## Task X: Construction of the Sierpinski triangle

Fractals are infinitely patterns that are self-similar across different scales. To construct a dynamic worksheet to create the first few patterns of the Sierpinski triangle.


Basic strategy for the creation:

1. Contract the present pattern.

2. Copy and translate the contracted pattern to create the next pattern.


| Steps | Objects to be created | Action |
| :---: | :---: | :---: |
| 1. | Slider n | - Select " $\stackrel{\square=2}{\bullet}$ Slider" button and click on the graphics window <br> - Set the name of the slider as $n ; \min =1 ; \max =3$; increment $=1$ <br> - Click "OK" |
| 2. | An equilateral triangle with length 1 (the 1st pattern) | - Type the following command in the input bar: <br> $\mathbf{P 1}=\operatorname{Polyg} \mathrm{n}(\mathbf{( 0 , 0}),(1,0), 3)$ <br> - In the Advanced Tab of the properties of polygon P1, input " $\mathrm{n}=1$ " as the condition to show object. |
| 3. | 3 translation vectors | - Type the following command in the input bar: $\mathrm{TV}=\{(0,0),(1 / 4, \operatorname{sqrt}(3) / 4),(1 / 2,0)\}$ <br> - Remark: <br> The points in TV correspond to the positions of the translation of the contracted patterns in the next few steps |


| 4. | The 2nd pattern | - Type the following command in the input bar: <br> P2 = Flatten(Sequence(Translate(Dilate(P1, $1 / 2$ ), $\operatorname{Element}($ PT, i)), i, 1, 3)) <br> - Remark: <br> Several functions are combined to create the next pattern: <br> - Dilate: enlarges or contracts an object <br> - Translate: translates an object by a vector <br> - Element: yields an element in a list <br> - Sequence: creates a list of objects followed by an index <br> - Flatten: combines all lists into one list <br> - In the Advanced Tab of the properties of polygon P 2 , input " $\mathrm{n}=2$ " as the condition to show object. |
| :---: | :---: | :---: |
| 5. | The 3rd pattern | - Type the following command in the input bar to create the 3rd pattern: <br> P3 = Flatten(Sequence(Translate(Dilate(P2, $1 / 2$ ), Element(PT, i)), i, 1, 3)) <br> - In the Advanced Tab of the properties of polygon P3, input " $\mathrm{n}=3$ " as the condition to show object. |
| 6. | Checking of the first 3 patterns | - Move the point on the slider of $n$ to check that the first 3 patterns can be shown correctly. |
| 7. | Adjust the max of slider of $n$ | - Change the max of slider of n to 6 . |
| 8. | The 4th, 5th and 6th pattern | - Type the following command in the input bar to create the 4th, 5th and 6th pattern respectively: P4 = Flatten(Sequence(Translate(Dilate(P3, $1 / 2$ ), $\operatorname{Element}($ PT, i) ), i, 1, 3)) <br> P5 = Flatten(Sequence(Translate(Dilate(P4, $1 / 2$ ), $\operatorname{Element}($ PT, i)), i, 1, 3)) <br> P6 = Flatten(Sequence(Translate(Dilate(P5, $1 / 2$ ), $\operatorname{Element}(\mathbf{P T}, ~ i)$ ), i, 1, 3)) <br> - In the Advanced Tab of the properties of polygon P4, P5 and P6, input " $n=4$ ", " $n=5$ " and " $n=6$ " respectively as the conditions to show objects. |

## Exercise:

Try to construct a GeoGebra file that can create the Sierpinski carpet.


