Logifaces: a Game with many Faces

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Abstract

Manipulatives have been used for increasing student activity and improving learning outcomes in the classroom for years. However, teachers still face problems selecting and implementing manipulatives in classes. After researching relevant literature, our journey with Logifaces started with asking teachers to assess the possibilities of using the manipulatives in class. We moved on to testing Logifaces among students and carried out a survey among their teachers. To get a clearer impression on how Logifaces can influence the development of maths skills, e.g. geometry skills, we used it in a classroom setting with children diagnosed with mathematics learning difficulties. The fun and joy of using Logifaces and being creative in a form of geometric art can encourage teachers to change their beliefs about games being useless in education.

Keywords: Art, Mathematics, 2 Dimension, 3 Dimension, Teaching method, Manipulatives, STEAM, Science, Interdisciplinary, Experience Based, Playful Learning

Introduction

The project "Analogue game for digital minds - Logifaces methodology"[1] is a STEAM-focused programme within the Erasmus+ programme. STEAM stands for the subjects of Science, Technology, Engineering, Arts and Mathematics. The Logifaces game integrates all subjects, and provides an interdisciplinary approach[2]. Mathematical skills have especially high value in education due to their high value in the technological transformation of society[3].

The European Council recommends certain skills as key competences for lifelong learning[4]. These recommendations were made to ensure accessible quality and inclusive education, training and lifelong learning. The aim is to develop and foster skills that allow active participation in society and working environments. The recommended skills include numerical, scientific and engineering skills, digital and technology-based competences, interpersonal skills, the ability to adopt new competences, entrepreneurship, as well as cultural awareness and expression. The application of 3D printing in education can help develop these skills and at the same time help develop ideas for use in maths education[5]. This is why we found this tool promising in our research. This specific technology is often used in prototyping or producing small quantities of products, such as manipulatives[6]. Manipulatives are physical objects that can be used to demonstrate or help investigate concepts, such as those taught in the mathematics classroom. Research suggests that using manipulatives may have a positive impact on student mathematics literacy and their understanding of mathematical concepts[7]. Furthermore, we believe that fun and motivation are crucial in early childhood education, so we chose a game as the object of our research. Studies have been performed that suggest that increased motivation and enjoyment have a positive effect on learning outcomes. Using games and manipulatives in the classroom is one possible way to achieve a motivational boost. One example of manipulatives that can be used in STEAM teaching is the game Logifaces, which is the subject of the project discussed in this text.

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[1] Erasmus+ 2019-1-HU01-KA201-061272

[2] M. S. Khine, (2019). Steam education. *Springer* Berlin Heidelberg.

[3] Q. Lippmann, & C. Senik. (2018). Math, girls and socialism. *Journal Of Comparative Economics, 46(3)*, 874-888. doi: 10.1016/j.jce.2018.07.013

[4] Council of the European Union (2018), Council Recommendation on key competences for lifelong learning. *Official Journal of the European Union*

[5] M. A. Collins, E. V. Laski (2015). Preschoolers' strategies for solving visual pattern tasks, *Early Childhood Research Quarterly 32 (2015) 204–214*, http://www.bclearninglab.bc.edu/downloads/Collins&Laski2015.pdf

[6] D. Lieban and Z. Lavicza. (2019). Dissecting a Cube as a Teaching Strategy for Enhancing Students' Spatial Reasoning: Combining Physical and Digital Resources. *In Bridges 2019 proceedings.* https://archive.bridgesmathart.org/2019/bridges2019-319.pdf.

[7] Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology*, *105*(2), 380.

The Context of this Research

Visualising abstract concepts is important for many teaching subjects, especially for mathematics. Manipulatives can be seen as a specific form of visualisation where students themselves are able to manipulate the representation of a concept in mathematics, for example, by holding, turning, rotating a 3D shape in their hands. Spatial reasoning is also an important process in geometry[8].

Manipulatives and games in education

As already established, manipulatives are physical objects that can be used to demonstrate or help investigate concepts, such as those taught in the mathematics classroom. Research suggests that using manipulatives can have a positive impact on student mathematics literacy and on the understanding of mathematical concepts. Usually, these manipulatives are provided by teachers and are not created by students[9]. In this case though, in the learning process, the students do not merely passively absorb information during lessons or from textbooks, they develop their own knowledge based on what they have already learned and experienced according to the theory of constructivist learning[10]. A student's experience of the environment is usually physical and three-dimensional, especially during preschool. Therefore, it is possible that certain types of mathematical manipulatives may be used by children before learning to read and, to some extent, these manipulatives can be created by students in a constructivist manner. Creating objects such as manipulatives requires a variety of skills and competencies, especially those in the field of mathematics.

Playing games and solving puzzles can foster strategic and critical thinking, while making them can foster creativity and assist in inquiry-based learning. When planning activities that involve the use of games, it is important to consider all these aspects in order to get the most out of the game[11]. Experiments in this direction were recently carried out in a private school in Canada, where students not only played, but also digitally remodelled existing physical games[12]. As the final outcome, a group of students used a 3D printer to physically create their digital models. The students examined existing materials, and tested different strategies by playing, or

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[8] Mix, K. S., & Battista, M. T. (2018). Visualizing mathematics: The role of spatial reasoning in mathematical thought. *Springer*.

[9] Fenyvesi, K., Budinski, N., Kaukolinna, M., Lakos, D., & Lavicza, Z. (2020). Playful Development of Mathematical Thinking Skills in Primary and Secondary School with the Logifaces STEAM Education Toolkit, *LUMAT Research Symposium, p40.*

[10] G. E. Hein, (1991). Constructivist learning theory. Institute for Inquiry. Available at:/http://www. exploratorium. edu/ifi/resources/constructivistlearning. htmlS.

[11] Olson, J. C. (2007). Developing students' mathematical reasoning through games. *Teaching Children Mathematics, 13(9)*, 464-471. https://doi.org/10.5951/tcm.13.9.0464

[12] D. Lieban, M. Barreto, S. Reichenberger and Z. Lavicza. (2018). Developing Mathematical and Technological Competencies of Students Through Remodeling Games and Puzzles. *In Bridges 2018 proceedings.* http://archive.bridgesmathart.org/2018/bridges2018-379.pdf.

understanding the code behind them. They also had to adapt them and create (or recreate) their own models.

In addition to understanding the rules, which is the first learning aspect of playing, other competencies can be developed from the use of games as a teaching and learning tool. Social skills such as resilience, self-expression, and collaboration can be improved when playing. A number of these competencies were identified by a group of psychologists, musicians and teachers/educators in a workshop in Brazil. Moreover, as already mentioned, they found that mathematical and technological skills can also be developed and improved. From this experience, the Brazilian study also found that the use of games as a teaching tool can be adapted as much as desired in differentiated learning/teaching, which shows the great value of using games in education. Currently we are considering extending our teaching research to different learning environments and classroom settings. One idea is to use various, more accessible technologies, such as smartphones and tablets, to complement the activities. Through this, we believe that a greater number of students may be reached and engaged in the learning process we describe in this paper.

Mathematical discussions influence the development of games on different levels. As a particular example explored in Lieban et al.[13], in the course of a study carried out by the GeoGebra project, another programme aiming at the development of mathematical skills using games, a student decided to change the geometrical basis of the game. Equilateral triangle prisms were used instead of squares, which became a great challenge: finding the centre of the triangle was necessary in the course of the game. Some trials with the tools in the GeoGebra platform guided the student to the precise solution. Nevertheless, when using 3D modelling software, Tinkercad, the student found only the grid as a support, not being able to use the same tools and strategies used in GeoGebra. As a consequence, the shape of the pieces was changed again, the square prism being chosen as the piece this time. This is a good example of the adaptability necessary in the interaction with mathematical games and how students are able to find solutions through their mathematical skills.

Examples of educational games

There is a wide range of educational games, and we selected two examples in the field of mathematics to provide context for the Logifaces game: how it follows the tradition of manipulatives to a certain extent while also offering something uniquely new. In the following we briefly explain the functions of Cuisenaire rods and PUSE. These games were selected because they are based on geometrical forms that are linked to mathematics and can also be used as manipulatives and can also be used interdisciplinary way for exercises in other subjects.

[13] D. Lieban and Z. Lavicza. (2019). Dissecting a Cube as a Teaching Strategy for Enhancing Students' Spatial Reasoning: Combining Physical and Digital Resources. *In Bridges 2019 proceedings.* https://archive.bridgesmathart.org/2019/bridges2019-319.pdf.

One of these originates from 1931 and the other was developed more recently, in 2010. Both combine mathematics and other subjects using a playful approach following the Montessori spirit and contain objects that can be assembled in configurations and forms, which can be created either as the students desire or following certain rules. These aspects can also be found in the Logifaces game, which combines mathematics and other subjects through assembling geometrical forms.

Cuisenaire rods

Cuisenaire rods are a kind of mathematical game and manipulatives. As can be observed in Figure 1, they consist of a set of ten rods with different colours and lengths. Each of the rods represents the numbers from one to ten with the smallest cube unit serving as the measure of the increase in height between each rod. This mathematics learning aid was invented by the Belgian primary school teacher Georges Cuisenaire and is used today as an interactive tool to discover mathematics and mathematical concepts[14].

Figure 1: *A set of Cuisenaire rods as used in school today*. I*mage source: https://www.abcschoolsupplies.ie/product/wooden-cuisenaire-rods-introductory-set-of-74*

Students that are challenged in understanding logical operations profit in particular when using Cuisenaire rods. The idea was born when Cuisenaire observed the difference in student enjoyment when teaching his two subjects, music and mathematics. He therefore tried to create a tool combining these subjects and spark the enjoyment students experience in music classes in mathematics classes too and used the rods as an aid. Although having been developed years earlier in 1931, Cuisenaire rods did not become broadly popular until the 1950s, when Caleb Gattegno, a British mathematician and specialist in maths education, popularised them and further developed their use. Since then they have been used in more than 100 countries and 10,000 schools all over the world. The areas of utilisation are extensive, for example they can be used to develop algebraic thinking, and understanding of sequences, patterns and counting[15]. The rods can be used to visualise addition, subtraction, multiplication and division, and even proportions, ratios and fractions, thereby developing the mathematic thinking in students. Another possible use for the Cuisenaire rods is in language teaching, more specifically in the so-called Silent Way or in connection with verb tenses[16].

The Poly-Universe in School Education (PUSE) methodology

The second example of the use of manipulatives in the context of maths teaching is the PUSE methodology. The acronym stands for Poly-Universe in School Education and is the name of an

[14] K. Delaney, (2001). Cuisenaire rods: 40 years on. Australian Primary Mathematics Classroom, 6(2), 26-31.
[15] M. Ollerton, H. Williams and S. Gregg. Cuisenaire: from Early Years to Adult (2018). From https://www.atm.org.uk/Shop/Cuisenaire---from-Early-Years-to-Adult/Cuisenaire---from-Early-Years-to-Adult-Book-and-Download/ACT103pk
[16] C. Gattegno, (2010). The common sense of teaching foreign languages. *Educational Solutions World.*

Erasmus+ project based on the Poly-Universe game[17], which was created by János Saxon-Szász. This approach also combines geometric shapes and art, however this time the art is visual rather than acoustic. PUSE includes triangles, squares with a missing corner and circles with straight edges.

Figure 2: *different PUSE sets*

Figure 3: *different combinations of PUSE elements. Image source: http://poly-universe.com/mathematics-set-up-and-combination-possibilities-of-the-poly-universe-game-family/*

Figure 4: *child playing with a PUSE set. Image source: http://poly-universe.com/#services*

The inventor of the Poly-Universe game has been involved in the PUSE Project since 2017. The essence of the game is to form an artistic and mathematical system by filling the two-dimensional plane with geometric forms, as shown in Figures 3 and 4. While creating a desire for discovery it also provides the experience of continuous success and enables the player to create their own task. Poly-Universe can help develop are art sensitivity, complex logical thinking, mathematical and combination skills, attention and concentration, visual memory and even social skills, such as openness and team spirit. The PUSE project describes the utilisation possibilities as follows: "The complexity emerging out of Poly-universe's simplicity makes it more than a game, more than art, more than mathematics: these elements come all together – creating synergy in education"[18].

Aspects of the above examples found in Logifaces

Both Cuisenaire rods and the PUSE methodology combine mathematical objects with arts using an interdisciplinary approach. The examples can also be used for teaching other subjects, including languages, engineering and science. These games can therefore be seen as STEAM games that use geometric forms as their basis, and constitute manipulatives. These geometric forms can be held in a student's hands, observed and combined to form different shapes and are generic, which is useful for the visualisation of concepts in other STEAM subjects. Both games were created with consideration to artistic creativity and motivational aspects and take a playful approach in the teaching of subjects commonly perceived as harder than art. They have certain shapes and a certain thickness but are in general used as two-dimensional visualisations and use colour coding as an additional dimension with meaning.

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[17] E. Stettner, & G. Emese, (2016). Teaching Combinatorics with "Poly-Universe". *In Proceedings of Bridges 2016: Mathematics, Music, Art, Architecture, Education, Culture (pp. 553-556).* Tessellations Publishing.
[18] E. Stettner, & G. Emese, (2016). Teaching Combinatorics with "Poly-Universe". *In Proceedings of Bridges 2016: Mathematics, Music, Art, Architecture, Education, Culture (pp. 553-556).* Tessellations Publishing.

Logifaces blocks are similar: the blocks can also be combined into various shapes that are either suggested by the game or decided by the students and, as it is also based on geometric forms, they can be used as manipulatives. The interdisciplinary background is another similarity; the Logifaces game is the result of an artistic design idea and has even received an award in this category. Consequently the Logifaces game has similar artistic and visualisation capabilities, being able to teach STEAM subjects in an integrative and interactive way and foster longer attention spans, combinatory skills, creativity and social skills such as communication. Just as in the previously mentioned examples, the developer of the game, Daniel Lakos, has a strong link to the arts with his Logifaces game receiving the Hungarian Design Award. While Cuisenaire has links to both mathematics and music, and his rods can be used to represent sounds, Saxon is a painter and created Polyuniverse with a high visual content, Daniel Lakos is an architect and created a game that has a three dimensional component. The Logifaces approach may have high motivation-creating qualities, like the other two examples, and can be used in educational settings for students with maths learning disabilities.

However, there is a distinct difference between the examples above and the Logifaces approach: the examples described are used in two dimensions and use colour coding as an additional data dimension, whereas Logifaces does not need colour coding as it consists of three dimensional blocks. This opens up numerous possibilities:

Firstly, not only children with special cognitive challenges, such as dyscalculia, can profit from this game but also children with special physical disabilities, such as the visually impaired, are able to profit more from this approach. The additional data dimension can also be perceived by other senses, such as touch, which adds to the blocks quality of being manipulatives.

Secondly, the game is more abstract than the other examples, but has the same number of data dimensions, which provide additional possibilities for use with STEAM subjects and, if needed, concepts that are more abstract can be communicated. Also, there is more freedom in the shapes that can be created because colour does not play a role, so all the blocks can be used. This can help create complex structures such as the representation of, for example, optics in physics. The colour dimension can still be added to code words and concepts, for example, if an additional data dimension is needed.

Thirdly, the three-dimensional shape itself may be of value in helping to foster visuospatial skills, identify orientation within geometric shapes and other important skills connected to three-dimensional thinking. The other two examples do not involve these skills in any way. Visuospatial skills are the ability to imagine the rotation of objects and the transition between 2D and 3D, and are connected to virtual 3D manipulation, which is required in engineering[19] [20].

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[19] Duffy, G., Sorby, S. A., & Bowe, B. (2016). Visualizing electric circuits: The role of spatial visualization skills in electrical engineering. 70th EDGD midyear conference, *American society for engineering education, 113– 117*

[20] Sorby, S. A., Metz, S., & Ribe, R. (2017). Implementing Training for Spatial Visualization Skills: Research and Best Practices for Engaging Future Engineers. Proceedings of the 2017 Conference for Industry and Education Collaboration, *American Society for Engineering Education, 1–7.*

The manipulation of 2D and 3D objects in two-dimensional representation requires understanding of drawing conventions and the ability to correctly reading a drawing[21]. Pittalis and Christou (2010) identify

- 3D geometry thinking
- the ability to manipulate a 3D model
- the ability to recognise and manipulate nets and 3D objects and compare them
- and the ability to calculate the volumes of solids as challenges for students
- creating and manipulating 3D objects in parallel and perpendicular directions

as great challenges for both young students and adults alike. Other researchers have observed that students lack visuospatial skills, especially when manipulating prisms in perpendicular and other directions[22]. These skills are not supported by the two examples discussed above, but do play an important role in science subjects and engineering. In the course of our investigation, we examined the topics of motivation, creativity, interdisciplinarity, three-dimensional thinking and support for students challenged in subjects connected to logic and computational thinking.

The Erasmus+ project

The main objective of the Logifaces methodology project is to prepare a new visual mathematics and art teaching system and adapt it to and disseminate it in EU-wide school education. The project is based on the Logifaces game, which is a spatial geometric, logic and sensorimotor skill development game. The target group of the project are primary and secondary school students, aged 6 to 18. Schools and institutes operating in the different education systems of various countries were invited to develop and test STEAM exercises using the game, involving students in both elementary and secondary school.

The introduction of the Logifaces teaching methodology aims to have a positive effect on school education because it involves the use of a game as a tool that switches dimensions, demonstrates spatial geometry, and develops sensorimotor and art skills. It creates fun and motivation in early childhood, provides support for children challenged by mathematics, interdisciplinary teaching exercises connected to mathematics and art, and support for developing cultural competence. Details of the additional skills that may be developed by this project :

- Flexible thinking - developing problem-solving skills by working on different solutions
- Creativity - Using imagination to finding unique solutions for open ended tasks

[21] Pittalis, M., & Christou, C. (2010). Types of reasoning in 3D geometry thinking and their relation with spatial ability. *Educational Studies in Mathematics, 75(2),* 191–212. https://doi.org/10.1007/s10649-010-9251-8
[22] Ma, H. L., Wu, D., Chen, J. W., & Hsieh, K. J. (2009). Mithelmore's development stages of the right rectangular prisms of elementary school students in Taiwan. *Proceedings of the 33rd conference of the international group for the psychology of mathematics education, 4,* 57–64

- Spatial and visuospatial abilities - Understanding the link between different objects and mental rotation of abstract objects - being able to imagine rotations in 2D and 3D
- Communication - Verbal skills and communication by planning and constructing projects in groups while using technical and artistic language
- Imagination - Most of the shapes built with Logifaces do not look quite like the real thing. Imagination has to fill in all the gaps. Imagination is a critical aspect of creative problem solving and abstract thinking
- Tactile senses - Logifaces is based on polygonal modelling, which helps students get a sense of virtual complexity at an early age by connecting the virtual and the physical
- Team building - Logifaces can be used freely or with the aim of building a predefined shape. Leadership and project discussion develop naturally
- Artistic inspiration - Exploring sculpture and design. The relief-like surfaces blend ancient aesthetics with contemporary design. Logifaces blocks have inspired artists, photographers, and even fashion designers
- Calculus - Exploration with simple blocks helps develop subconscious skills in mathematics and logic. Children practice geometry and build an understanding of numbers and how units interrelate

Play-based learning in the early years is fundamental as we absorb communication and language skills in both early numeracy and literacy learning. This naturally leads to the acquisition of social skills, such as turn-taking and sharing, and to the desire to discover more. Play activities that are simple, magic and fun are vital. Later, during the years students spend in primary education play-based learning is important for acquiring numeracy and literacy skills, but above all for developing the desire to keep learning. In secondary school positive stimulation to learn and a vision of where this is all leading are primary. Hands-on, open-ended activities help stimulate constant reflection and inquiry-based thinking, and we believe Logifaces is a suitable tool for achieving these goals.

Logifaces aims to help teachers introduce and practice a variety of mathematical, scientific, artistic and other concepts. It was created by Hungarian architect and designer Daniel Lakos, who is also an assistant professor at Moholy-Nagy University, Budapest (MOME). A group of teachers, researchers and designers from Austria, Finland, Hungary, and Serbia joined forces to investigate and develop examples of education activities in the project entitled The Logifaces Methodology: analogue game for digital minds[23]. Many activities revolve around using the game blocks as manipulatives, such as the proposed workshop involving the creation of 3D models of

[23] The coordinator of the Logifaces project is Planbureau Kft located in Budapest, Hungary. Other partners in the project are the Akademische Gymnasium in Vienna, Austria, the Experience Workshop in Jyväskylä, Finland, Johannes Kepler Universität Linz (JKU), Austria, Lauder Javne Zsidó Közösségi Óvoda, Általános Iskola, Középiskola és Zenei Alapfokú Művészeti Iskola in Hungary, and Osnovna i srednja skola sa domom ucenika Petro Kuzmjak in Serbia.

the blocks themselves. Using the blocks as a point of inspiration and motivation to engage students in mathematics and multidisciplinary learning projects is in line with the skill development recommended by the European Council. The geometry of the blocks begs their use as manipulatives in various STEAM areas revolving around spatial, visual, and computational thinking.

Exercises are created and collected based on using the game as a teaching tool[24]. The Logifaces project is aimed at creating a catalogue of exercises in STEAM subject education using the Logifaces game and at investigating the effects of such exercises. For this it is crucial to use the game in a teaching context to find different utilizations. Another core element within the goals of the Logifaces method is enhancing students' spatial reasoning and three-dimensional thinking. Lieban (2018) gives the example of cube-puzzles, arguably a kind of manipulative, to improve spatial reasoning in students by addressing the problem in two as well as in three dimensions[25]. Lieban (2018) discusses how knowing one representation of a problem can help students visualise and comprehend other representations. Not only may switching between dimensions be trained by the game, but more mathematical, artistic, and STEAM-related subjects can be supported.

Mathematical concepts involving the jumps between the planar and the spatial, spatial geometry, combinatorics, algebra, figurating numbers and tessellation can be introduced and connected to artistic thinking, such as analysing the light-shadow effect on different angles and slopes. Also graph theory can be explained connecting to, for example, computer related skills, like network science and polygonal modelling.

Art is taught through encouraging thinking about design, relief structures, light and shadow effect, and basic figurative forms. Social skills can also be taught, such as problem solving, cooperation when learning with peers, and non-verbal communication. In addition, the game may help inclusivity through collaborative activities with visually impaired children.

The exercises created will be available online at https://www.geogebra.org/m/pghjyunt and a paper version will also be available.

[24] Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in mathematics*, *47*(2), 175-197.
[25] D. Lieban, M. Barreto, S. Reichenberger and Z. Lavicza. (2018). Developing Mathematical and Technological Competencies of Students Through Remodeling Games and Puzzles. *In Bridges 2018 proceedings.* http://archive.bridgesmathart.org/2018/bridges2018-379.pdf.

The Logifaces Methodology

The Logifaces game consists of blocks constituting prisms with a triangular top and base and different heights at the corners. This leads to various slopes and areas of the sides of each block. The goal of the game is to form shapes with a continuous top surface. To create this, blocks with a similar side surface have to be selected and arranged accordingly. In the section on the Logifaces methodology the design and structure of the game will be explained in more detail. There is a broad variety of activities possible with Logifaces, which offer challenges with a wide range of difficulty. This wide range provides fun activities that are suitable for many age groups.

The Logifaces game

Two different Logifaces sets were used in the exercise book including either nine or 16 elements. The elements are called blocks or pieces depending on the context of the exercise. The blocks are geometrical objects with equilateral triangles as bases and the heights of the corners can vary. This results in block sides with different slopes and area sizes, so called triangular based prisms and truncated prisms. In total, there are eleven different shapes. With each of the sets, as well by combining the sets to use 25 pieces, it is possible to create a larger equilateral triangle as a triangular tessellation with a continuous top surface. Figure 5 shows some of the Logifaces blocks and how they can form a continuous surface. Pieces have three height levels and should be assembled to form a continuous surface.

(a) (b)

Figure 5: *(a) Logifaces blocks assembled incorrectly to form a non-continuous surface (b) Logifaces blocks assembled correctly to form a continuous surface*

There are two solutions for solving the triangle with nine pieces, 22 solutions with 16 blocks, but 4942 solutions with 25 blocks. This seemingly simple task provides logical and creative challenges that can be used in a variety of exercises during maths classes, e.g. problem solving activities or relaxing the atmosphere of the lesson through an entertaining game. The broad variety of possible activities provides challenges with a wide range of difficulty level, as mentioned in [26]. Patterns play a significant role in early childhood education. With Logifaces, students can engage with patterns in at least three different dimensions, as listed in[27]:

- Structure: for example, prisms and truncated prisms
- Content: with their different materials, shapes and colours,
- and Complexity.

[26] Erasmus+ 2019-1-HU01-KA201-061272

[27] K. J. Carbonneau, S.C. Marley, J. P. Selig, (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology, 105(2),* 380.

The connection of Logifaces to art is not restricted to the creation of patterns. Just like photographs consist of squared pixels as the smallest unit, three-dimensional models consist of triangles, since a triangle is the smallest possible way to represent a plane. A plane can also be called a face. This was the inspiration for its name and implies the idea of using Logifaces as the smallest unit to create three-dimensional artworks. Geometric abstraction also plays a role in art, in fine arts as well as in cultural arts and applied arts. The prime example of this is Cubism, which revolutionised the fine arts, painting and sculpture, in the early 20th century. It also influenced other movements in art and architecture that followed, like abstract art, futurism, suprematism, constructivism, De Stijl and Art Deco, just to name a few.

Exercises for almost all occasions

An approach that reaches beyond the boundaries in the classroom is not only useful but also necessary, according to the European Union and many experts in the field. An interdisciplinary approach help foster key competencies and furthermore: we simply do not know the challenges students will face in the future, so we have to try to prepare them for anything.

Preliminary literature research was carried out in November 2019. This was used to create an overview about what to expect, which topics could be covered and which methods can be used in the Logifaces project. An interview was conducted with the developer of the game to understand its possible value for education and the features that could be used by teachers. To identify methods and collect data from multiple schools and at various events, we investigated how the Logifaces game was already being used by some teachers. Teachers entered their ideas for possible exercises in a spreadsheet table, which was created based on the gathered information. This table was later very helpful when constructing the content of our measurement tools.

The following data collection measurement tools were identified. Firstly we developed a questionnaire to be used after teaching with Logifaces in class (or at festivals if possible). Secondly interview questions were written for a survey with the possibility of giving open answers to determine multiple ways of using the game in multiple subjects. We will discuss the relevant findings later on. Thirdly we planned to observe classes, which was made impossible by the global Covid-19 pandemic.

The literature review was later refined towards a wider variety of topics and a more detailed picture. The research fields were manipulatives, problem solving, teacher beliefs, open learning, geometry, and methods in the teaching subjects of art, physics, mathematics and information technology. We found a great many studies and collated them. Though the literature review is an on-going process, the basis was defined at the time and is only updated whenever new research is found to be useful. With this in mind, many teachers created exercises to use Logifaces in their lessons. Here is one example of a lesson that took place in ninth grade in Vienna.

In a 9th grade (±14 year old pupils) computer science class the teacher used Logifaces in an interesting way. The task was to use Excel to write a program that checks whether two Logifaces blocks fit together or not. It was conducted in pair work or in small groups of three. Each student could use a computer, had access to the Internet for research and each group received a Logifaces set (16 pieces). The classes were held twice, one double lesson each (2x50 minutes) with a total of 41 students participating in the educational experiment. There were always two teachers present during the lessons.

Here are some of the things the students said:

> *The good thing was that you had to think for yourself before working.*
>
> *You learn exactly the things you need to solve the problem - otherwise we would probably not have learned these things.*
>
> *It's fun to experiment with the stones and the program - and you can also check if the program is correct.*
>
> *It's good that you can think for yourself and have time to think.*
>
> *It is good that there are several solutions and that you do not have to look for only one solution.*

Using Logifaces for online and hybrid teaching

Online and hybrid teaching gained importance during the Covid-19 pandemic. However, Logifaces blocks are physical manipulatives and, therefore, using them in online or in hybrid teaching situations is challenging. Students would need a locally available set of blocks to be able to participate. However, virtual manipulatives and emerging technologies, such as Augmented Reality and 3D printing, can help with the transition between virtual and physical worlds.

3D modelling and printing has the potential to help develop and train STEAM skills in students and help teachers in their lessons and teaching process. In addition to skill development for the students' later careers, it is not only valuable to understand this technology but it also supports the development of skills. In particular, skills that can be used for educational purposes and can provide teachers with tools to motivate students to consider career paths in STEAM education. Using 3D modelling and printing, three dimensional thinking, problem solving competencies and more skills can be developed in STEAM classes, such as an understanding of the connection between the virtual and physical worlds as well as between 2D and 3D representations. 3D modelling and printing can therefore open up new possibilities to teaching these skills and help transport concepts from, for example, mathematics[28]. The GeoGebra app was created to help the creation of a personalised Logifaces block for printing. Some pieces were created and tested. This GeoGebra resource may represent a basis for activities in schools. Logifaces pieces using GeoGebra: this GeoGebra resource contains the information and tools of the workshop https://www.geogebra.org/classic/ymmmehdf.

The free GeoGebra platform, which was mentioned briefly above, allows the creation of maths-based applets for creating graphs, investigating geometrical objects, using algebra and more purposes such as 3D modelling. 3D models can be downloaded in a 3D printable format meaning a GeoGebra applet can be used to create exercises and support activities. One example of an activity inspired by the Logifaces game is the creation of personalised prism-based blocks. Such blocks can either be created by using an applet developed by the authors, where each side can be manipulated by a student, as can be seen in Figure 6.

Figure 6: *Creating one Logifaces block with two sides set to low and one side set to medium height*

For more complexity and more advanced training of mathematical skills and computational thinking the GeoGebra functionality can be used in the classic version of Logifaces. For example by drawing a polygon and extruding it to 3D. The created block with features also used in Logifaces can then be downloaded and used to introduce students to 3D printing.

Figure 7: *Example blocks similar to Logifaces created by the applet in and downloaded from the GeoGebra and 3D printed using the Fused Filament Fabrication technology*

A workshop concept was created to develop maths, art, and social skills using Logifaces. The aforementioned skills described by the European Council and the motivation created and the exercise possibilities using the Logifaces game can be combined into this workshop idea. Initially, students are asked to imagine and draw the side areas with the triangle corners at different heights. Later, the students can play with the GeoGebra applet, which allows the Logifaces-like blocks to be freely rotated. Then they are asked to investigate how setting the sides to the heights they thought of before changes the surface of the other sides of the prism.

A shape is presented to the students that has to be filled with blocks to create a continuous top surface. The students are asked to find out which blocks fill this shape. Next, they are asked to create prism blocks themselves and to download them for production. They can either use features of the GeoGebra tool by themselves constructing their previously drawn block or by creating a version of their block using the applet. Now, the students are asked to load their prisms to a 3D printer to produce their solutions and put them to the test. This step can take some time so we recommend that only small blocks be produced. High blocks should be turned

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[28] D. Lieban, M. Barreto, S. Reichenberger and Z. Lavicza. (2018). Developing Mathematical and Technological Competencies of Students Through Remodeling Games and Puzzles. *In Bridges 2018 proceedings.* http://archive.bridgesmathart.org/2018/bridges2018-379.pdf.

to one of the side surfaces to avoid knocking them over during production and to have them finished at the same time to avoid unnecessary waiting time. Additionally, the students can find other shapes that could be formed by their solution.

Figure 8: *Example printouts with a ruler showing blocks with a maximum height of 3 cm*

Skills in mathematics, digital and technology based competencies, as well as communication and cultural awareness and expression can be taught through these steps. The students may also be able to use the produced manipulatives for other exercises revolving Logifaces, for example, by combining the mathematical activities with art, such as adding drawings to the shapes produced with Logifaces.

Figure 9: *Logifaces blocks arranged into various shapes and decorated with drawings*

Furthermore, the skills gained can be used for a follow up workshop where students are asked to develop their own mathematical puzzles, such as printable cube puzzles. This follows the European Council's recommendation for the development of competencies that foster entrepreneurship.

The role of art in the exercises

Art may also be viewed as a fundamental element in holistic skill development education. In connection with the European Council requirement for European citizens to be able to communicate and be culturally aware, interdisciplinary knowledge about understanding and embracing various cultures also includes art. STEAM fuses the sciences and the arts, combines logical and intuitive thinking and brings together systematic exploration and revealing insights[29]. The STEAM education methods include experimenting, being open to change, improvisation, and creating freely. The resulting new structures, in turn, aid development and construction of personality[30]. As such, STEAM education can be thought of as a compound way of gaining knowledge. Science and art can be seen as being connected by STEAM education methods – openness to change and creating freely can be trained through art lectures as well as within all other subjects, especially in an interdisciplinary STEAM approach. This, in turn, helps students achieve the European Council skills.

(a) (b) (c)

Figure 10: *Using Logifaces as inspiration and as a part of artistic projects*

[29] Colucci-Gray, L., Burnard, P., Gray, D., and Cooke, C. (2019). A critical review of STEAM (Science, technology, engineering, arts, and mathematics). *Oxford Research Encyclopedia of Education.* https://doi.org/10.1093/acrefore/9780190264093.013.398

[30] I. Nahalka (2013). Konstruktivizmus és nevelés. *Neveléstudomány 2013/4.* http://nevelestudomany.elte.hu/downloads/2013/nevelestudomany_2013_4_21-33.pdf

Besides tactile experiences we also investigated integrating whole body movement into Logifaces exercises, based on the following considerations:

1. Whole body experiences are an integral part of learning[31], so by getting involved in whole body movement connected to Logifaces tasks students are able to internalise the quality and structure of the Logifaces pieces and constructions at a deeper level, and so they become more efficient when carrying out other Logifaces tasks, and are able to internalise and connect the knowledge they obtain in any field through engaging with Logifaces.
2. Physical activity has physiological effects (e.g. increasing the supply of oxygen to the brain) and psychological effects (e.g. increasing motivation) that enhance learning[32], so movement may act as a catalyst to reaching the educational objectives of Logifaces tasks.
3. Engaging in an adequate amount of physical activity is crucial for physical and mental health[33], meaning that integrating movement into Logifaces tasks has health benefits for students.

The innovation of Logifaces lies in the fact that it takes the process of digital design from the screen and places it in tactile reality, bridging the gap between immaterial shapes and solid physical reality. Correspondingly, the range of methods embraced by art education has widened to include 3D modelling and digital creation, for example, which means that new interdisciplinary connections are formed.

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[31] M. Johnson. (1991). Knowing through the body, *Philosophical Psychology*, 4:1, 3-18, DOI: 10.1080/09515089108573009

[32] C. Bedard, L. St John, E. Bremer, G. Jeffre, J Cairney (2019). A systematic review and meta-analysis on the effects of physically active classrooms on educational and enjoyment outcomes in school age children. *PLoS One,* 14(6), DOI: https://doi.org/10.1371/journal.pone.0218633

[33] V. Beserra, M. Nussbaum, M. Navarrete, D. Alvares. (2021). Teaching through dance: An opportunity to introduce physically active academic lessons, *Teaching and Teacher Education*, 106 / 2021, DOI: https://doi.org/10.1016/j.tate.2021.103450

Feedback from Teachers

Teachers and students are the most important stakeholders in the Logifaces project. Therefore, it is crucial to listen to and implement the feedback given by them about using Logifaces in class.

Teachers who are experienced with Logifaces

After defining interviews and questionnaires as the tools needed to collect data, a list of general and detailed questions was developed. Multiple questionnaires were based on the literature topics previously mentioned and a general interview guideline was used for first interviews. The people working with us were asked to take notes on their experiences.

The first two interviews were conducted with teachers in German since this is also the language of the authors of this study. The interviews were conducted using a collection of about 50 questions that were defined beforehand, and following the line of the discussion, an additional around 10 open questions were asked. The questions revolved around inquiring about exercise ideas and past implementations versus new ideas occurring during the school closures. Also the use and usefulness of Logifaces as a manipulative was investigated. Additionally, questions about crossover lessons with other subjects and the possibility of using Logifaces virtually were asked and answered.

Through the interviewed teachers it was revealed that a German interview may work in this manner but English interviews could have difficulties due to the language barrier. To overcome these difficulties, first about 10 test interviews were conducted with teachers who were not part of the Logifaces team to identify good strategies. Secondly, carefully aimed towards the goal of the study, selected questions were sent to teachers in advance in order to allow them to translate the questions and think about their answers before the actual interview. The teachers were also able to answer the questions in written form beforehand. The interview again went through the questions and answers to identify if the study authors and those interviewed understood everything in the same way and that all was understood correctly. In these cases it was also possible to ask questions in more depth about certain interesting points in the answers given.

Moreover, if there was an interesting point that was surprising or unconventional the interviewed people were asked to elaborate. For example, one of the teachers thought beforehand that the students would be distracted by the game during lessons or even thought the game could be too boring due to the simple design. In contrast, the teacher found out later that the children loved it and it was an excellent tool for creating exercises. This was found out by carefully asking what exactly was surprising for the teacher during classes.

The analysis of the interviews, which were also partly recorded, was evaluated using a qualitative approach. The interviews themselves were conducted during the Sars-CoV-2 outbreak and thus were conducted via online video tools, such as Zoom, Whereby, or Google Meetup. Interesting points were selected as anecdotes.

We asked the teachers participating in the project about their opinion of the game. Two male and five female teachers from Serbia, Hungary and Austria provided answers. The teachers' subjects were science, art, maths, and languages.

The aim was to learn about their expectations, their backgrounds and their experiences using the game during their lessons. Our questions revolved around their motivations and their perceptions of the interactions of the children. Interesting recurring statements were collected to give an impression of the mindset of the teachers.

When asked about their motivation to use Logifaces during their lessons the following aspects were brought up. Motivating students to connect maths to the real world and to foster creativity was one straightforward answer we obtained multiple times. Since innovation had high value for some, they said that the combination of art and maths was beneficial. It seemed to provide the possibility to work with other teachers by combining subjects that seemed motivational and inspirational. Also inspiring children to think in a nonlinear way had much value to some teachers.

We then went on to ask about the setting they used the game in. The teachers used it physically in classrooms mostly. They often use it either in small groups or with each student by themselves. The age of the students was mostly between 12 and 18 years.

Here some of the things the teachers said:

> *"In my educational practice I am interested in STEM activities and the creative teaching process. Logifaces is an opportunity to satisfy my interests." Could you explain that further? "I like working together with other teachers, we try to do something new. We combine different subjects, for example, with physics or other subjects."*
>
> *"The thinking and perception of contemporary youth operate on the principle of network thinking. Logifaces now provides an opportunity to deepen nonlinear and rhizomatic thinking. I find it very exciting!"*
>
> *"Usually, I use them for my workshops, where students revise content learned in traditional lessons. Logifaces needs a connection to the subject and the topics; you have to adjust the students to the new way of learning, it is a process."*
>
> *"In the 8th grade, we experimented with the topic of Object and Space design. They made objects and practical spaces from their own combinations."*

Afterwards we investigated the experiences of the teachers themselves. Some reported that it was interesting to see that the students were only either very excited or slightly annoyed when using the game, with there being no other opinions. One teacher even expected students to be annoyed but in contrast they were excited. Students in science classes seemed to gain a deeper understanding of processes using the game. Also a general experience was that the simple rules of the game were not always easy to follow.

> *Interviewer: "Why did you think they wouldn't like it?" - Interviewee: "It is so simple, maybe they lose interest fast. They even asked for more Logifaces in the lessons!"*

> *"The use of Logifaces sets in my biology classes helps students to visualise certain processes and thereby make proper conclusions. The students listen to explanations, but they gain a deeper understanding with the game."*

When asked about the duration of the prepared exercises and what kind of instructions they needed from an exercise book, the teachers often prepared for either 45 or 90 minutes. Some reported that the plan was usually too ambitious and the exercises took longer than that, others said that the planning and the time overlapped nicely. One teacher pointed out that a strict planning process needs to take personal aptitudes, skills, and rhythm into account and that they usually try to let students find their own solutions to a problem. This can make planning obsolete.

> *"My activities usually last for 45 minutes, and usually everything goes as planned. 50 minutes in Austria; you need some time for introduction."*

> *"The planning of a process is never too strict. It is important to consider individual aptitudes, skills and rhythm. Cooperative group work is therefore beneficial because everyone has more chances of success. Solutions can be of many kinds!"*

Topics were collected that teachers were able to teach using Logifaces. They named topics such as geometry, reflections, translations, volume and area calculations as well as combinatorics. Furthermore they were able to teach about pressure, density, space design and talent management, as well as about filmmaking and perform investigations into culture and traditions and even exhibition planning. Many teachers also used the game within body and mind workshops to improve students' brainpower and to foster thinking out of the box.

Questionnaires for future studies

Another questionnaire was developed in addition to the interview questionnaire. It aims at obtaining feedback from teachers on their experiences with Logifaces as manipulatives in general during their lessons but also on how the exercises developed for the game work in class. It will be conducted as soon as teachers have the possibility to try out examples, which is not happening for the time being due to the Covid situation.

First, we prepared a general questionnaire. We want to collect the teacher's first impressions of the game and their observations when students use Logifaces in class. We are especially interested in the skills that students develop when using the game. The next part of the questionnaire consists of pairs of contrasting attributes that may apply to Logifaces. The circles between the attributes represent gradations between the opposites as is common for a Likert scale. Teachers can express agreement with the attributes by ticking the circle that most closely reflects their impression. Teachers should decide spontaneously and not think too long about their decision to make sure that they convey their original impression. One example is the opposites dull and creative. In the last part of the questionnaire we ask about personal information and the environment in which Logifaces was tested, like class and subjects.

Secondly, we created another questionnaire to test for specific exercises. For each exercise the teachers conduct, they are asked to fill out one of these questionnaires. This will help us get to know what specific environment the exercise was tested in, for example, if it was a school project or a regular lesson. We were also interested in how the exercise was introduced to the students. During the Johannes Kepler University summer workshops we were lucky to have very positive experiences with an introduction of fractals and mathematics in nature to boost student creativity.

Workshops with teachers without experience with Logifaces

Two kinds of workshop were conducted to gain better understanding of teachers' needs and the views and beliefs of teachers related to Logifaces. One was carried out in small groups with five Austrian Steiner Waldorf school teachers in person while a second workshop was conducted online with around 50 teachers from Indonesia at a later point.

Workshops with ideas from Steiner Waldorf teaching in Austria

Five teachers from a Steiner Waldorf school were asked to play with the game to get familiar with it and then brainstorm ideas about when and how the game or the game's content could be used. Two of them teach foreign language, one is a class teacher with a botany focus, another an arts and crafts teacher and then also a class teacher with a focus on mathematics. The teachers were handed both 3D printed versions as well as concrete versions of the game. We collected their impressions on the versions made with both material and also asked them about the augmented reality version as a warm-up and let them play with the game.

Due to the pandemic, not all teachers were always present during the workshops at the same time and we distributed the workshop time among the individual teachers for both data collection as well as due to the legal restrictions of the amount of people in one room during the pandemic.

The warm-up feedback about the material gave us the result that all materials had their benefits and drawbacks. The teachers had varying preferences about the materials, which we believe originates in their way of using the game and current constraints. The teachers were comfortable discussing their thoughts about the use of the game and its advantages and limitations in lessons and gave us ideas how, when and for which purpose they believe the shapes can be inspiring and useful for their students.

Figure 10: *A language teacher playing with 3D printed Logifaces blocks*

The original version made out of concrete was preferred by one class teacher because, as they explained, the children appreciate the weight and sturdiness of the material. Younger children sometimes use building blocks to create walls or objects and they would be familiar with wood or heavy blocks in the school rather than plastic and virtual objects. They themselves created many prisms out of two blocks on top of each other and then finally a hexagonal shape on top of another hexagonal shape making comments about how this would keep her smartest students busy while they attended to other students.

One of the language teachers, who can be seen playing in Figure 10, clearly favoured the 3D printed versions due to the possibility of marking them and removing stains easily as well as the possibility to disinfect them. Moreover, they were intrigued by the possibility of mixing multiple colours and thought that the game was best used in training already existing knowledge.

The other language teacher also played with a concrete version and she liked the feel of the blocks but for her the game was just too logical. She said that maybe grammar rules could be taught using the game but she felt limited by the rules and might only use the game to keep students occupied, agreeing with the class teacher that liked creating hexagons sitting next to her. Seeing the created hexagons, they stated this had to be a maths game due its complexity.

The arts and crafts teacher played with the concrete as well as the 3D printed version and had the idea to create personalised Logifaces blocks right away. They were reminded of crystalline structures and came up with a variety of ideas that could be used for crafting. After a short while playing, they were talking about how exciting a room would be consisting of triangles or how interesting walking on a floor of Logifaces would be for students. According to them Logifaces could also be used to explain rainbows.

The other class teacher did not prefer any material. It was the shapes they found to be very interesting. They appreciated that due to the various slopes and angles, the game could be used in a great variety of ways to teach mathematics, geography, or even history. After being given the game to play with at home, they had many ideas for exercises for younger students.

The teachers testing the game were given some 3D printed versions to use at the school as a thank you and are happy to improve their lessons using their own exercises. We took the input of the tangible and physical advantages of the Logifaces blocks and, as the next step, investigated how this could be transferred to virtuality to be applicable in distance learning.

Workshop with Asian Teachers

About 50 teachers from Southeast Asia, mainly Indonesia but also neighbouring countries such as Laos, the Philippines, and Thailand, participated in an online workshop about the Logifaces game and ways to use the game in a distance learning environment.

We first presented the game and the game's rules to the participants. We explained the Logifaces project and introduced the inventor of the game, Daniel Lakos. We later went on to show them how they could use online micro-games from GeoGebra. After the introduction of the platform we asked them to download, print, and fold a prepared Logifaces block folding exercise to make the form of the blocks clearer to the audience as can be seen in Figure 11. The instructions as well as the game's rules were collected in a GeoGebra book that was accessible to the teachers.

(a) (b)

Figure 11: *Paper folding Logifaces blocks with Daniel Lakos (a) presenting the exercise to teachers from Asia using paper folding printouts (b)*

(a) (b)

Figure 12: *a teacher from Asia folding Logifaces from paper*

The folding of the block helped to convey to the teachers how the game works because they had no 3D printer available to print a small set of the game. They were much more easily able to understand the rules of the game, which were demonstrated by two researchers, Imam Rachmadi and Eva Ulbrich, collaboratively creating shapes, such as a cat, using two sets of Logifaces. While creating the shapes they were constantly interacting with the audience as they posed questions about the game and the researchers explained their thoughts and talked about the difficulties to each other and the teachers.

The teachers also were informed about the basic advantages of the game and that exercises had already been created and collected in the GeoGebra book containing the folding instructions, information about the game and other online resources. One of these resources was a 3D app that provides teachers with the possibility to display a Logifaces block into their environment using Augmented Reality. They were shown that all 3D objects from GeoGebra were able to be used for this purpose or to be 3D printed as well.

We sent a feedback form about the workshop to the participants that they were invited to complete later on. From the 21 completed forms, 16 of the teachers were women and five were men between the ages of 29 and 46. Ten of them teach multiple subjects as class teachers and 11 teach mathematics or other subjects. Two thirds already knew about GeoGebra beforehand.

We asked several questions that allowed the teachers to reply in free text form and some additional questions that required them to choose from a predefined set of answers, such as their previous knowledge about GeoGebra, their gender, their age. They mostly answered in English but sometimes in their own respective languages, which then had to be translated by the authors.

First, we wanted to know whether the teachers used technology or games during their lessons, because we wanted to determine their prior knowledge in these fields ("Do you usually use technology and/or games during your lessons? If not, what is the reason? If yes, how do you use them?"). 14 of them rarely or never use technology in their classes, four of them use technology regularly. The rest use technology only occasionally. The reasons given for not using any technology during lessons included that the teachers lack the knowledge or the schools are not equipped with smartphones or computers. The reasons given by the teachers that use technology were that it is very motivating for students. Games were used by seven teachers regularly, sometimes by two and never by twelve.

An exemplary answer was: *"I usually use games but not technology. Games make students interested in lessons and are effective so I usually use games described in our teacher's manual and also my own creations. I can't use tech in my school because there is no power supply and other modern things like computers."*

Later we asked about how they thought Logifaces could be implemented in their lessons. Many replied with ideas about which topics or concepts they could teach: volume, patterns, tessellation, areas, edges, crafting, triangles, 3D, counting, geometry, perimeters, volume, measurement, art, shapes, colour, texture, 3D printing, to name a few. Others explained the context of when and how they would use it, for example connections to daily life, as a distraction, with a specific technical operating system environment such as Android or teaching to use gadgets, constructing things, after the lesson, or to have students go from two dimensional pictures to forms.

One teacher said: *"I think I can use Logifaces to teach various topics in maths like net of pattern or shape, tessellations, area, edges and so on."*

We also gained some insight into teachers' opinions about how games can be beneficial in teaching situations. The opinions included that Logifaces can help with the development of problem-solving skills and critical thinking, the training of imagination and creativity, inspire motivation, create fun, and develop understanding of mathematical concepts such as geometry, and foster a safe environment that can help to prevent psychological problems, for example.

“There is a lot besides, it will increase the enthusiasm of students in learning, but it can also help me as a teacher to more easily convey the subject matter to students as well as to make students easily understand the material.”

“Because it can cover the 4c's...collaboration, critical thinking, creativity, and communication ”

In general, the teachers expressed surprise at how games can support lessons and add fun to topics that might be less known for being entertaining.

“I think it will be boring idea [sic]. But as the time flew, this boredom was thrown away. So many new things for me are shown here. It's out of my expectation. Really the best. Actually I'm a type of traditional teacher, didn't know the new world.”

This indicates that the teachers' beliefs changed during the workshop. Other statements support the idea that the benefits of Logifaces can be easily made clear to teachers:

“This workshop changed my mindset of mathematics, I thought mathematics is only counting and counting, but I got a lot of think [sic]here and change my mindset about mathematics.”

“The game as I know that is something that can make us fun [sic], and more games make us lazy. But after I saw the explanation and other interesting games, wow it's amazing. Of course, the game can be applied in mathematics and science. It's something new for me.”

Figure 13: *Feedback from Asian teachers about the benefits of Logifaces*

We conclude that the Logifaces game can have benefits in lessons far beyond teaching mathematics and art and can even be used to change the beliefs of teacher who are usually more traditionally thinking and are initially opposed to games and technology in class.

Feedback from students

Science Holidays at Johannes Kepler University

We had the opportunity to conduct workshops with about 60 students during the Covid pandemic in the summer of 2020. In total about 200 children had access to the game. We first let the students between 9 and 12 years of age explore a 2D open game called Points and Edges and afterwards, we tested some easier exercises from the mathematics part of the Logifaces exercise book.

Figure 14: *Students during the JKU Summer Workshops creating Logifaces blocks with their bodies*

Figure 15: *Explaining the game to students so they can create their own artwork in pairs or alone*

Figure 16: *Proud students presenting their artwork*

We first started introducing mathematics by showing the students where mathematics surrounds them in their daily lives. We asked them to tell us about examples of where they suspect mathematics is involved and then showed them how mathematics can be found in nature. We emphasised the connection between mathematics and vegetables such as broccoli or leaves or the growing of trees or the shape of the human body. Then we showed them how games can support maths or how other subjects such as art can be connected to mathematics. In the end, we first asked them to get comfortable with a 2D maths game with open solutions called Points and Edges created by Julia Handl and then gave them Logifaces, first just for playing with and then later for doing some of the exercises developed for the Logifaces project.

Figure 17: *The lesson plan was tested and put in operation during the Johannes Kepler University Summer Workshops in August*

The students were asked to fill out a survey after the workshop to help us understand how they perceived using Logifaces as a game and in the exercises. A total of 28 students between 8 and 12 years of age gave us feedback on their experiences, especially with the exercises. 12 of them were boys and 14 were girls, two did not want to state their gender.

Students were very engaged and gave us the following feedback:

A surprising number of 10 students answered that mathematics was one of their favourite school subjects. Only physical education was more liked, being mentioned 12 times. After mathematics were arts and crafts with seven mentions. One student wrote that they previously did not count mathematics as one of their favourite subjects but now they did not perceive it as "that bad" anymore ("Deutsch und englisch aber jetzt finde ich Mathe auch nicht mehr so schlecht/German and English but now I think maths is not so bad either").

We also asked questions about the exercises and the maths experience they had. 18 Students answered that they fully agreed with the statement that using Logifaces was a fun way to learn maths and eight answered that they partly agreed. Only one student stated that they did not think the game was a fun way to learn maths and another did not want to give a comment.

In addition, we asked them about their favourite exercises and about what they think they learned during this workshop. They answered that they loved creating forms, especially pyramids, enjoyed building things and were also happy about the group work. Many of them answered that they think they learned a lot about geometry, about patience, about focus and concentration, and many answered that they learned that mathematics is part of everything around us.

We created an online workshop with this information in mind to give more children the chance to get to know Logifaces, learn about mathematics, and to communicate and discuss mathematical concepts during the pandemic. It can be used for remote teaching or individually by students.

Figure 18: *The lesson plan was tested and put in use during the Johannes Kepler University Summer Workshops in August*

Special Needs

Discovering the myriad of interactions between disciplines and applying them to the real world is one major focus in early childhood education[34]. Arts, mathematics and sciences are taught daily and teachers try to combine these different subjects in activities such as completing patterns, searching for shapes and forms in paintings or experimenting with different materials. In this research study, we also worked with students with diagnosed mathematical learning disabilities (MLD). MLD is a generic term for a range of developmental difficulties related to mathematics, with each student facing different challenges. Due to the known specific educational needs of MLD students, we presumed that the development of their mathematical skills would be more easily detectable by the teacher. This will help to find out if and in which areas the use of puzzles and games as manipulatives can be beneficial. Some students may have difficulties learning certain skills if they are not motivated or have cognitive difficulties in an area. Thus, manipulatives can be used in education to support children to develop skills.

[34] Fenyvesi, K., Budinski, N., Kaukolinna, M., Lakos, D., & Lavicza, Z. (2020). Playful Development of Mathematical Thinking Skills in Primary and Secondary School with the Logifaces STEAM Education Toolkit, *LUMAT Research Symposium,* p40.

We used Logifaces to explore its possible impact on spatial reasoning and pattern recognition of students with MLD in early childhood education. We worked with nine students aged 4 to 6 year and their teachers in an experimental setting to create visual art pieces and recreate patterns[35]. The first findings of this research were increased motivation and engagement in discussions about mathematics and arts during and after the pupils solved open-ended tasks using the game.

The research questions of our qualitative approach were:

1. What impact does using manipulatives, such as the Logifaces game, have on students with mathematical disabilities?
2. Which tasks can enhance the development of students' spatial reasoning and recognising patterns in early childhood education while using the game Logifaces?

We started to collect data through task-based interviews[36] and additional field observations for five weeks in Luxembourg with the help of nine children in early childhood. We aimed to develop tasks that can be carried out in future studies by early childhood teachers.

We proposed different game sets and materials to the students at the beginning of our project. Apart from the original game, which is made of concrete and is in different colours, we printed several Logifaces sets with a 3D printer using with PLA and PET-G in different colours. First we let students experiment with the puzzles in a collaborative way with the students in groups of two to three. The differences in how the students experienced the materials were very interesting to observe. Thus, the Logifaces made of concrete were reported by the students as being heavier and easier to manipulate. However, they were more fragile and edges broke when they were dropped. The 3D printed Logifaces were lighter and more robust, but more difficult to use in different combinations since they did not stay in place.

Students tried to organise the Logifaces blocks in the first exercises. Some organised them by colour, some by similar face and some connected two or three Logifaces with the correct side. Depending on the student, the process of categorisation was more or less developed. This could be related to their personal or classroom experience. However, Logifaces incited active discussion among students on how to organise the different pieces. On their own, the students connected the geometrical shapes of Logifaces to several art pieces students discovered incidentally in class, such as paintings by Kandinsky on geometric shapes or architecture like the Louvre's entrance, the pyramid shape. It was important for us to let students discover the materials and try out applications in an open setting to create first connections and indicate possible manipulations to us through task-based interviews. Moreover, students explained how

[35] B. Haas,Y. Kreis, Z. & Lavicza, (2020). Connecting the real world to mathematical models in elementary schools in Luxemburg. In R. Marks (Hrsg.), *Proceedings of the British Society for Research into Learning Mathematics: Bd. 40 (2)* (1–6). https://bsrlm.org.uk/wp-content/uploads/2020/10/BSRLM-CP-40-2-06.pdf

[36] G.A. Goldin, (2000). A Scientific Perspective on Structured, Task-Based Interviews in Mathematics Education Research. In A. E. Kelly & R. A. Lesh (Hrsg.), *Handbook of Research Design in Mathematics and Science Education* (S. 517–545). Routledge. https://doi.org/10.4324/9781410602725.ch19

and why they categorised the Logifaces and why they were similar or different to the referred art pieces. One could say these meta-discussions on the connections between mathematics and art were similar to those described by Krauthausen, where students actively explained mathematical concepts and discarded the properties of shapes, forms and patterns[37].

(a) (b) (c)

Figure 19: *Students manipulating the 3D printed blocks (a) creating smooth surfaces, (b) creating animals, (c) creating their own games within the game.*

In the following exercises, students created a series of combinations, this time with the focus on connecting the correct sides. The combinations ranged from two to five pieces, where many creations were made using one axial approach, for instance, one vertical or horizontal row. Only a few students created biaxial works. Within the peer approach, students discussed and invented patterns for their creations, for example, based on colour, shape or surface. We noticed that students developed new strategies in spatial reasoning and laying patterns out while manipulating the Logifaces. Thus, the creations gained complexity regarding the colours and the number of pieces they used in their patterns, driven by aesthetic appeal. Students also became faster in creating these mathematical art pieces. In our last manipulation sessions with Logifaces, the students discussed developing their creations further, by adding new colours with 3D printed blocks and increasing the number of disposable pieces to create landscapes.

The children displayed development in their mathematics and art skills. Due to the students' iterative testing of the materials using a creative approach, over time they felt more confident about solving the open-ended task. They discovered more and more mathematical features of the blocks, thus their development of geometric reasoning and spatial organisation improved. This teachers found this out by listening to the lively conversations of students had among themselves.

Aesthetics played an important role in the patterns the students made. The students developed their artistic skills further in the process; it was just like students were drawing with the blocks, fully in flow in their creative process. The students recognised and created patterns with increasing complexity and figured out the one rule of the game, to always create a smooth surface, by themselves, because they made choices to support their aesthetic preferences.

Other positive findings included the peer interactions that lead to a visible development of student meta-language, and thus allowing different paces for the students. All students showed a high level of motivation and engagement. They enjoyed playing and learning with the blocks so much that they repeatedly requested to use them again in class. The teachers also rated the learning experience with Logifaces very highly and plan on continuing to work with the game.

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[37] G. Krauthausen, (2018). Einführung in die Mathematikdidaktik - Grundschule (4. Aufl.. ed.). Berlin: *Springer Spektrum.*

The positive findings spread to other teachers in the school, who are now also happy to use the game in their lessons.

Discussion and Outlook

The data collected so far in discussions with teachers and interactions with students is encouraging. It indicates that Logifaces can be a useful tool to support educational environments in multiple ways. As became apparent in the teacher interviews there is not necessarily a widespread use of games or technology in classrooms, live or digital. Bringing teachers and students in contact with Logifaces may be the first step to teachers treating games with a more positive attitude and as an innovative tool for students. After gaining knowledge about teachers' beliefs and practices we want to lay the foundations for gamification gaining access to more educational spaces.

The classroom work with MLD students shows that they associated Logifaces not only with mathematics but also with art and they naturally connected the two subjects. Moreover, exploring the possibilities of these manipulatives fostered collaboration between students immensely, which we did not anticipate to the extent experienced. This was also the case in the summer school workshop program at Johannes Kepler University. Both aspects are interesting subjects for further investigation and research and students could, for example, use Logifaces in art history as a tool for understanding and perceiving art more fully. We want to produce high-quality, valid teaching materials for all teachers, not only those involved with MLD students.

The exercises with children with special needs in mathematics and the experience of language teachers show that exercises using the game hold much potential. Children with special needs can be supported from a very young age and language skills can be taught using the Cuisenaire rod example. Colours can be added as an additional data dimension, as used in some experiments we have mentioned that provide additional motivation. The strength of the game lies in its flexibility as a tool, it being fun to play in all age groups and combining different fields of mathematics with art and social interaction. So in the Logifaces project, a wide range of exercises are being created to suit different age groups, school types and school subjects, with each meeting different student requirements.

It is our goal to test these in the field. The Covid-19 situation makes testing the methodology difficult at the moment, but we hope to circumvent this by creating exercises for home-schooling as well. As soon as the situation improves, combining this work with 3D printing and other puzzles and games used as manipulatives will be another interesting direction for our research. The GeoGebra platform may be a useful tool and partner in this too. More research will deepen the knowledge we gained during this project about manipulatives and help us to apply it.