II. For each of the following functions find:

1) $f(x) = x^3 - 6x^2 + 9x + 1$

a) Domain $(-\infty, \infty)$

b) Derivative of $f(x) = 3x^2 - 12x + 9$

c) Critical Values $x = 3, x = 1$

d) Maximum and minimum coordinates Max $(1, 5)$ Min $(3, 1)$

e) Intervals where the function is increasing $(-\infty, 1) \cup (3, \infty)$

f) Intervals where the function is decreasing $(1, 3)$

$0 = 3(x^2 - 4x + 3)$

$0 = (x-3)(x-1)$

$f'(x)$	$x = 3$	$x = 1$	
	$(-\infty, 1)$	$(1, 3)$	$(3, \infty)$
	0	2	4
	+	−	+
	Increase	Decrease	Increase

Max $(1, 5)$ Min $(3, 1)$

2) $y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x + 8$

a) Domain $(-\infty, \infty)$

b) Derivative of $f(x)$ $y' = x^2 + x - 6$

c) Critical Values $x = 2 \quad x = -3$

d) Maximum and minimum coordinates Max $(-3, 21)$ Min $(2, \frac{2}{3})$

e) Intervals where the function is increasing $(-\infty, -3) \cup (2, \infty)$

f) Intervals where the function is decreasing $(-3, 2)$

$0 = x^2 + x - 6$

$x \quad -2$

$x \quad +3$

	$(-\infty, -3)$	$(-3, 2)$	$(2, \infty)$
	−4	0	3
	+	−	+

Max $(-3, \frac{43}{2})$ Min $(2, \frac{2}{3})$

3) $f(x) = x^3 + x^2 - 5x - 5$

a) Domain $(-\infty, \infty)$

b) Derivative of $f(x)$ $f'(x) = 3x^2 + 2x - 5$

c) Critical Values $x = 1 \quad x = -\frac{5}{3}$

d) Maximum and minimum coordinates Max $(-\frac{5}{3}, \frac{40}{27})$ Min $(1, -8)$

e) Intervals where the function is increasing $(-\infty, -\frac{5}{3}) \cup (1, \infty)$

f) Intervals where the function is decreasing $(-\frac{5}{3}, 1)$

$0 = 3x^2 + 2x - 5$

$3 \quad 5 \quad -3$

$1 \quad -1 \quad +5$

$0 = (x-1)(3x+5)$

$x = 1 \quad x = -\frac{5}{3}$

	$(-\infty, -\frac{5}{3})$	$(-\frac{5}{3}, 1)$	$(1, \infty)$
	−2	0	2
	+	−	+

Max $(-\frac{5}{3}, \frac{40}{27})$ Min $(1, -8)$

4) $f(x) = x^4 - 8x^2 + 1$

a) Domain $(-\infty, \infty)$

b) Derivative of $f(x)$ $f'(x) = 4x^3 - 16x$

c) Critical Values $x = 0, x = 2, x = -2$

d) Maximum and minimum coordinates Max $(0, 1)$ Min $(-2, -15) (2, -15)$

e) Intervals where the function is increasing $(-2, 0) \cup (2, \infty)$

f) Intervals where the function is decreasing $(-\infty, -2) \cup (0, 2)$

$0 = 4x^3 - 16x$

$0 = 4x(x^2 - 4)$

$x = 0 \quad x = 2 \quad x = -2$

	$(-\infty, -2)$	$(-2, 0)$	$(0, 2)$	$(2, \infty)$
	−3	−1	1	3
	−	+	−	+

Min Max Min $(-2, -15) (0, 1)$

5) $g(x) = \frac{x^4}{4} - \frac{x^3}{3} - 3x^2 + 1$

a) Domain $(-\infty, \infty)$

b) Derivative of $f(x)$ $f'(x) = x^3 - x^2 - 6x$

c) Critical Values $x = 0 \quad x = -2 \quad x = 3$

d) Maximum and minimum coordinates Max $(0, 1)$ Min $(-2, \frac{35}{3}) (3, \frac{-59}{4})$

e) Intervals where the function is increasing $(-2, 0) \cup (3, \infty)$

f) Intervals where the function is decreasing $(-\infty, -2) \cup (0, 3)$

$0 = x^3 - x^2 - 6x$

$0 = x(x^2 - x - 6)$

$x \quad +2 \quad -3$

$x = 0 \quad x = -2 \quad x = 3$

	$(-\infty, -2)$	$(-2, 0)$	$(0, 3)$	$(3, \infty)$
	−3	−1	1	4
	−	+	−	+

Min $(3, \frac{-59}{4})$ Min $(-2, \frac{35}{3})$ Max $(0, 1)$