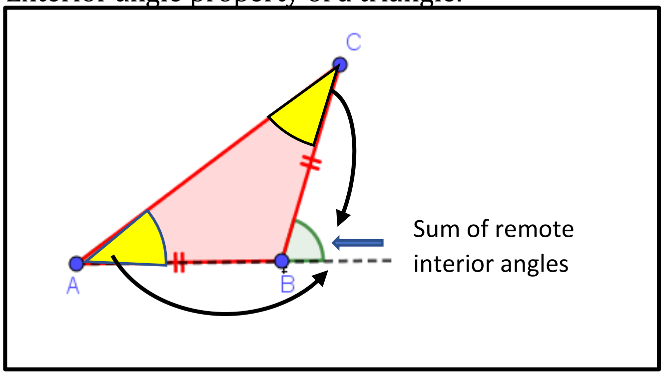
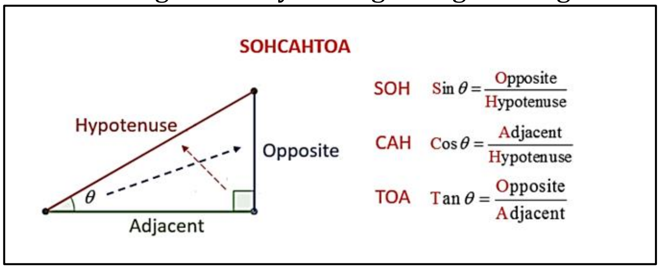
Investigate Double Angle Identities (Sine and Cosine)

Pre-Required Knowledge:

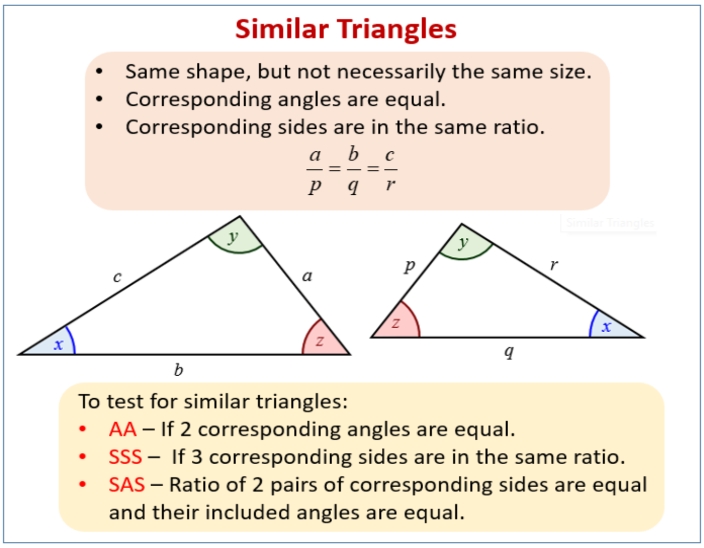
- Exterior angle property of a triangle.



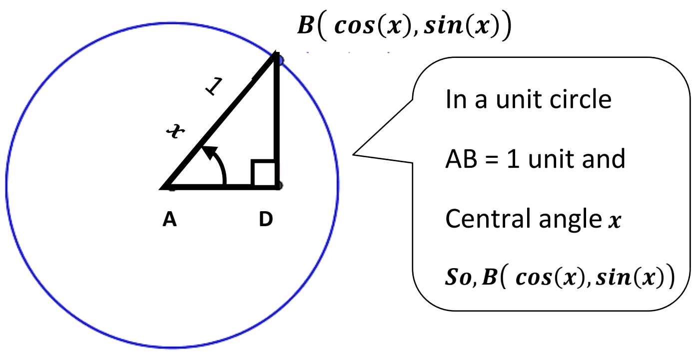
- Basics of Trigonometry in a right-angle triangle.



- Pythagoras theorem
- Similarity of triangles.



- Conversion of coordinates into sine and cosine with given angle using unit circle.



Objectives:

Big Idea:
Learners will investigate the sine and cosine double angle relationships and derive three or four identities.

During the Activity:
Learners will

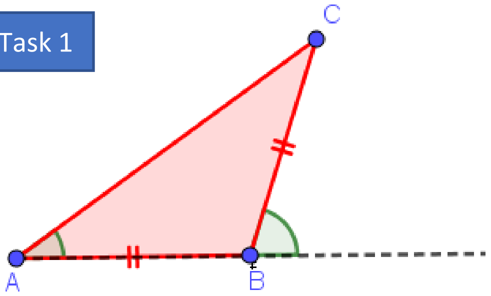
- Apply exterior angle property to see the relationship between α *and* β.
  Student´s discovery:
  $$\alpha = 2\,\beta$$
- Apply SOHCOHTOA to convert sides in terms of α *and* β.
  Student´s discovery:
  from ΔDEF,
  $$DF = \sin(\alpha),$$
  $$EF = \cos(\alpha)$$
  from ΔCGE
  $$CG = \cos(\,\beta)$$
  $$GE = \sin(\,\beta)$$
- Apply Pythagoras theorem in one triangle then in the other (big) to come up with one identity of cos $(2\,\beta)$
  Student´s discovery:
  from ΔDCF,
  $$DC^2 = DF^2 + CF^2$$
  where:
  $$DC = 2\text{CG} = 2\cos(\beta)\ \ \&\ CF = 1 + \cos\,(\alpha)$$
  Simplify to get
  $$\cos(\alpha = 2\beta) = 2\cos^2(\beta) - 1$$
- Apply similarity of triangles and use previous results to derive sin $(2\,\beta)$ identity.

  Student´s discovery:

  $\Delta DCF \sim \Delta ECG, which\ means$
  $$\frac{DC}{EC} = \frac{DF}{EG}$$
  Simplify to get
  $$\sin(\alpha = 2\beta) = 2\cos(\beta)\ sin\beta$$
  $$\cos(\alpha = 2\beta) = 1 - 2\sin^2(\beta)$$
  Or Generalize to
  $$\cos(2x) = \cos^2(x) - \sin^2(x)$$

Starter for the Day:

Task 1



If AB= BC and Angle BAC = x
then What is exterior angle of ABC

Any relationship between angles BAC and BCA ?

Is it always true for all triangles?

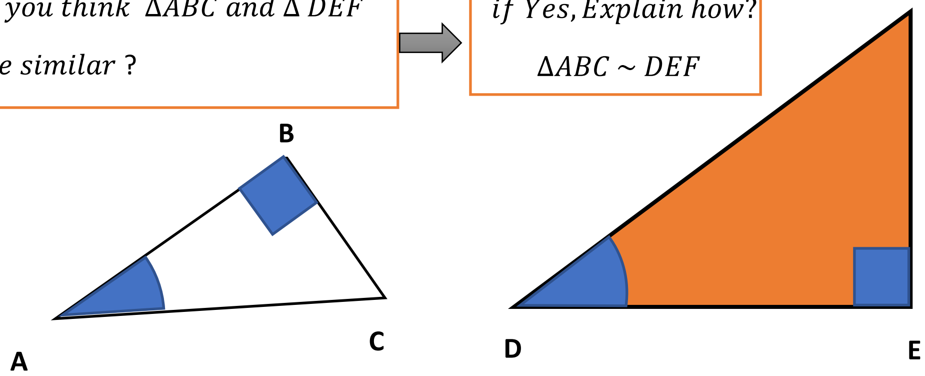
In this special case AB= BC what is the relationship between exterior angle of ABC and angle BAC ?

Do you think ΔABC and $\Delta\ DEF$ are similar ?

$if\ Yes, Explain\ how?$

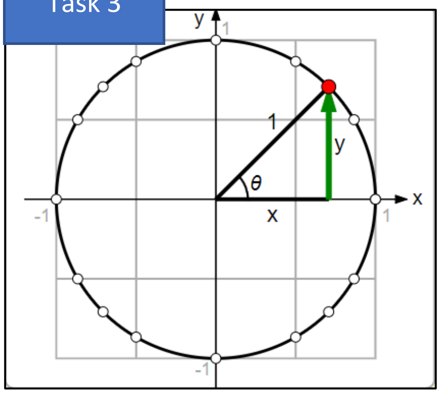
$\Delta ABC \sim DEF$

Task 2



$what\ can\ be\ the$ $outcome\ if\ \ \Delta ABC \sim DEF$

Task 3



Is there any relationship between x, y and 1

Can you write x and y in terms of θ

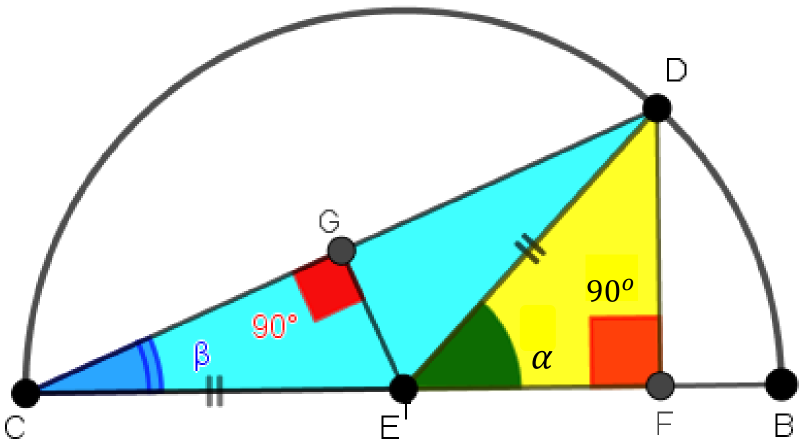
$x =$

$y =$

Your Notes or Any Question:

Investigation:

Geogebra online link for the Activity: Online Investigation



A semicircle is given in the diagram whose centre is at E and radius CE=ED= 1 unit

ΔDFE and ΔCGE are two right angled triangles.

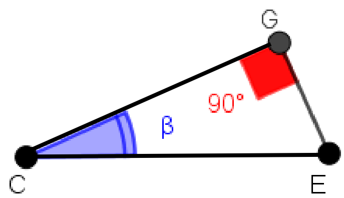
Task 1

Apply exterior angle property in ΔCED to work out the relationship between α and β

Your Outcome:

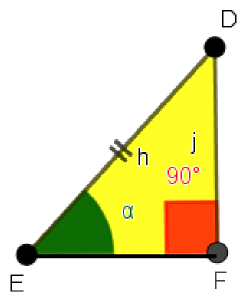
Task 2

$a)$ Apply SOHCAHTOA in ΔCGE to work out the lengths of CG and CE in terms of β



Your Outcome: CE= CG=

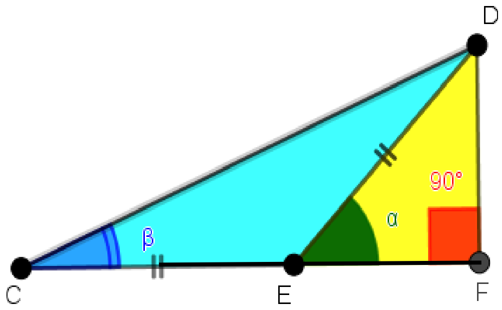
$b)$: Apply SOHCAHTOA in ΔDEF to work outh the lengths of DF and EF in terms of α



Your Outcome: DF= EF=

Task 3

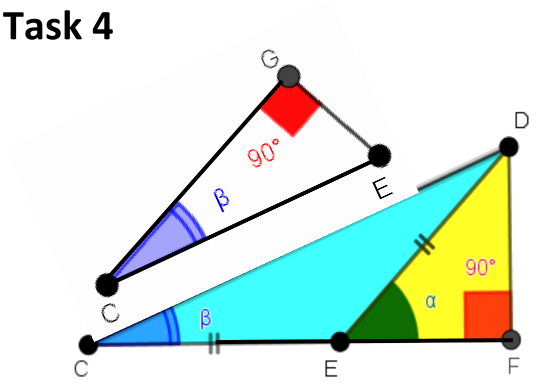
Apply Pythagoras theorem in ΔCDF *to investigate the relationship between* $\cos(\alpha)$ *and* $\cos(\beta)$



Your Outcome: $\cos(\alpha = 2\beta) =$

Task 4

Apply similarity in ΔDCF *and* ΔECG *to investigate the relationship between* $\sin(\alpha)$ *and* $\cos(\beta)$



Your Outcome: $\sin(\alpha = 2\beta) =$

Task 5

As you know $\alpha = 2(\beta)$, $if\ you\ consider\ \beta = x\ then\ \alpha = 2x$

$a)\ using\ the\ results\ of\ task\ 4\ rewrite\ the\ relationship\ in\ terms\ of\ x$

$\sin(2x) =$

$b)\ using\ the\ results\ of\ task\ 3\ rewrite\ the\ relationship\ in\ terms\ of\ x$

$\cos(2x) =$ $-----(\ in\ terms\ of\ cosine)$

$\cos(2x) =$ $------(\ in\ terms\ of\ sine)$

$\cos(2x) =$ $---(in\ terms\ of\ cosine\ and\ sine)$

Task 6:

$$Verify\ above\ four\ identites\ with\ x = 30^o\ or\ \frac{\pi}{6}$$

Extension: Task 7— — $tan(x) = \frac{\sin(x)}{\cos(x)}$	
Open ended	**Structured**
Establish a relationship between $tan(\alpha)$ *and* $tan(\beta)$	$if \quad tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)}$ 1. Use task 3 and 4 to rewrite $\sin(\alpha)$ *and* $\cos(\alpha)$ *in terms of* β $tan(\alpha) =$ ______ 2. *convert everything in terms of* $\tan(\beta)$

Can you rewrite first and last identity (task 5) in terms of tan(x),

$$\sin(2x) = 2\sin(x)\cos(x) = \frac{2\sin(x)\cos(x)}{1} = \frac{2\sin(x)\cos(x)}{\sin^2 x + \cos^2 x} = \frac{2\sin(x)\cos(x) / \cos^2 x}{\sin^2 x + \cos^2 x / \cos^2 x}$$

$\sin(2x) =$

$$\cos(2x) = \cos^2 x - sin^2 x = \frac{\cos^2 x - \sin^2 x}{1} =$$

$\cos(2x) =$