

INTERNATIONAL BACCALAUREATE  
Mathematics: analysis and approaches

**MAA**

**EXERCISES [MAA 1.6]**  
**BINOMIAL THEOREM**  
Compiled by Christos Nikolaidis

**O. Practice questions**

1. [Maximum mark: 9] **[without GDC]**

Write down the expansions of

- (a)  $(1 \pm x)^3$  [3]  
(b)  $(1 \pm x)^4$  [3]  
(c)  $(1 \pm x)^5$  [3]

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3. [Maximum mark: 5] **[with GDC]**

Find the term in  $x^7$  in the expansion of  $(1-x)^{10}$

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4. [Maximum mark: 5] **[with GDC]**

Find the term in  $x^{16}$  in the expansion of  $(1-x^2)^{10}$

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5. [Maximum mark: 10] [with GDC]

(a) Expand (i)  $\left(x - \frac{1}{x}\right)^3$  (ii)  $\left(x - \frac{1}{x}\right)^4$  [5]

(b) In the expansion of  $\left(x - \frac{1}{x}\right)^{10}$

(i) find the constant term

(ii) find the term in  $x^2$

(iii) find the term in  $x^{-2}$ . [5]

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6\*. [Maximum mark: 10] **[with GDC]**

(a) Expand  $(2x + 1)^4$  [2]

**Hence, find**

(b) the term in  $x^2$  in the expansion of  $x(2x + 1)^4$  [2]

(c) the term in  $x^2$  in the expansion of  $(x + 1)(2x + 1)^4$  [3]

(d) the term in  $x^2$  in the expansion of  $(3 - x^2)(2x + 1)^4$ . [3]

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7. [Maximum mark: 6] **[without GDC]**

(a) Show that

(i)  $\binom{8}{7} = 8$ ,   (ii)  $\binom{8}{6} = 28$ ,   (iii)  $\binom{8}{5} = 56$  [3]

(b) Find

(i)  $\binom{10}{9}$ ,   (ii)  $\binom{10}{8}$ ,   (iii)  $\binom{10}{7}$  [3]

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9\*. [Maximum mark: 9] **[with GDC]**

In the expansion of  $\left(3x - \frac{2}{x}\right)^{12}$  find

- (a) the constant term; [3]
- (b) the coefficient of  $x^4$ ; [3]
- (c) the first three terms in descending powers of  $x$ . [3]

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**A. Exam style questions (SHORT)**

**12.** [Maximum mark: 4]     ***[without GDC]***

Express  $(\sqrt{3} + \sqrt{2})^3$  in the form  $a\sqrt{2} + b\sqrt{3}$ , where  $a, b \in \mathbb{Z}$ .

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**13.** [Maximum mark: 4]     ***[without GDC]***

Express  $(\sqrt{3} + \sqrt{2})^4$  in the form  $a + b\sqrt{6}$ , where  $a, b \in \mathbb{Z}$ .

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14. [Maximum mark: 6] **[without GDC]**

Express  $(\sqrt{3} - 2)^3$  in the form  $a\sqrt{3} + b$ , where  $a, b \in \mathbb{Z}$ .

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15. [Maximum mark: 6] **[with / without GDC]**

Given that  $(3 + \sqrt{7})^3 = p + q\sqrt{7}$  where  $p$  and  $q$  are integers, find

- (a)  $p$ ;            (b)  $q$ .

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17. [Maximum mark: 6] **[with / without GDC]**

Complete the following expansion.

$$(2 + ax)^4 = 16 + 32ax + \dots$$

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18. [Maximum mark: 4] **[with GDC]**

Use the binomial theorem to complete this expansion.

$$(3x + 2y)^4 = 81x^4 + 216x^3y + \dots$$

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21. [Maximum mark: 4] **[with / without GDC]**

Find the coefficient of  $a^5b^7$  in the expansion of  $(a + b)^{12}$ .

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22. [Maximum mark: 4] **[with / without GDC]**

Find the coefficient of  $a^3b^4$  in the expansion of  $(5a + b)^7$ .

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**23.** [Maximum mark: 6] **[without GDC]**

The fifth term in the expansion of the binomial  $(a + b)^n$  is given by  $\binom{10}{4} p^6 (2q)^4$ .

- (a) Write down the value of  $n$ . [1]
- (b) Write down  $a$  and  $b$ , in terms of  $p$  and/or  $q$ . [2]
- (c) Write down an expression for the sixth term in the expansion. [3]

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**24.** [Maximum mark: 5] **[with GDC]**

Find the coefficient of  $x^7$  in the expansion of  $(2 + 3x)^{10}$ , giving your answer as a whole number.

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25. [Maximum mark: 5] **[with / without GDC]**

Consider the expansion of  $(x + 2)^{11}$ .

(a) Write down the number of terms in this expansion. [1]

(b) Find the term containing  $x^2$ . [4]

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26. [Maximum mark: 6] **[with / without GDC]**

Find the coefficient of  $x^3$  in the expansion of  $(2 - x)^5$ .

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27. [Maximum mark: 4] **[with GDC]**

Find the coefficient of  $x^5$  in the expansion of  $(3x - 2)^8$

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28. [Maximum mark: 6] **[with GDC]**

Find the term containing  $x^3$  in the expansion of  $(2 - 3x)^8$ .

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29. [Maximum mark: 6] **[with GDC]**

Find the coefficient of  $x^3$  in the binomial expansion of  $\left(1 - \frac{1}{2}x\right)^8$ .

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30. [Maximum mark: 6] **[with GDC]**

Find the coefficient of the  $x^3$  term in the expansion of  $\left(2 - \frac{3x}{2}\right)^6$ .

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31. [Maximum mark: 5] **[with GDC]**

Find the term in  $x^3$  in the expansion of  $\left(\frac{2}{3}x - 3\right)^8$ .

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32. [Maximum mark: 6] **[with / without GDC]**

One of the terms of the expansion of  $(x + 2y)^{10}$  is  $ax^8y^2$ . Find the value of  $a$ .

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33. [Maximum mark: 6] **[with GDC]**

Find the term containing  $x^{10}$  in the expansion of  $(5 + 2x^2)^7$ .

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34. [Maximum mark: 6] **[with GDC]**

Consider the expansion of  $(x^2 - 2)^5$ .

(a) Write down the number of terms in this expansion. [1]

(b) The first four terms of the expansion in descending powers of  $x$  are

$$x^{10} - 10x^8 + 40x^6 + Ax^4 + \dots$$

Find the value of  $A$ . [5]

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**35.** [Maximum mark: 6] **[with GDC]**

Consider the expansion of the expression  $(x^3 - 3x)^6$ .

(a) Write down the number of terms in this expansion. [1]

(b) Find the term in  $x^{12}$ . [5]

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**36.** [Maximum mark: 6] **[with / without GDC]**

Find the term in  $x^4$  in the expansion of  $\left(3x^2 - \frac{2}{x}\right)^5$ .

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37. [Maximum mark: 6] **[with GDC]**

Consider the expansion of  $\left(3x^2 - \frac{1}{x}\right)^9$ .

- (a) How many terms are there in this expansion? [1]
- (b) Find the constant term in this expansion. [5]

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38. [Maximum mark: 4] **[with GDC]**

Determine the constant term in the expansion of  $\left(x - \frac{2}{x^2}\right)^9$ .

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39. [Maximum mark: 6] **[without GDC]**

Find the constant term in the expansion of  $\left(x^2 - \frac{2}{x^2}\right)^6$ .

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40. [Maximum mark: 6] **[with / without GDC]**

Find the coefficient of  $x$  in the expansion of  $\left(3x - \frac{2}{x}\right)^5$ .

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41. [Maximum mark: 6] **[with GDC]**

When the expression  $(2 + ax)^{10}$  is expanded, the coefficient of the term in  $x^3$  is 414 720.

Find the value of  $a$ .

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42. [Maximum mark: 6] **[with GDC]**

The coefficient of  $x$  in the expansion of  $\left(x + \frac{1}{ax^2}\right)^7$  is  $\frac{7}{3}$ . Find the possible values of  $a$ .

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**43.** [Maximum mark: 6]     ***[without GDC]***

(a) Expand  $(x - 2)^4$  and simplify your result. [3]

(b) Find the term in  $x^3$  in  $(3x + 4)(x - 2)^4$ . [3]

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**44\***. [Maximum mark: 6] ***[without GDC]***

(a) Expand  $(2+x)^4$  and simplify your result. [3]

(b) Hence, find the term in  $x^2$  in  $(2+x)^4 \left(1+\frac{1}{x^2}\right)$ . [3]

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**45\***. [Maximum mark: 8]     **[without GDC]**

- (a) Find the expansion of  $(2 + x)^5$ , giving your answer in ascending powers of  $x$ .                     [3]
- (b) By letting  $x = 0.01$  or otherwise, find the **exact** value of  $2.01^5$ .                                     [2]
- (c) Find the **exact** value of  $1.99^5$ .     [3]

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47\*\*. [Maximum mark: 6]     ***[without GDC]***

Given that

$$(1+x)^5 (1+ax)^6 \equiv 1+bx+10x^2+\cdots+a^6x^{11},$$

find the values of  $a, b \in \mathbb{Z}^*$ .

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48\*. [Maximum mark: 6] ***[without GDC]***

Find the term in  $x^2$  in  $(1+x)^4 \left(3 + \frac{2}{x^2}\right)$

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