202404中考数学二模卷普陀区参考答案

一、选择题：(本大题共6题，每题4分、满分24分)

1. $(\mathbf{D})$　2. $(\mathbf{C})$　3. $(\mathbf{B})$　4. $(\mathbf{A})$　5. $(\mathbf{D})$　6. $(\mathbf{B})$

二、填空题：(本大题共 12 题，每题4分，满分48分)

7、$9a^6$;　8、$-3(\sqrt{2}+1)$;　9、$-2<x<\frac{1}{2}$;　10、$k<1$;　11、150°;　12、$\frac{3}{4}$;
13、$-\frac{3}{2}$;　14、$(2,3),A(m,n),B(m+4,n+6),A+B=(0,0)$;　15、$27,40:15=72:x$;　16、$\frac{4}{3}\vec{a}+\frac{2}{3}\vec{b}$;　$\overrightarrow{BE}=2\overrightarrow{EC}=2\overrightarrow{AD}=2\vec{a}$

$\overrightarrow{AE}=\overrightarrow{AB}+\overrightarrow{BE}=\vec{b}+2\vec{a}$

$\because AD//BC \therefore \Delta ADF\sim\Delta EBF$

$\therefore AD:BE=AF:FE, AD=EC, \therefore AF:FE=1:2$

$\therefore FE=\frac{2}{3}AE$

$\therefore \overrightarrow{FE}=\frac{2}{3}(\vec{b}+2\vec{a})=\frac{4}{3}\vec{a}+\frac{2}{3}\vec{b}$

三、解答题：(本大题共7题，满分78分)

17、解:(1a) 如图1, E 在线段 BC 上时, 易证 $\Delta AEC\sim\Delta FEA \impliedby \angle AEC=\angle FEA,\angle ACE=\angle FAE=45^\circ$

$\therefore AE:FE=EC:EA(=AC:FA)\ \sqrt{17}:(3+x)=3:\sqrt{17},x=\boxed{\frac{8}{3}}$

(1b) 如图2, E 在线段 CB 的延长线上时, 同上易证

$\Delta AEF\sim\Delta CEA \impliedby \angle E=\angle E,\angle EAF=\angle ECA=45^\circ$

$\therefore \frac{AE}{CE}=\frac{FE}{AE},AE^2=CE\cdot FE,17=5\times(5-x),x=\boxed{\frac{8}{5}}$

(2a) 题目没有标识 $ABCD$ 顺序, 可能还可以是顺时针 $ABCD$, 故还有两个解. E 在B 的左侧:

$\Delta ACF\sim\Delta EAF,AC:EA=CF:AF=AF:EF$

$AF^2=CF\cdot EF,AF^2=AB^2+BF^2$

设$BF=x$, 则 $(4+x)(1+x)=4^2+x^2,x=\frac{12}{5},CF=\boxed{\frac{27}{5}}$;

(2b) E 在B 的右侧:

$\Delta AEF\sim\Delta CAF,AE:CA=EF:AF=AF:CF$

$AF^2=CF\cdot EF,AF^2=AB^2+BF^2$

设$EF=x$, 则 $(x+5)x=4^2+(1+x)^2,x=\frac{17}{3},CF=\boxed{\frac{32}{3}}$;

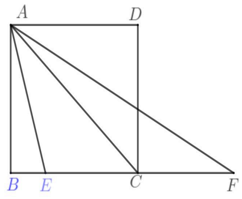


图1

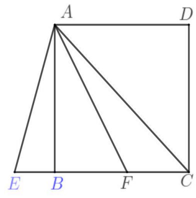


图2

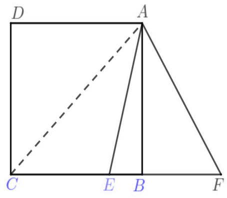


图3

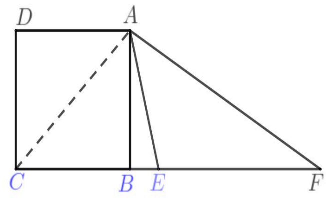


图4

18、解: 作 $AH \perp BC$, 交点垂足为 H, 设 $BD = x$, $\because \cos B = BH/AB, AB = 5 \therefore$ $AH = 3, BH = 4, AB'$ 与 BC 交于点 K,

$\because \angle B = \angle C = \angle DB'A \therefore A, D, B', C$ 四点共圆;

由**托勒密定理**可得 $AB' \cdot CD = AD \cdot B'C + DB' \cdot AC$

$5(8-x) = 5x + 2AD, AD^2 = AH^2 + HD^2 = 9 + (4-x)^2$

$\therefore 20 - 5x = AD, 5^2(4-x)^2 = 9 + (4-x)^2$

$|4-x| = \dfrac{\sqrt{6}}{4}, x = 4 \pm \dfrac{\sqrt{6}}{4}$ 其实两个解分别对应了D在H的左右两旁, 各有一个位置满足. 由于题目中要求AB'与AB相交,故只有一个解满足, $BD = 4 - \dfrac{\sqrt{6}}{4}$

解法二. 如果不用竞赛的托勒密定理, 用已知了三边的三角形ABC作为桥梁, 解三角形ADB.

$AB = AB' = AC = 5, B'C = 2,$

$\therefore \cos\angle AB'C = \dfrac{1}{5}, \sin\angle AB'C = \dfrac{2\sqrt{6}}{5}, \tan\angle AB'C = 2\sqrt{6},$

$\because \angle AB'C = \angle ADB', \therefore \tan\angle ADB' = AH : DH = 2\sqrt{6}$

$\therefore 3 = 2\sqrt{6}(4-x) \implies x = 4 - \dfrac{\sqrt{6}}{4};$

当 $BD = x > 4$ 时, $\because \angle ADC + \angle AB'C = 180^\circ, \therefore \tan\angle ADC = -\tan\angle AB'C = -2\sqrt{6}$

$\therefore 3 = -2\sqrt{6}(4-x) \implies x = 4 + \dfrac{\sqrt{6}}{4}$(此时线段AB'与BC不相交,舍去)

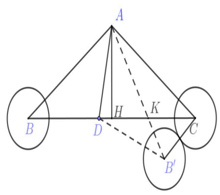


图1

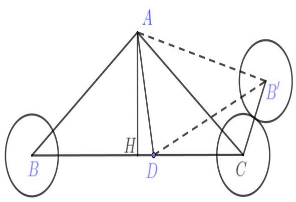


图2

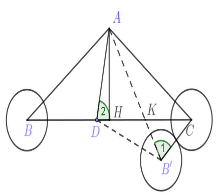


图3

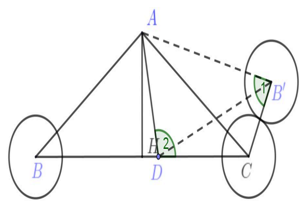


图4

19、解: $-4 + 2\sqrt{2} + 4^2 - (2 + \sqrt{2}) = 10 + \sqrt{2}$

20、解: $6x + x(x-3) = 2(x^2 - 9), x_1 = 6, x_2 = -3$(增根,舍去)

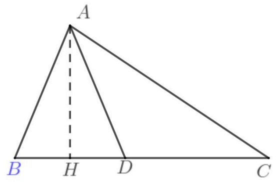
21、解: $\because AB = AD \therefore \angle B = \angle ADB = 2\angle C$

$\because \angle ADB = \angle C + \angle CAD \therefore \angle C = \angle CAD, AD = DC = 13;$

(1) $\therefore BD = 10;$

(2) 过点 A 作 $AH \perp BC$, 垂足为点H, 则 $BH = HD = 5$ (等腰三角形三线合一)

$\therefore AH=\sqrt{AB^2-BH^2}=12, \therefore \tan\angle C=\frac{AH}{HC}=\frac{12}{5+13}=\frac{2}{3}$



22、解: 乙平台上小王的月工资单信息,可以算出月送单天数,设为 x 天. 则有

$(50+6\times 61)x-32\times 10=8832 \implies x=22$(天)

小张在甲外面平台日均送单数为 $\frac{2\times(54+58+65)+3\times 65+6\times 60}{2\times 3+3+6}=60$(单)

月违规送单数为 $\frac{10+11+15+2\times(9+13+14)+3\times 12}{1\times 3+2\times 3+3\times 1}=12$(单)

由以上信息计算

(1) 小张在甲平台的税前月工资收入为

$(70+5.5\times 60)\times 22-10\times 12=8680$(元)

(2) 小张在乙平台的税前月工资收入为

$(50+6\times 60)\times 22-32\times 12=8636$(元)

结论: 因为 $8680>8636$, 所以不用跳槽.

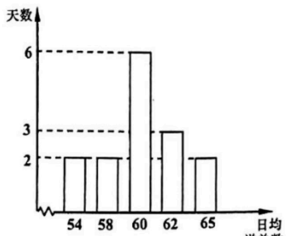


图 6-1

第22题图1

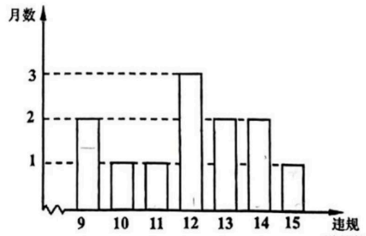


图 6-2

第22题图2

23、(1)证明: $\because AB//CD, \therefore \triangle AFE\sim\triangle DCE$(横8字型)

$\therefore FA:CD=AE:ED$, 且 $FA:AB=AE:ED$(已知)

$\therefore AB=CD$, 且 $AB//CD$(已知)

$\therefore ABCD$是平行四边形(一组对边平行且相等的四边形是平行四边形)

(2)证明: 在$\triangle FCD$ 和 $\triangle FGC$ 中:

$FC:FG=FD:FC$(已知), $\angle CFD=\angle GFC$(公共角)

$\therefore \triangle FCD\sim\triangle FGC \quad (\mathrm{SAS})$

$\therefore \angle FCG=\angle G;$

$\because AF//CD \therefore \angle BFC=\angle FCD=\angle G$

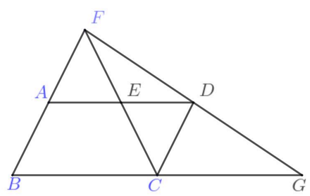
$\because AB//CD \therefore \angle B=\angle DCG$

$\therefore \triangle BFC\sim\triangle OGD \quad (\mathrm{AAA})$

$\therefore BF:BC=CG:CD$

平行四边形ABCD中, $AD=BC$

$\therefore BF\cdot CD=BC\cdot CD=AD\cdot CD.$



24、解:(1) $\because P(4,3), \angle APB=90^\circ$, 由抛物线的对称性可知 $A(4-3,0), B(4+3,0)$, 可设抛物线解析式为 $y=a(x-1)(x-7)$, 顶点坐标代入得到

$$y=-\frac{1}{3}(x-1)(x-7)$$

(2) $A(m-n,0)$代入方程得到

$$0=a(m-n-m)^2+n, an^2+n=0, \because n\neq 0 \therefore an=-1, n=-\frac{1}{a}\ (a\neq 0)$$

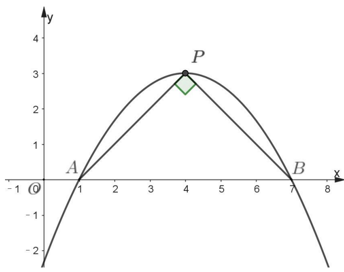
(3) 直线 $y=x+n/2$ 交x轴于点 $J(-n/2,0)=(-1/(2a),0)$, 做$PI\perp MN$, 交点为 I, $P(m,n)=P(2n,n)=(-2/a,-1/a), A(m-n,0)=(n,0)=(-1/a,0), B(m+n,0)=(3n,0)=(-3/a,0)$

$AP=\sqrt{2}n=-\sqrt{2}/a, AB=2n=-2/a, JB=3n+n/2=7n/2=-7/(2a)$ $y=x+n/2$ 与 AP 直线的斜率相同, 都是1, 故 MN 与x 轴正方向的夹角为 45°,

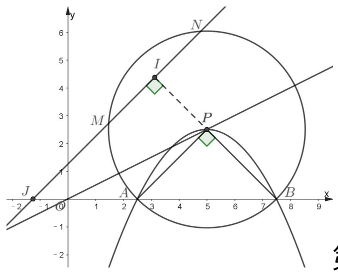
故ΔJBI 为等腰直角三角形, $AP//IJ \implies AP:IJ=AB:JB, r=PA, r:(r+PI)=2n:7n/2=4:7, r:PI=4:3, PI=\frac{3}{4}r, PM=r, MI=\sqrt{MP^2-PI^2}$

$$MI=\frac{\sqrt{7}r}{4}, MN=2MI=\frac{\sqrt{7}r}{2}=-\frac{\sqrt{14}}{2a}$$

或者用比例式 $AB:JA=PB:PI$ 会更加简单一些.



第24题图1



第24题图2

25、(1) 设 $\angle BCD=\alpha$, 连接DF, 则CB=CD=CF, $\because AD//BC, \therefore AF//BC$

$\therefore \angle BCD=\angle CDE=\angle DFC$,

ΔDCF 是等边三角形,$\therefore \angle BCD=60^\circ$;

(2) ① 作 $DG\perp BC$, 交 BC 于点 $G, AB^2=DG^2=6^2-(6-x)^2=12x-x^2$,

$\therefore CA^2=AB^2+BC^2=12x-x^2+36$

$\therefore \angle ECA=\angle DCB, CA=CE, CB=CD=6$

$\therefore \Delta CAE\sim\Delta CBD\quad(\mathbf{SAS})$

$$\therefore y:BD=CA:CB \implies \frac{y}{\sqrt{x^2+12x-x^2}}=\frac{\sqrt{-x^2+12x+36}}{6}$$

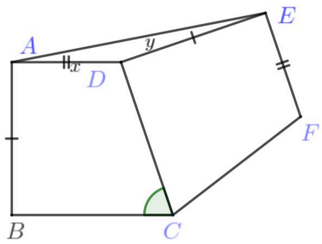
$$y=\frac{1}{3}\sqrt{-3x^3+36x^2+108x}$$

② 由①知道 $\Delta CAE\sim\Delta CBD, \therefore \frac{S_{\Delta CBD}}{S_{\Delta CAE}}=\frac{4}{5}=\left(\frac{CB}{CA}\right)^2$

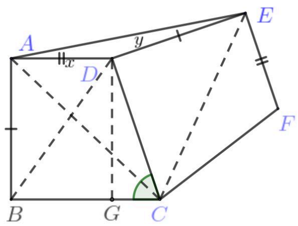
$\therefore CA=3\sqrt{5}$;

$\therefore AB=DG=\sqrt{CA^2-CB^2}=3, \sin\angle BCD=\frac{3}{6}=\frac{1}{2}, \therefore \angle BCD=30^\circ; \angle BCF$ 是一个正多边形的中心角, 故可以将点C 看成是正多边形的中心,

故正多边形的边数为 $\frac{360^\circ}{30^\circ}=12$



第25题图1



第25题图2