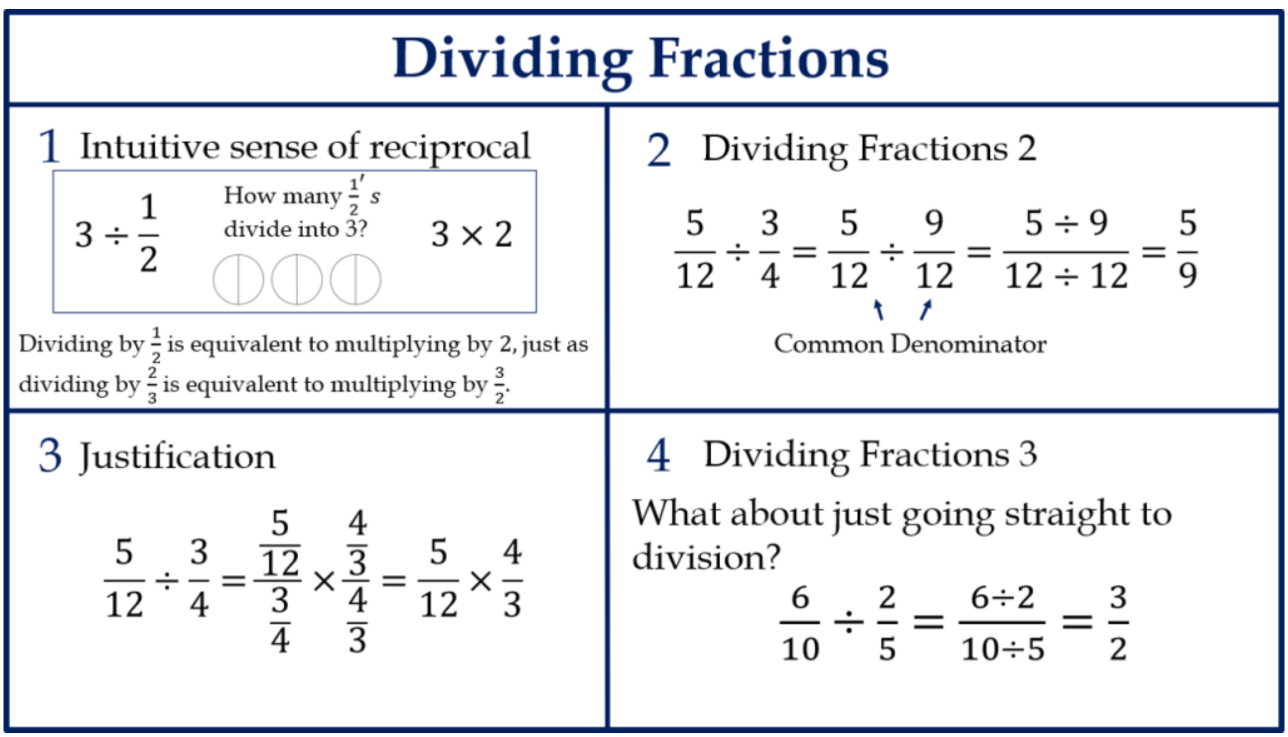
Method 2:

Method 3:

Method 1:

Method 2:

Method 3:



https://danpearcy.com/2021/11/05/prompt-36-dividing-fractions/

1) https://www.geogebra.org/m/txmnpkme
2) https://www.geogebra.org/m/dbsq7x9g

Keep the first fraction
Change the ÷ operation to ×

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Reciprocal of the second fraction ==> use the reciprocal of divisor

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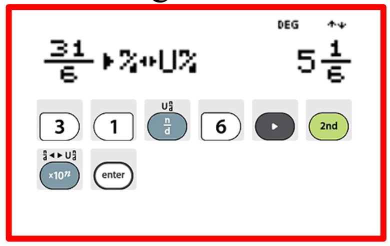
Mixed Number Operations

Addition

A mixed number occurs when *an integer is added to a fraction* or when the *numerator is greater than the denominator* which when simplified becomes the sum of an integer and a fraction.



Example of an <u>improper fraction</u> which simplifies to a <u>mixed number</u>.

$$\frac{31}{6} = \frac{30}{6} + \frac{1}{6} = 5 + \frac{1}{6} = 5\frac{1}{6}$$

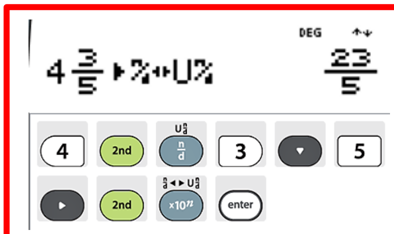
While the above example is used in finalizing work, for many problems we need to reverse the process. Most of these steps are seldom repeated in daily work, but students need to recall this to understand the processes.

$$4\frac{3}{5} = 4 + \frac{3}{5} = 1 + 1 + 1 + 1 + \frac{3}{5}$$

We know we need to have a common denominator:

$$= \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{3}{5}$$

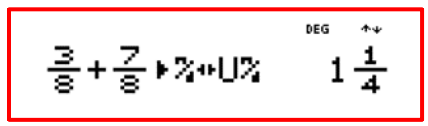
$$= \frac{4 \times 5}{5} + \frac{3}{5} = \frac{(4 \times 5) + 3}{5} = \frac{23}{5}$$



Calculator Instructions

Adding fractions whose sum is greater than one give us improper fractions/mixed numbers.

$$\frac{3}{8} + \frac{7}{8} = \frac{10}{8} = \frac{5}{4} = 1\frac{1}{4}$$



Adding a fraction to a mixed number.

$$2\frac{3}{4} + \frac{5}{8} =$$

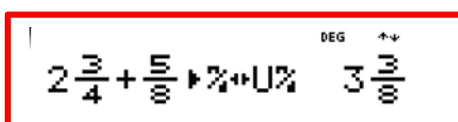
Find the common denominator between 4 and 8; since 4 divides 8 exactly, the LCM is 8.

$$2\frac{3 \times 2}{4 \times 2} + \frac{5}{8} = 2\frac{6}{8} + \frac{5}{8} = 2\frac{11}{8}$$

However, this is not simplified since the fraction part is an improper fraction. A better way to write this line is:

$$2 + \frac{3 \times 2}{4 \times 2} + \frac{5}{8} = 2 + \frac{6}{8} + \frac{5}{8} = 2 + \frac{11}{8} = 2 + 1 + \frac{3}{8}$$

$$= 2 + 1 + \frac{3}{8} = 3\frac{3}{8}$$



Even if you know how to do this using technology, you may have to be able to show evidence of an in-between step on the exam. So you need to know how to do problems without technology. The <u>**purpose of technology**</u> is not to replace student knowledge and expected abilities; it is to facilitate speed is completing repetitive problem solving.

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There are many correct ways to add two or more numbers containing fractions; the following is an example of one way using the associative property to separate the integer and fraction parts.

Method 1:

$$12\frac{3}{5} + 13\frac{5}{8} + 8\frac{7}{10} =$$

$$12 + \frac{3}{5} + 13 + \frac{5}{8} + 8 + \frac{7}{10} =$$

$$12 + 13 + 8 + \frac{3}{5} + \frac{5}{8} + \frac{7}{10} =$$

The LCD of 5, 8, 10 is 40.

$$33 + \frac{3 \times 8}{5 \times 8} + \frac{5 \times 5}{8 \times 5} + \frac{7 \times 4}{10 \times 4} =$$

$$33 + \frac{24}{40} + \frac{25}{40} + \frac{28}{40} = 33 + \frac{77}{40}$$

Since 77 and 40 have no common factors, we rewrite the fraction as a mixed number and add the integer to 33.

$$= 33 + \frac{77}{40} = 33 + 1\frac{37}{40} = 34\frac{37}{40}$$

While many do not show their work in this manner, all of the steps taken are illustrated in most people's solutions.

Method 2:

One of the most common methods used to do this addition problem is as follows:

$$12\frac{3}{5} + 13\frac{5}{8} + 8\frac{7}{10} =$$

The LCD of 5, 8, 10 is 40.

$$12\frac{3 \times 8}{5 \times 8} + 13\frac{5 \times 5}{8 \times 5} + 8\frac{7 \times 4}{10 \times 4} =$$

$$12\frac{24}{40} + 13\frac{25}{40} + 8\frac{28}{40} =$$

$$33\frac{77}{40} = 33 + 1\frac{37}{40} = 34\frac{37}{40}$$

Method 3: Another common method of adding mixed numbers:

$$12\frac{3}{5} + 13\frac{5}{8} + 8\frac{7}{10} =$$

$$\frac{63}{5} + \frac{109}{8} + \frac{87}{10} =$$

The LCD of 5, 8, 10 is 40.

$$\frac{63 \times 8}{5 \times 8} + \frac{109 \times 5}{8 \times 5} + \frac{87 \times 4}{10 \times 4} =$$

$$\frac{504}{40} + \frac{545}{40} + \frac{348}{40} = \frac{1397}{40} \text{ or } 34\frac{37}{40}$$

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Subtraction

The second method shown for addition is really the best technique to use for subtraction problems as you may **need to borrow** from the whole number value.

$$5\frac{1}{4} - 2\frac{3}{4} =$$

There is already a common denominator; however, the numerator of the first fraction needs to get help from the whole number. We are going to borrow 1 from the 5, making it a 4.

$$\left(5 + \frac{1}{4}\right) - 2\frac{3}{4} =$$

$$\left(\overset{4+1}{4 + \frac{4}{4}} + \frac{1}{4}\right) - 2\frac{3}{4} =$$

$$\left(4 + \frac{5}{4}\right) - 2\frac{3}{4} =$$

$$4\frac{5}{4} - 2\frac{3}{4} = 2\frac{2}{4} = 2\frac{1}{2}$$

Break up 5 into 4 + 1, change the 1 to the fraction $\frac{4}{4}$, which is a 1. This allows us to **borrow** from the 5 so we can subtract the $\frac{3}{4}$.

Now, let's look at the problem of needing to find a LCD.

$$14\frac{17}{24} - 8\frac{5}{18} =$$

Neither fraction will reduce, so finding the LCD of 24 and 18 can be found by multiplying the two values $24 \times 18 = 432$. This is a large number, let's try this…

$$24 = 4 \times 6 = 2 \times 2 \times 2 \times 3$$
$$18 = 3 \times 6 = 3 \times 2 \times 3$$

This method shows multiplying by 3 and 4 to find the common denominator.

Both number share a 6, so the LCD(24, 18) = $2 \times 2 \times 3 \times 6 = 72$.

Another way to find the LCD is:

18, 36, 54, 72, 90, 108, …
24, 48, 72, 110, 158, …

So, working the problem

$$14\frac{17 \times 3}{24 \times 3} - 8\frac{5 \times 4}{18 \times 4} =$$

$$14\frac{51}{72} - 8\frac{20}{72} = 6\frac{31}{72}$$

If you are subtracting negative mixed numbers, remember **a – b = a + (-b)**.

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Learning how to Multiply Mixed Numbers

Solving mixed number multiplication problems:

$$4\frac{1}{2} \times 2\frac{1}{3} = \frac{(4 \times 2) + 1}{2} \times \frac{(2 \times 3) + 1}{3}$$

$$\frac{8+1}{2} \times \frac{6+1}{3} = \frac{9}{2} \times \frac{7}{3}$$

$$\frac{\overset{3}{\cancel{9}}}{2} \times \frac{7}{\underset{1}{\cancel{3}}} = \frac{3}{2} \times \frac{7}{1} = \frac{21}{2} = 10\frac{1}{2}$$

Reasoning for the steps in the work.

$$4\tfrac{1}{2} = 4 + \tfrac{1}{2} =$$

$$1 + 1 + 1 + 1 + \frac{1}{2} =$$

$$\frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{1}{2} =$$

$$\frac{4 \times 2}{2} + \frac{1}{2} = \frac{(4 \times 2) + 1}{2} = \frac{8+1}{2} = \frac{9}{2}$$

The above shows why we do the steps which we short-cut in lessons, the **red steps** are usually not needed by those who build good mental arithmetic skills.

Learning how to Divide Mixed Numbers

Dividing mixed numbers have many steps which are the same as multiplying them.

$$4\frac{1}{2} \div 2\frac{1}{3} = \frac{(4 \times 2) + 1}{2} \div \frac{(2 \times 3) + 1}{3}$$

$$\frac{8+1}{2} \div \frac{6+1}{3} = \frac{9}{2} \div \frac{7}{3}$$

$$\frac{9}{2} \div \frac{7}{3} = \frac{9}{2} \times \frac{3}{7} = \frac{27}{14} = 1\frac{13}{14}$$

If you carefully observe these two problems, you will notice the first two lines are almost identical. The difference in the last line where the division is changed into multiplication of the reciprocal.

Here is a totally new division problem:

$$14\frac{3}{8} \div 6\frac{2}{3} = \frac{(14 \times 8) + 2}{8} \div \frac{(6 \times 3) + 2}{3}$$

The numerator and denominator of the first fraction reduce by dividing out 2.

$$\frac{114}{8} \div \frac{20}{3} = \frac{57}{4} \div \frac{20}{3}$$

$$\frac{57}{4} \div \frac{20}{3} = \frac{57}{4} \times \frac{3}{20} = \frac{171}{80} = 2\frac{11}{80}$$

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