Grades 9-12 (AS)

Duration: 30 min

Tools: one Logifaces Set / class

Individual / Group work

Keywords: Right-angled Trigonometry, Angles, Trapezium, Rectangle, Triangle

535 - Calculating Angles

MATHS / TRIGONOMETRY

LOGIFACES METHODOLOGY

Erasmus+

TEACHER

Logifaces

2019-1-HU01-KA201-0612722019-1

DESCRIPTION

Students choose a block from the 9 pcs or 16 pcs Set, draw the polygons that make it up, then

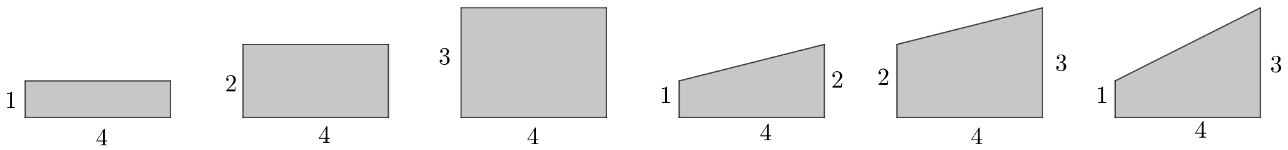
a) calculate the angles of the quadrilateral faces.

b) calculate the angles of the triangular faces.

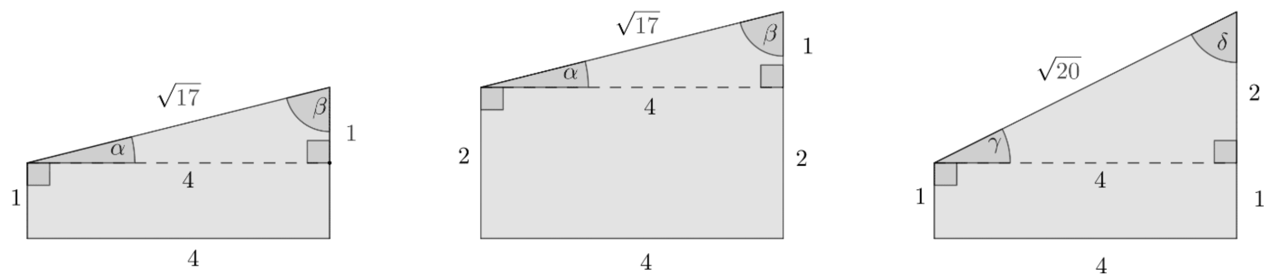
SOLUTIONS / EXAMPLES

The definitions of the trigonometric ratios in a right-angled triangle or the cosine rule can be used to calculate the angles.

a) The types of quadrilateral faces that are possible are listed in the figure below (see the reasons in 401 - Quadrilateral Faces):



All the angles of the rectangles and the angles on the 4 unit length legs of the trapeziums are 90°.
The angles on the other leg of the trapeziums can be calculated as follows. Draw parallels with the 4 unit length leg as shown in the figure. The hypotenuse of the resulting right-angled triangle is calculated by the Pythagorean theorem. (Calculations of the length of the edges can be found in exercise 404 - Top Edges). Use trigonometric ratios to calculate each angle.



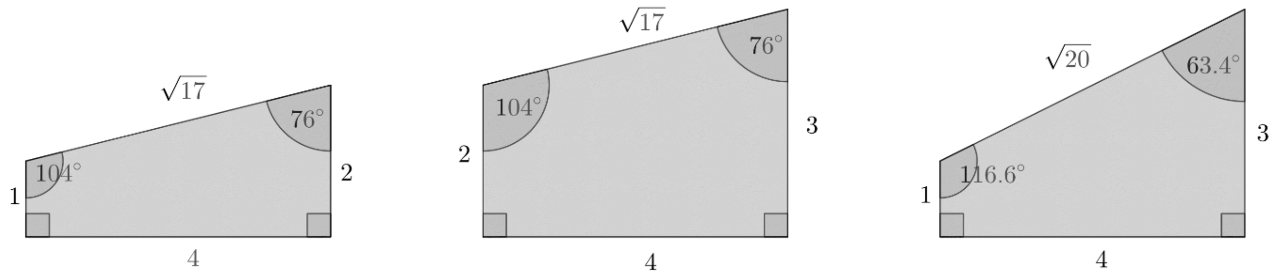
$\alpha = sin^{-1}\left(\frac{1}{\sqrt{17}}\right) = cos^{-1}\left(\frac{4}{\sqrt{17}}\right) = tan^{-1}\left(\frac{1}{4}\right) \approx 14.0°$ and $\beta = 90° - \alpha \approx 90° - 14.0° = 76.0°$

$\beta = sin^{-1}\left(\frac{4}{\sqrt{17}}\right) = cos^{-1}\left(\frac{1}{\sqrt{17}}\right) = tan^{-1}\left(\frac{4}{1}\right) \approx 76.0°$ and $\alpha = 90° - \beta \approx 90° - 76.0° = 14.0°$

$\gamma = sin^{-1}\left(\frac{2}{\sqrt{20}}\right) = cos^{-1}\left(\frac{4}{\sqrt{20}}\right) = tan^{-1}\left(\frac{2}{4}\right) \approx 26.6°$ and $\delta = 90° - \gamma \approx 90° - 26.6° = 63.4°$

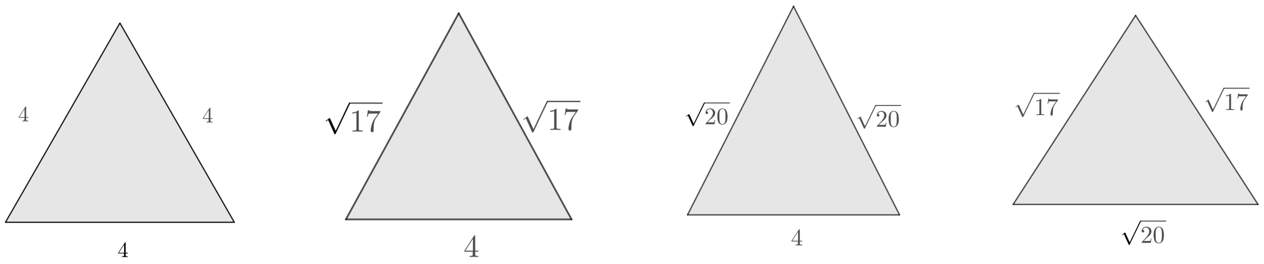
$\delta = sin^{-1}\left(\frac{4}{\sqrt{20}}\right) = cos^{-1}\left(\frac{2}{\sqrt{20}}\right) = tan^{-1}\left(\frac{4}{2}\right) \approx 63.4°$ and $\gamma = 90° - \delta \approx 90° - 63.4° = 26.6°$

All the angles are marked in the figure below:

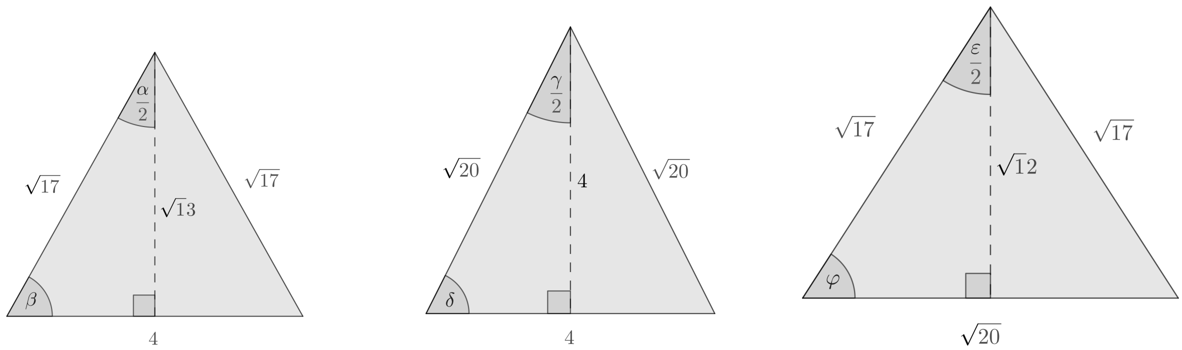


b) All of the base faces are equilateral triangles, and the top faces are either equilateral triangles or isosceles triangles. The types of top faces are listed in the table below (see the reasons in exercise 404 - Top Edges):

Blocks: 111, 222, 333	Blocks: 112, 221, 223, 332	Blocks: 113, 331	Blocks: 123, 132



The equilateral triangles have equal angles, all of them are 60°. For the isosceles triangles: draw the altitudes and use trigonometric ratios in the arising right-angled triangle to calculate each angle. (See exercise 411 - Area of Triangles for the calculations of the altitudes):



$\alpha = sin^{-1}\left(\frac{2}{\sqrt{17}}\right) = cos^{-1}\left(\frac{\sqrt{13}}{\sqrt{17}}\right) = tan^{-1}\left(\frac{2}{\sqrt{13}}\right) \approx 29.0$ and $\beta = 90° - \alpha \approx 90° - 29.0° = 61°$

$\beta = sin^{-1}\left(\frac{\sqrt{13}}{\sqrt{17}}\right) = cos^{-1}\left(\frac{2}{\sqrt{17}}\right) = tan^{-1}\left(\frac{\sqrt{13}}{2}\right) \approx 61.0$ and $\alpha = 90° - \beta \approx 90° - 61.0° = 29°$

$\gamma = sin^{-1}\left(\frac{2}{\sqrt{20}}\right) = cos^{-1}\left(\frac{4}{\sqrt{20}}\right) = tan^{-1}\left(\frac{2}{4}\right) \approx 26.6$ and $\delta = 90° - \gamma \approx 90° - 26.6° = 63.4°$

$\delta = sin^{-1}\left(\frac{4}{\sqrt{20}}\right) = cos^{-1}\left(\frac{2}{\sqrt{20}}\right) = tan^{-1}\left(\frac{4}{2}\right) \approx 63.4$ and $\gamma = 90° - \delta \approx 90° - 63.4° = 26.6°$

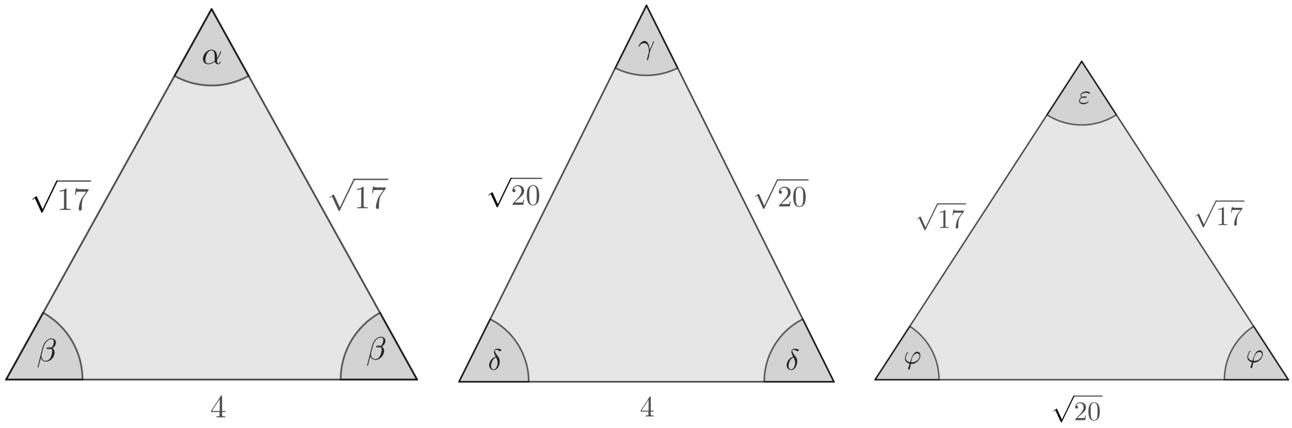
$\varepsilon = sin^{-1}\left(\frac{\frac{\sqrt{20}}{2}}{\sqrt{17}}\right) = cos^{-1}\left(\frac{\sqrt{12}}{\sqrt{17}}\right) = tan^{-1}\left(\frac{\frac{\sqrt{20}}{2}}{\sqrt{12}}\right) \approx 22.8$ and $\varphi = 90° - \varepsilon \approx 90° - 22.8° = 67.2°$

$\varphi = sin^{-1}\left(\frac{\sqrt{12}}{\sqrt{17}}\right) = cos^{-1}\left(\frac{\frac{\sqrt{20}}{2}}{\sqrt{17}}\right) = tan^{-1}\left(\frac{\sqrt{12}}{\frac{\sqrt{20}}{2}}\right) \approx 67.2$ and $\varepsilon = 90° - \varphi \approx 90° - 67.2° = 22.8°$

Alternative solution: cosine rule $c^2 = a^2 + b^2 - 2ab \times cos(\gamma)$

To calculate the angle rearrange the formula: $cos(\gamma) = \frac{a^2+b^2-c^2}{2ab}$ and hence $\gamma = cos^{-1}\left(\frac{a^2+b^2-c^2}{2ab}\right)$

Use the following notations:



$$\alpha = cos^{-1}\left(\frac{(\sqrt{17})^2+(\sqrt{17})^2-4^2}{2\sqrt{17}\times\sqrt{17}}\right) = cos^{-1}\left(\frac{17+17-16}{2\times17}\right) = cos^{-1}\left(\frac{18}{34}\right) \approx 58.0°$$

$$\beta = cos^{-1}\left(\frac{(\sqrt{17})^2+4^2-(\sqrt{17})^2}{2\sqrt{17}\times4}\right) = cos^{-1}\left(\frac{16}{8\times\sqrt{17}}\right) \approx 61.0°$$

$$\gamma = cos^{-1}\left(\frac{(\sqrt{20})^2+(\sqrt{20})^2-4^2}{2\sqrt{20}\times\sqrt{20}}\right) = cos^{-1}\left(\frac{20+20-16}{2\times20}\right) = cos^{-1}\left(\frac{24}{40}\right) \approx 53.1°$$

$$\delta = cos^{-1}\left(\frac{(\sqrt{20})^2+4^2-(\sqrt{20})^2}{2\sqrt{20}\times4}\right) = cos^{-1}\left(\frac{16}{8\times\sqrt{20}}\right) \approx 63.4.0°$$

$$\varepsilon = cos^{-1}\left(\frac{(\sqrt{17})^2+(\sqrt{17})^2-(\sqrt{20})^2}{2\sqrt{17}\times\sqrt{17}}\right) = cos^{-1}\left(\frac{17+17-20}{2\times17}\right) = cos^{-1}\left(\frac{14}{34}\right) \approx 65.7°$$

$$\varphi = cos^{-1}\left(\frac{(\sqrt{17})^2+(\sqrt{20})^2-(\sqrt{17})^2}{2\sqrt{17}\times\sqrt{20}}\right) = cos^{-1}\left(\frac{20}{2\sqrt{17}\times\sqrt{20}}\right) \approx 57.2.0°$$

PRIOR KNOWLEDGE

Properties of isosceles triangles, Trapeziums, Equilateral triangles, Rectangles, Right-angled Trigonometry

RECOMMENDATIONS / COMMENTS

This exercise is suitable for differentiation. Each team can be given a block, different students use different trigonometric functions or rules to calculate the angles. They can compare the results and check whether they are consistent.

The calculations can be verified using GeoGebra, see exercise 528 - Read the Results in GeoGebra.