Taak: goniometrie

Virga Jessecollege **Toets WISKUNDE**

Hasselt **leerkracht: Karel Appeltans**

Datum: **schooljaar……**

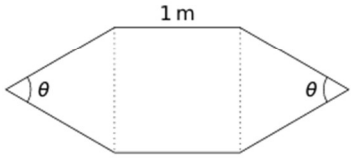
Klas: VI **Afdeling:**

Naam:…… **Aantal uren wiskunde: 6**

1	**8.** Evaluate $\lim_{x\to 0} \frac{3\sin(x) - \sin(3x)}{x - \sin(x)}$. a. 24 b. 20 c. 16 d. 12 e. 10 f. 9 g. 8 h. 6
2	Show that $y = \frac{3-\cos x}{x}$ satisfies the differential equation: $xy' + y = \sin x$
3	**Problem 5.** *(10 points total)* Is there a value of c that will make $f(x) = \begin{cases} \frac{\sin^2(3x)}{x^2} & \text{if } x \neq 0 \\ c & \text{if } x = 0 \end{cases}$ continuous at $x = 0$? Explain.
4	**Problem 6.** *(10 points)* Consider the curve $y = \frac{\pi \sin x}{x}$. Show that the line tangent to this curve at the point where $x = \pi$ and the line tangent to this curve at the point where $x = -\pi$ intersect at a right angle. *Solution: we have* $f(x) = \frac{\pi \sin x}{x}, \quad f'(x) = \pi \frac{x\cos x - \sin x}{x^2},$ *and at points* $x = \pi$ *and* $x = -\pi$, *we have* $f'(\pi) = -1, \quad f'(-\pi) = 1.$
5	$\lim_{x\to 0} \frac{1 - \cos x^2}{x^2 \sin x^2} =$

6

Een verplaatsbare dierenren bestaat uit 6 hekjes van elk 1 m lang die aan mekaar bevestigd kunnen worden. Construeer met deze hekjes een opstelling waarbij het bovenaanzicht een rechthoek vormt met 2 driehoeken aan 2 overstaande zijden, zoals weergegeven in de figuur. De hoek $\theta \in]0, \pi[$ wordt uitgedrukt in radialen. Voor welke waarde van θ wordt de oppervlakte van het bovenaanzicht maximaal?



(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{2\pi}{3}$

7

Find $\lim_{x \to \pi} \frac{(x-\pi)\sin(x)}{1+\cos(x)}$.

8

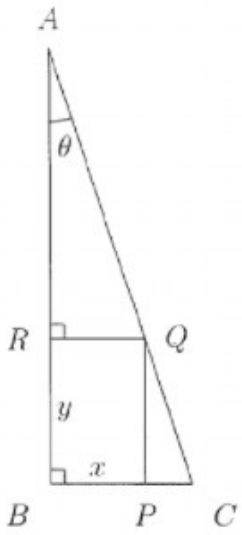
6. Consider the right-angle triangle ABC appearing in the picture. The length of the side AB is 6, and the length of the side BC is 2.
Picking a point Q on the hypotenuse AC gives us a rectangle $BPQR$ as in the picture. Let x be the length of BP and y the length of BR.

(i) Looking at the angle θ in the triangle ABC, calculate $\tan(\theta)$.

(ii) Looking at the angle θ in the triangle ARQ, calculate $\tan(\theta)$ in terms of x and y.

(iii) Use the results of (i) and (ii) to write y in terms of x.

(iv) Find the values of x and y that maximise the area of the rectangle $BPQR$.



9

Find the equation of the tangent to the curve $y = \frac{2\cos 2x}{2+\sin 2x}$ at the point (0, 1).

10

5. Compute the derivative of

$$f(x) = \sin^{-1}(\sqrt{1-x}).$$

A. $f'(x) = \frac{-1}{\sqrt{x}}$

B. $f'(x) = \frac{1}{2\sqrt{(1-x^2)(1-x)}}$

C. $f'(x) = \frac{-1}{2\sqrt{(1-x^2)(1-x)}}$

D. $f'(x) = \frac{1}{2\sqrt{x(1-x)}}$

E. $f'(x) = \frac{-1}{2\sqrt{x(1-x)}}$

11

Compute $\lim_{x\to 0}[\cos(2x)]^{1/x^2}$.

A. e^2

B. e^{-1}

C. e

D. 1

E. e^{-2}

12

(20 points) Consider the piecewise function $f(x)$ defined below. Where is this function continuous? Where is it not continuous? Explain why.

$$f(x)=\begin{cases}\sqrt{1+\cos(x)} & x\geq 0\\ -2x, & x<0\end{cases}.$$

13

Evaluate $\lim_{x\to 0^+}(\cos x)^{5/x^2}$

14

The function $f:\mathbb{R}\to\mathbb{R}$ is given by

$$f(x)=\begin{cases}\tan\left(x+\frac{\pi}{3}\right) & \text{if } x\geq 0,\\ ax+b & \text{if } x<0.\end{cases}$$

a) For which values of a and b is f continuous at $x=0$?

b) For which values of a and b is f differentiable at $x=0$?

[Explain your answers!]

15

Find an equation for the tangent line to the graph of $y=\tan\left(\frac{\pi}{\sqrt{25-x^2}}\right)$ at the point with $x=3$.

16

4. Let $y=\sin(2x)\cos^2 x$. Find $y'\left(\frac{\pi}{4}\right)$.

A) -2 E) 0

B) $-\sqrt{2}$ F) 1

C) -1 G) 2

D) $\frac{-1}{2}$ H) $\frac{\sqrt{2}}{2}$

17

If $y=x+\sin^{-1}x$, prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+x=0$.

18

Find a, b such that the following function is differentiable at $x = 0$. [9]

$$f(x) = \begin{cases} a\sin 2x + 2b\cos 3x + 1 & x < 0 \\ b\sin 2x + a\cos 3x & x \geq 0 \end{cases}$$

19

$$\lim_{x\to 0^+} (1 - \sin x)^{\frac{1}{x}}.$$

20

4. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by:

$$f(x) = \begin{cases} \sin(3x - \frac{1}{3}\pi) + 1 & \text{als } x > 0 \\ ax + b & \text{als } x \leq 0. \end{cases}$$

a) For which values of a and b is f continuous at 0? Explain your answer.

b) For which values of a and b is f differentiable at 0? Explain your answer.

21

Onderzoek de afleidbaarheid van $f(x) = Bgsin\left(\frac{1}{x}\right)$ voor x=1

22

De functie $f : \mathbb{R} \to \mathbb{R}$ heeft als voorschrift $f(x) = \cos\left(2x + \frac{\pi}{2}\right)$. Definieer de functie $g : \mathbb{R} \to \mathbb{R}$ door $g(x) = 2f(f(x))$. Bepaal $g'\left(\frac{\pi}{2}\right)$.

(A) -8 (B) -4 (C) 4 (D) 8

23

9. Let

$$f(x) = \sin^{-1}(2e^x)$$

What is $f'\left(\ln\left(\frac{3}{10}\right)\right)$?

A) 0 B) $\frac{3}{16}$ C) $\frac{1}{4}$ D) $\frac{1}{2}$ E) $\frac{3}{4}$ F) $\frac{13}{16}$ G) 1 H) $\frac{1}{e}$

24

Gegeven is de functie $f : \mathbb{R} \to \mathbb{R}$ met voorschrift $f(x) = 3 - e^{\sin(1-x)}$. Bepaal de waarde van de parameter p zodat de rechte met vergelijking $y = p(x-1) + 2$ een raaklijn is aan de grafiek van de functie f in het punt $(1, 2)$.

(A) -1 (B) 0 (C) 1 (D) e

25

$$\lim_{x\to 0^+} (4x + 1)^{\cot 3x}$$

26	If $g(x) = \sqrt{1-x^2} + x\sin^{-1}x$, find $g'(\sqrt{2}/2)$. a. 0 b. $\pi/6$ c. $\pi/4$ d. $\pi/3$ e. 1 f. $\pi/2$ g. π h. -1
27	(b) (i) The function f is continuous on $[a, b]$, differentiable on $]a, b[$ and $f'(x) = 0$ for all $x \in]a, b[$. Show that $f(x)$ is constant on $[a, b]$. (ii) Hence, prove that for $x \in [0, 1]$, $2\arccos x + \arccos\left(1-2x^2\right) = \pi$.
28	$\lim_{x\to\infty} (1-\cos(3x))^x$
29	A curve has the equation $y = x^2\cos x$. Show that the equation of the tangent to the curve at the point $\left(\pi, -\pi^2\right)$ is $y + 2\pi x = \pi^2$
30	**Exercise 1. (Level 2)** The function f is continuous at $x = 0$ and is defined for $-1 < x < 1$ by $$f(x) = \begin{cases} \frac{2a}{x}\ln(1+x) & \text{if } -1 < x < 0 \\ b & \text{if } x = 0 \\ \frac{x^2\cos x}{1-\sqrt{1-x^2}} & \text{if } 0 < x < 1. \end{cases}$$ Determine the values of the constants a and b.