Sección 1,5

(27)

$$(x + ye^{y})\frac{dy}{dx} = 1$$

$$x + ye^{y} = \frac{dx}{dy}$$

$$ye^{y} = \frac{dx}{dy} - x$$

$$e^{-y} ye^{y} = e^{-y}\frac{dx}{dy} - e^{-y}x$$

$$y = \frac{d(e^{-y}x)}{dy}$$

$$\int y\,dy = \int d(e^{-y}x)$$

$$\frac{y^2}{2} = e^{-y}x + c$$

$$x = \left[e^{y}\frac{y}{2} - e^{y}c\right]$$

$$Q(y) = \frac{dx}{dy} + P(y)x$$

$$Q(y) = ye^{y}$$

$$P(y) = -1$$

$$f(y) = e^{\int P(y)dy} = e^{-y}$$

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$$\frac{dy}{dx} = 1 + 2xy \qquad Q(x) = \frac{dy}{dx} + P(x)\,y$$

$$1 = \frac{dy}{dx} - 2xy \qquad Q(x) = 1$$

$$P(x) = -2x$$

$$e^{-x^2} = e^{-x^2}\frac{dy}{dx} - e^{-x^2}2xy \qquad \rho(x) = e^{\int P(x)dx}$$

$$= e^{-x^2}$$

$$e^{-x^2} = \frac{d(e^{-x^2}y)}{dx}$$

$$\int e^{-x^2}dx = \int d(e^{-x^2}y) \qquad \frac{\sqrt{\pi}}{2}\text{erf}(x) = \frac{2}{\sqrt{\pi}}\int e^{-x^2}dx \,\frac{\sqrt{\pi}}{2}$$

$$\int e^{-x^2}dx = e^{-x^2}y + c$$

$$y = e^{x^2}\left[\int e^{-x^2}dx - c\right] \qquad \boxed{y = e^{x^2}\left[\frac{\sqrt{\pi}}{2}\text{erf}(x) - c\right]}$$

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$$2x\frac{dy}{dx} = y + 2x\cos(x) \qquad Q(x) = \frac{dy}{dx} + P(x)\,y$$

$$\frac{dy}{dx} = \frac{y}{2x} + \cos(x) \qquad Q(x) = \cos(x)$$

$$\cos(x) = \frac{dy}{dx} - \frac{y}{2x} \qquad P(x) = -\frac{1}{2x}$$

$$\rho(x) = e^{\int P(x)dx}$$

$$x^{-1/2}\cos(x) = x^{-1/2}\frac{dy}{dx} - x^{-1/2}\frac{y}{2x} \qquad = e^{-\frac{1}{2}\ln x} = x^{-1/2}$$

$$x^{-1/2}\cos(x) = \frac{d(y\,x^{-1/2})}{dx}$$

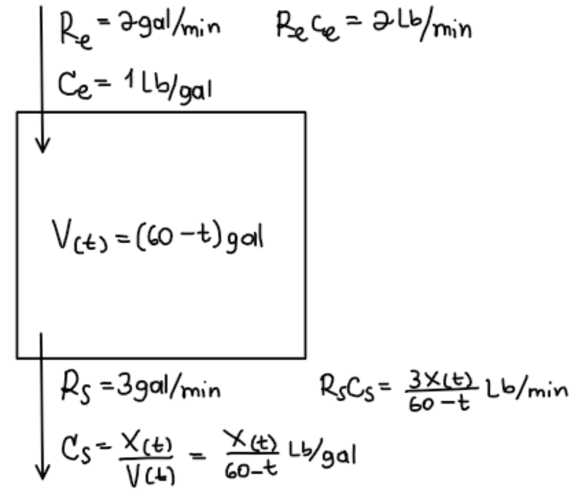
$$\int x^{-1/2}\cos(x)\,dx = \int d(y\,x^{-1/2})$$

$$\boxed{y = x^{1/2}\left[\int x^{-1/2}\cos(x)\,dx\right]}$$

(36)

$X_{(t)} \longrightarrow$ Cantidad de sal en t minutos

$X_{(0)}=0$



$$\frac{dx}{dt} = R_eC_e - R_sC_s = 2\,Lb/min - \frac{3X(t)}{60-t}\,Lb/min$$

$$\frac{dx}{dt} + \frac{3x}{60-t} = 2$$

$Q_{(t)} = 2$

$P_{(t)} = \frac{3}{60-t}$

$\rho_{(t)} = e^{\int P(t)dt} = e^{-3\ln|60-t|} = (60-t)^{-3}$

$$\frac{dx}{dt}(60-t)^{-3} + \frac{3x}{60-t}(60-t)^{-3} = 2(60-t)^{-3}$$

$$\frac{d(x(60-t)^{-3})}{dt} = 2(60-t)^{-3}$$

$$\int d(x(60-t)^{-3}) = \int 2(60-t)^{-3}dt$$

$$x(60-t)^{-3} = (60-t)^{-2} + C$$

$$x = \frac{(60-t)^{-2} + C}{(60-t)^{-3}}$$

$X = (60-t) + (60-t)^3 C$ [PVI(0,0)]

$0 = (60-0) + (60-0)^3 C$

$$C = -\frac{60}{60^3} = -\frac{1}{60^2}$$

(a) $X = (60-t) - (60-t)^3 \frac{1}{60^2}$

(b) $\frac{dX}{dt} = -1 + \frac{3}{60^2}(60-t)^2$

$$\frac{dx}{dt} = -1 + 3\left(1 - \frac{t}{60}\right)^2$$

$$0 = -1 + 3\left(1 - \frac{t}{60}\right)^2$$

$$\sqrt{\frac{1}{3}} = \sqrt{\left(1 - \frac{t}{60}\right)^2}$$

$$\frac{t}{60} = 1 - \sqrt{\frac{1}{3}}$$

$$t = 60 - 60\sqrt{\frac{1}{3}} = 60 \pm 20\sqrt{3}$$

$$X = (60 - 60 \pm 20\sqrt{3}) - (60 - 60 \pm 20\sqrt{3})^3 \frac{1}{60^2}$$

$X = 23{,}09$ Lb Sal

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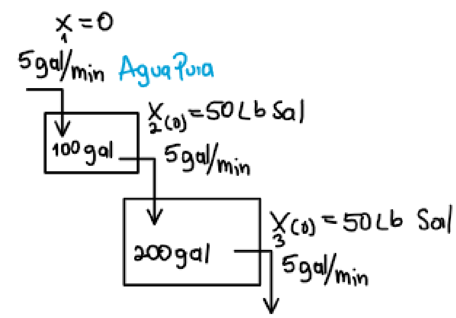

$$\frac{dx_2}{dt} = \frac{5gal}{min}(0) - \frac{5gal}{min}\,\frac{x_2}{100gal} = \frac{-x_2}{20\,min}$$

$$-\int \frac{dx_2}{x_2} = \frac{1}{20}\int dt$$

$$-\ln(x_2) = \frac{t}{20} + C \qquad (0,50) \qquad x_2 = e^{-\frac{t}{20}}\,50$$

$$-\ln(50) = \frac{0}{20} + C \qquad C = -3{,}912$$

$$\frac{dx_3}{dt} = \frac{5gal}{min}\,\frac{x_2}{100gal} - \frac{5gal}{min}\,\frac{x_3}{200gal} = \frac{50}{20}\,\frac{e^{-\frac{t}{20}}}{min} - \frac{x_3}{40min}$$

$$\frac{dx_3}{dt} + \frac{x_3}{40min} = \frac{5}{2}\,\frac{e^{-t/20}}{min}$$

$$Q_{(t)} = \frac{5}{2}\,\frac{e^{-t/20}}{min}$$

$$P_{(t)} = \frac{1}{40}$$

$$f_{(t)} = e^{\int P(t)dt} = e^{t/40}$$

$$\frac{dx_3}{dt}e^{t/40} + e^{t/40}\frac{x_3}{40min} = \frac{5}{2}\,\frac{e^{-t/20}}{min}\,e^{t/40}$$

$$\frac{d(x_3 e^{t/40})}{dx} = \frac{5}{2}\,\frac{e^{-t/20}}{min}\,e^{t/40}$$

$$\int d(x_3 e^{t/40}) = \int \frac{5}{2} e^{-t/40} dt$$

$$x_3 e^{t/40} = \frac{5}{2}(-40 e^{-t/40}) + C$$

$$x_3 = \frac{-100 e^{-t/40}}{e^{t/40}} + \frac{C}{e^{t/40}}$$

$$x_3 = -100 e^{-t/20} + Ce^{-t/40} \qquad \begin{matrix} t & x_3 \\ (0, & 50) \end{matrix} \qquad C = 150$$

$$x_3 = 150\,e^{-t/40} - 100 e^{-t/20}$$

$$\frac{dx_3}{dt} = -\frac{150}{40}e^{-t/40} + \frac{100}{20}e^{-t/20} = 0$$

$$\ln\left|5 e^{-t/20}\right| = \ln\left|\frac{15}{4} e^{-t/40}\right|$$

$$\ln|5| - \frac{t}{20} = \ln\left|\frac{15}{4}\right| - \frac{t}{40}$$

$$\ln|5| - \ln\left|\frac{15}{4}\right| = -\frac{t}{40} + \frac{2t}{40}$$

$$0{,}29 \times 40 = t = 11{,}6$$

$$x_3 = 150\,e^{-11{,}6/40} - 100 e^{-11{,}6/20}$$

$$x_3 = 112{,}24 - 55{,}99 = 56{,}25\ Lb$$

(44)

$2x\,Sen(y)\,Cos(y)\,\frac{dy}{dx}\,dx = (4x^2 + Sen^2(y))\,dx$

$(-4x^2 - Sen^2(y))\,dx + 2x\,Sen(y)\,Cos(y)\,dy = 0$

$M = -4x^2 - Sen^2(y)$ $\quad$ $N = 2x\,Sen(y)\,Cos(y)$

$M_y = -2Sen(y)\,Cos(y)$ $\quad$ $N_x = 2Sen(y)\,Cos(y)$

$x^{-2}(-4x^2 - Sen^2(y))\,dx + x^{-2}\,2x\,Sen(y)\,Cos(y)\,dy = 0$

$(-4 - Sen^2(y)\,x^{-2})\,dx + \frac{2}{x}\,Sen(y)\,Cos(y)\,dy = 0$

$M = -4 - Sen^2(y)\,x^{-2}$ $\quad$ $N = \frac{2}{x}\,Sen(y)\,Cos(y)$

$M_y = -2x^{-2}\,Sen(y)\,Cos(y)$ $\quad$ $N_x = -2x^{-2}\,Sen(y)\,Cos(y)$

$M\,dx + N\,dy = 0$ $\quad$ $M_y = N_x$

$\mu(x) = e^{\int \frac{M_y - N_x}{N}dx} = e^{\int \frac{-2Sen(y)Cos(y) - 2Sen(y)Cos(y)}{2x\,Sen(y)Cos(y)}dx}$

$\mu(x) = e^{\int \frac{-4Sen(y)Cos(y)}{2x\,Sen(y)Cos(y)}dx} = e^{-2\ln|x|} = x^{-2}$

$F(x,y) = \int M\,\partial x = c$

$F(x,y) = \int -4 - Sen^2(y)\,x^{-2}\,\partial x$

$F(x,y) = -4x + \frac{Sen^2(y)}{x} + \varphi(y)$

$\frac{\partial F}{\partial y} = \frac{2}{x}Sen(y)\,Cos(y) + \varphi'(y)$

$\varphi'(y) = N - \frac{2}{x}Sen(y)\,Cos(y)$

$\varphi(y) = \int \frac{2}{x}Sen(y)\,Cos(y) - \frac{2}{x}Sen(y)\,Cos(y)$

$\varphi(y) = C_1$

$K = -4x + \frac{Sen^2(y)}{x}$

$Kx + 4x^2 = Sen^2(y)$

Sección 1,6
(29)

$2x\,Sen(y)\,Cos(y)\,\frac{dy}{dx}\,dx = (4x^2 + Sen^2(y))\,dx$

$(-4x^2 - Sen^2(y))\,dx + 2x\,Sen(y)\,Cos(y)\,dy = 0$

$M = -4x^2 - Sen^2(y)$ $\quad$ $N = 2x\,Sen(y)\,Cos(y)$

$M_y = -2Sen(y)\,Cos(y)$ $\quad$ $N_x = 2Sen(y)\,Cos(y)$

$x^{-2}(-4x^2 - Sen^2(y))\,dx + x^{-2}\,2x\,Sen(y)\,Cos(y)\,dy = 0$

$(-4 - Sen^2(y)\,x^{-2})\,dx + \frac{2}{x}\,Sen(y)\,Cos(y)\,dy = 0$

$M = -4 - Sen^2(y)\,x^{-2}$ $\quad$ $N = \frac{2}{x}\,Sen(y)\,Cos(y)$

$M_y = -2x^{-2}\,Sen(y)\,Cos(y)$ $\quad$ $N_x = -2x^{-2}\,Sen(y)\,Cos(y)$

$M\,dx + N\,dy = 0$ $\quad$ $M_y = N_x$

$\mu(x) = e^{\int \frac{M_y - N_x}{N}dx} = e^{\int \frac{-2Sen(y)Cos(y) - 2Sen(y)Cos(y)}{2x\,Sen(y)Cos(y)}dx}$

$\mu(x) = e^{\int \frac{-4Sen(y)Cos(y)}{2x\,Sen(y)Cos(y)}dx} = e^{-2\ln|x|} = x^{-2}$

$F(x,y) = \int M\,\partial x = c$

$F(x,y) = \int -4 - Sen^2(y)\,x^{-2}\,\partial x$

$F(x,y) = -4x + \frac{Sen^2(y)}{x} + \varphi(y)$

$\frac{\partial F}{\partial y} = \frac{2}{x}Sen(y)\,Cos(y) + \varphi'(y)$

$\varphi'(y) = N - \frac{2}{x}Sen(y)\,Cos(y)$

$\varphi(y) = \int \frac{2}{x}Sen(y)\,Cos(y) - \frac{2}{x}Sen(y)\,Cos(y)$

$\varphi(y) = C_1$

$K = -4x + \frac{Sen^2(y)}{x}$

$Kx + 4x^2 = Sen^2(y)$

(30)

$$(x+e^y)\frac{dy}{dx} = xe^{-y} - 1\,\frac{e^{-y}}{e^{-y}}$$

$$(x+e^y)\frac{dy}{dx}\,dx = e^{-y}(x-e^y)\,dx$$

$$\left(\frac{x}{e^{-y}} + \frac{e^y}{e^{-y}}\right)dy = (x-e^y)\,dx$$

$$(xe^y + e^{2y})\,dy = (x-e^y)\,dx$$

$$(x-e^y)\,dx + (-xe^y - e^{2y})\,dy = 0$$

$$M\,dx + N\,dy = 0$$

$$M_y = N_x \qquad F(x,y) = C$$

$$M = x - e^y \qquad N = -xe^y - e^{2y}$$

$$M_y = -e^y \qquad N_x = -e^y$$

$$M = \frac{\partial F}{\partial x} \qquad \int M\,\partial x = \int \partial F$$

$$F = \int x - e^y\,\partial x$$

$$F = \frac{x^2}{2} - e^y x + \varphi(y) = \frac{x^2}{2} - e^y x - \frac{e^{2y}}{2} + C$$

$$\boxed{K = x^2 - 2e^y x - e^{2y}}$$

$$\frac{\partial F}{\partial y} = -xe^y + \varphi'(y)$$

$$\varphi'(y) = N + xe^y$$

$$\varphi(y) = \int -xe^y - e^{2y} + xe^y\,dy$$

$$\varphi(y) = -\frac{e^{2y}}{2} + C$$

(46)

$$xy'' + y' = 4x$$

$$x\frac{dP}{dx} + P = 4x$$

$$\frac{d(Px)}{dx} = 4x$$

$$\int d(Px) = \int 4x\,dx$$

$$Px = 4\frac{x^2}{2} + C \quad (4/2 \to 2)$$

$$P = \frac{dy}{dx} = \frac{2x^2}{x} + \frac{C}{x}$$

$$\int dy = \int\left(2x + \frac{C}{x}\right)dx$$

$$\boxed{y = 2\frac{x^2}{2} + C\ln|x| + C_1}$$

$$y' = \frac{dy}{dx} = P$$

$$y'' = \frac{dy'}{dx} = \frac{dP}{dx}$$

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$$yy'' + (y')^2 = yy'$$

$$yp\frac{dp}{dy} + p^2 = yp$$

$$y\frac{p}{p}\frac{dp}{dy} + \frac{p^2}{p} = y\frac{p}{p}$$

$$\frac{d(yp)}{dy} = y$$

$$\int d(yp) = \int y\,dy$$

$$yp = y\frac{dy}{dx} = \frac{y^2}{2} + C_1$$

$$\int \frac{y}{y^2/2 + C_1}dy = \int dx$$

$$\int \frac{du}{u} = \int dx \qquad u = \frac{y^2}{2} + C_1, \quad du = y\,dy$$

$$\ln\left|\frac{y^2}{2} + C_1\right| = x + C_2$$

$$\frac{y^2}{2} + C_1 = e^x C_2$$

$$\boxed{\sqrt{y^2} = \sqrt{2(e^x C_2 - C_1)}}$$

(50)

$$y'' = (x + y')^2$$

$$\frac{dp}{dx} = (x + p)^2$$

$$\frac{dp}{dx} = v^2 \qquad v = x + p, \quad \frac{dv}{dx} = \frac{dx}{dx} + \frac{dp}{dx}$$

$$\frac{dv}{dx} = v^2 + 1$$

$$\int \frac{dv}{v^2+1} = \int dx$$

$$\tan^{-1}(v) = x + c$$

$$v = \tan(x+c)$$

$$x + p = \tan(x+c)$$

$$\frac{dy}{dx} = \tan(x+c) - x$$

$$\int dy = \int \tan(x+c) - x\,dx$$

$$\boxed{y = -\ln|\cos(x+c)| - \frac{x^2}{2} + C_1}$$