[MAA 2.8] EXPONENTS

SOLUTIONS

Compiled by: Christos Nikolaidis

O. Practice questions

1.

$3^3 = 27$	$3^{-2} = \dfrac{1}{9}$	$3^{-3} = \dfrac{1}{27}$
$\left(\dfrac{1}{3}\right)^2 = \dfrac{1}{9}$	$\left(\dfrac{2}{3}\right)^2 = \dfrac{4}{9}$	$\left(\dfrac{2}{3}\right)^3 = \dfrac{8}{27}$
$\left(\dfrac{1}{3}\right)^{-2} = 9$	$\left(\dfrac{2}{3}\right)^{-2} = \dfrac{9}{4}$	$\left(\dfrac{2}{3}\right)^{-3} = \dfrac{27}{8}$
$4^{\frac{1}{2}} = 2$	$4^{\frac{3}{2}} = 8$	$4^{-\frac{1}{2}} = \dfrac{1}{2}$
$25^{\frac{1}{2}} = 5$	$25^{-\frac{1}{2}} = \dfrac{1}{5}$	$\left(\dfrac{25}{4}\right)^{\frac{1}{2}} = \dfrac{5}{2}$

2.

$3^2 \times 2^3 = 72$	$3^5 \times 3^{-3} = 9$	$3^{-5} \times 3^3 = \dfrac{1}{9}$
$3^0 + 0^3 = 1$	$\dfrac{5^4 \times 2^3}{2^2 \times 5^3} = 10$	$\dfrac{7^4 \times 7^2}{7 \times 7^7} = 1$
$3^{-1} \times 2^3 = \dfrac{8}{3}$	$3 \times 2^{-3} = \dfrac{3}{8}$	$\dfrac{2^{-1}}{3^{-1}} = \dfrac{3}{2}$

3.

$\dfrac{30^2}{15^2} = 4$	$\dfrac{15^{-2}}{30^{-2}} = 4$	$\dfrac{12^{\frac{1}{2}}}{3^{\frac{1}{2}}} = 2$
$7^{0.3} \times 7^{0.7} = 7$	$7^{\frac{1}{2}} \times 7^{\frac{3}{2}} = 49$	$7^{-2} \times 7 = \dfrac{1}{7}$
$7^2 \times 7^{-1} = 7$	$\dfrac{2^3 \times 3^3}{6^3} = 1$	$\dfrac{4^3 \times 3^3}{6^3} = 8$

4.

$a^6 a^2 = a^8$	$a^6 a = a^7$	$(a^{\frac{1}{2}})^6 = a^3$
$\left(a^6\right)^2 = a^{12}$	$\left(a^2\right)^6 = a^{12}$	$\dfrac{a^6}{a^2} = a^4$
$a^2 a^3 a = a^6$	$\dfrac{a^2 a^6}{a^5} = a^3$	$\dfrac{a^2 b^6}{b^2 a^{-3} b^4} = a^5$
$\left(a^{\frac{3}{5}}\right)^5 = a^3$	$\left(a^{\frac{3}{5}}\right)^{10} = a^6$	$\left(a^{-1}\right)^{-2} = a^2$
$a^{x+3} a^{1-x} = a^4$	$\dfrac{a^{n+5}}{a^{n+3}} = a^2$	$\dfrac{a^{-8}}{a^{-10}} = a^2$

5.

$\dfrac{A^6 B^3 C^{10}}{C^5 A^2 B} = A^4 B^2 C^5$
$\dfrac{A^2 B + AB^3}{AB} = A + B^2$
$\dfrac{2A + A(4B) + (2A)^2}{2A} = 1 + 2B + 2A$
$\dfrac{A^4 B^3 + A^3 B^4}{A + B} = A^3 B^3$

6.

$\left(e^x\right)^2$	$\dfrac{1}{e^x}$	$\dfrac{e^{2x}}{e^x}$	$\dfrac{1}{e^2}$	$\sqrt{e}$	$\dfrac{e^x}{e^{-2}}$	$\sqrt{e^x}$
F. e^{2x}	**D.** e^{-x}	**C.** e^x	**A.** e^{-2}	**B.** $e^{\frac{1}{2}}$	**G.** e^{x+2}	**E.** $e^{x/2}$

7. (a)

$y = 2^x$	$y = 5^x$	$y = e^x$
(c)	(a)	(b)

$y = 2^{-x}$	$y = 5^{-x}$	$y = e^{-x}$
(d)	(f)	(e)

(b)

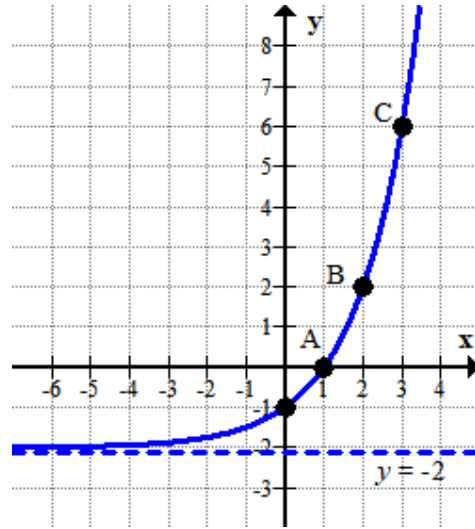| y – intercept	$y = 1$
Horizontal asymptote.	$y = 0$
Domain	$x \in \mathrm{R}$
Range	$y > 0$

8. (a) $y = 1$ (b) 0.25 (c) $b = 4$ $c = 16$ (d) $y = 0$ (f) $x \in \mathrm{R}$ $y > 0$

9. (a) $a = 4, b = 6, c = 10$

(b) y-intercept: $(0,3)$, horizonta asymptote: $y = 2$
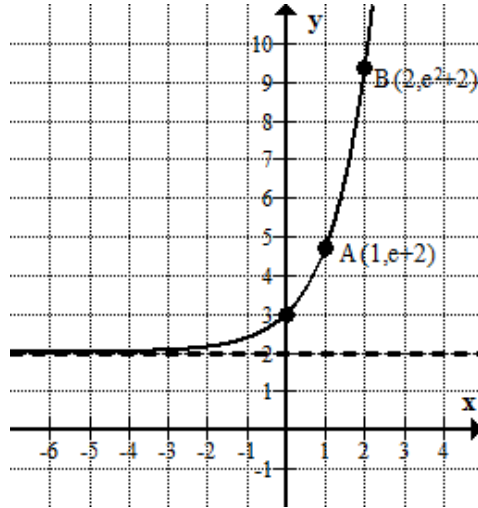
10. (a) $a = 0, b = 2, c = 6$

    (b) y-intercept: $(0, -1)$,   horizonta asymptote: $y = -2$

11. (a) $2^{2x} = 2^{1-x} \Leftrightarrow 2x = 1 - x \Leftrightarrow 3x = 1 \Leftrightarrow x = 1/3$

    (b) $2^{2x} = 8^{1-x} \Leftrightarrow 2^{2x} = 2^{3-3x} \Leftrightarrow 2x = 3 - 3x \Leftrightarrow 5x = 3 \Leftrightarrow x = 3/5$

    (c) $4^{2x} = 8^{1-x} \Leftrightarrow 2^{4x} = 2^{3-3x} \Leftrightarrow 4x = 3 - 3x \Leftrightarrow 7x = 3 \Leftrightarrow x = 3/7$

    (d) $8^{x+3} = 16^x \Leftrightarrow 2^{3x+9} = 2^{4x} \Leftrightarrow 3x + 9 = 4x \Leftrightarrow x = 9$

    (e) $\dfrac{1}{2^x} = 4^{x-3} \Leftrightarrow 2^{-x} = 2^{2x-6} \Leftrightarrow -x = 2x - 6 \Leftrightarrow 6 = 3x \Leftrightarrow x = 2$

    (f) $\sqrt{2^x} = 4^{1-x} \Leftrightarrow 2^{\frac{x}{2}} = 2^{2-2x} \Leftrightarrow \dfrac{x}{2} = 2 - 2x \Leftrightarrow x = 4 - 4x \Leftrightarrow 5x = 4 \Leftrightarrow x = 4/5$

12. (a) $25^{x+1} = 5^3 \Leftrightarrow 5^{2x+2} = 5^3 \Leftrightarrow 2x + 2 = 3 \Leftrightarrow 2x = 1 \Leftrightarrow x = 1/2$

    (b) $25^{x+1} = \dfrac{1}{5^x} \Leftrightarrow 5^{2x+2} = 5^{-x} \Leftrightarrow 2x + 2 = -x \Leftrightarrow 3x = -2 \Leftrightarrow x = -2/3$

    (c) $25^{x+1} = \sqrt{5^x} \Leftrightarrow 5^{2x+2} = 5^{\frac{x}{2}} \Leftrightarrow 2x + 2 = \dfrac{x}{2} \Leftrightarrow 4x + 4 = x \Leftrightarrow 3x = -4 \Leftrightarrow x = -4/3$

    (d) $25^{x^2} = 125^x \Leftrightarrow 5^{2x^2} = 5^{3x} \Leftrightarrow 2x^2 = 3x \Leftrightarrow x(2x - 3) = 0$

         $\Leftrightarrow x = 3/2$ or $x = 0$

    (e) $7^{x^2-5x} = 1 \Leftrightarrow 7^{x^2-5x} = 7^0 \Leftrightarrow x^2 - 5x = 0 \Leftrightarrow x(x - 5) = 0$

         $\Leftrightarrow x = 0$ or $x = 5$

13.   (a) (i) $b = e^2 + 2$ (ii) $b \cong 9.39$

(b) $f(1) = e + 2 \cong 4.72$, $f^{-1}(6) = 1.386294... \cong 1.39$

(c) y-intercept: $(0,3)$,   horizontal asymptote: $y = 2$

(d) Domain: $x \in R$,   Range: $y > 2$

14. $9^{2x} = 27^{(1-x)} \Leftrightarrow 3^{4x} = 3^{3-3x} \Leftrightarrow 4x = 3 - 3x \Leftrightarrow 7x = 3 \Leftrightarrow \cdot x = \dfrac{3}{7}$

15. $9^{x-1} = \left(\dfrac{1}{3}\right)^{2x} \Leftrightarrow 3^{2x-2} = 3^{-2x}$

$2x - 2 = -2x \Leftrightarrow x = \dfrac{1}{2}$

16. $25^{x^2} = \sqrt{5} \Leftrightarrow 5^{2x^2} = 5^{\frac{1}{2}} \Leftrightarrow 2x^2 = \dfrac{1}{2} \Leftrightarrow x^2 = \dfrac{1}{4}$

$x = \pm\dfrac{1}{2}$

17. $4^{x^2} = 8^x \Leftrightarrow 2^{2x^2} = 2^{3x} \Leftrightarrow 2x^2 = 3x \Leftrightarrow x(2x - 3) = 0$
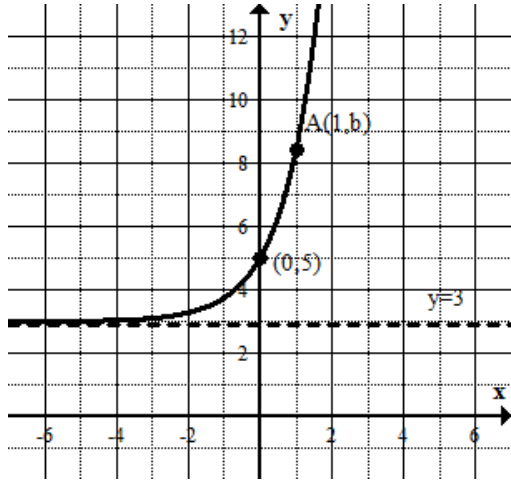
$x = 3/2$ or $x = 0$

18. $5^{x^2-3} = \left(\dfrac{1}{25}\right)^{2x-1} \Leftrightarrow 5^{x^2-3} = 5^{-4x+2} \Leftrightarrow x^2 - 3 = -4x + 2 \Leftrightarrow x^2 + 4x - 5 = 0$
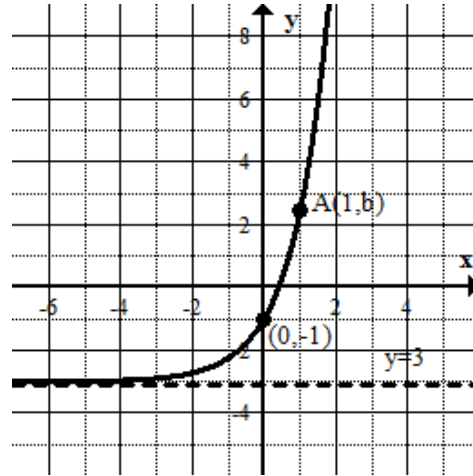
$x = -5$ or $x = 1$

19.

(a)

y- intercept: (0,5)	
h. asymptote: y=3	value of *b*: 2e+3
Domain: x∈R	Range: y>3

(b)

y- intercept: (0,-1)	
h. asymptote: y= -3	value of *b*: 2e-3
Domain: x∈R	Range: y> -3

20. (a) $x \in R$

(b) $y = 15$

(c) (i) $f(5) \cong 49.8$ (ii) $x \cong 7.50$

(d) $x > 8.14$ so $x = 9$

(e) $y \cong 5.02$, the horizontal line is $y = 5$

(f) $y > 5$