SOS HERMANN GMEINER SCHOOLS, NEPAL

Joint SEE Preparatory Examination- 2079
Subject: Optional Mathematics   **Full Marks: 100**
Time: 3:00 Hrs.

Attempt all the questions. All the working must be shown.

Group: A [5×(1+1) =10]

1. a. Under what condition does the inverse of a function exist?
   b. What is the inequality whose boundary line is 2x+3y =10 and the solution set does not contain (− 1, 3)?

2. a. At which point is the function $f(x) = \frac{2x+3}{3x-2}$ discontinuous?
   b. Find the value of x if $\begin{vmatrix} -3 & 5 \\ x & -5 \end{vmatrix} = 0$.

3. a. If the intersecting plane is parallel to the axis of cone, then which conic does it form?
   b. What is the condition for the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ to be parallel?

4. a. Write the formula to transform $2\sin\theta\cos\alpha$ into sum or difference.
   b. Solve: $3\tan^2\theta – 1 = 0$ $(0^0 \leq \theta \leq 90^0)$

5. a. The position vectors of the vertices of Δ ABC are $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively. If $\vec{g}$ is the position vector of centroid of Δ ABC then what is value of $\vec{g}$ in terms of $\vec{a}$, $\vec{b}$ and $\vec{c}$?
   b. What is the inverse of point P (x, y) with respect to the inversion circle $(x – h)^2 + (y – k)^2 = r^2$?

Group: B [13×2=26]

6. a. If f: x→2x – 3, find the value of $f^{-1}(2)$.
   b. If $g = \{(1, 2), (3, 5), (4, 1)\}$ and $f = \{(5, 1), (1, 3), (2, 3)\}$ then show the function f_og in an arrow diagram and write it in ordered pair form.
   c. What is vertex and axis of symmetry of parabola $y = x^2 – 2x – 5$.

7. a. Find the inverse of matrix $\begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}$.
   b. If $\begin{pmatrix} 3 & 1 \\ y & 2 \end{pmatrix}$ and $\begin{pmatrix} 2 & x \\ -5 & 3 \end{pmatrix}$ are inverses of each other, then find the value of x and y.

8. a. Calculate the value of m if the line 3x – my =19 makes an angle of 45^0 with the line 3x + 5y = 7.
   b. If the centre of the circle $x^2 + y^2 – px – qy – 12 = 0$ is (2, 3), find the value of p and q.

9. a. Prove that: $\frac{1+\cos2\theta+\sin2\theta}{1-\cos2\theta+\sin2\theta} = \cot\theta$
   b. If $\sin\frac{A}{3} = \frac{4}{5}$, find the value of cosA.
   c. If $\sin\theta – \cos\theta = 0$, find the value of θ under 0° ≤θ ≤ 360°.

10. a. If $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = 6$, $|\vec{b}| = 10$ and $|\vec{c}| = 14$, find the angle between $\vec{a}$ and $\vec{b}$.
    b. The position vectors of the points A and B are $8\vec{i} + 6\vec{j}$ and $3\vec{i} + \vec{j}$ respectively. Find the position vector of the points C, which divides AB in the ratio of 2:3 internally.
    c. The coefficient of quartile deviation of a grouped data is 0.25. If upper quartile is 75, find the lower quartile and quartile deviation.

Group: C [11×4 =44]

11. Solve: $x^3 – 3x – 2 = 0$
12. Maximize the objective function f = 6x + 5y, subject to the constraints x + y ≤ 6, x – y ≤ 4, x ≥ 0 and y ≥ 0.

13. Examine the continuity or discontinuity of the function:

$$f(x) = \begin{cases} 5x + 2 & x \le 3 \\ 7x - 4 & x > 3 \end{cases} \text{ at x=3.}$$

14. Solve by Cramer's rule

$$2x = 3(6 - y) \text{ and } 3x = 2y + 1$$

15. Find the equation of pair of lines passing through the point (2, 3) and making 90^0 to the pair of lines $2x^2 - xy - 6y^2 + 4x + 6y = 0$.

16. Prove that: $4(\cos^6\theta - \sin^6\theta) - \cos^3 2\theta = 3\cos 2\theta$

17. If $A + B + C = \pi^c$, prove that

$$\cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2} = 4\cos\frac{\pi^c - A}{4}\cos\frac{\pi^c - B}{4}\cos\frac{\pi^c - C}{4}$$

18. The angle of elevation of the top of a tower observed from two points on the same plane are found to be complementary. If the points are at a distance of 20 meters and 40 meters from the foot of the tower, find the height of the tower.

19. Find a 2×2 transformation matrix which transforms a unit square OABC into the parallelogram O'A'B'C' with vertices O'(0, 0), A'(3, 0), B'(4, 1) and C'(1, 1).

20. Calculate the mean deviation from median and its coefficient from the given data

Marks obtained	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	5	8	15	16	6

21. Find the standard deviation and its coefficient from the data given bellow:

Class Interval	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20	20 – 24
Frequency	7	7	10	15	7	6

Group: D [4×5 =20]

22. To pay a loan amount Rs 40800 for farming from a Finance Company, it takes 2 years to pay in monthly installment system. If each installment is Rs 100 more than the previous one, find the amount that should be paid in first and last installment. Also, find which installment is Rs 1650.

23. Two circles are concentric and equation of great circle is $x^2 + y^2 - 4x - 6y - 3 = 0$. If the small circle touches Y – axis, find the equation of the smaller circle.

24. Prove by vector method that the middle point M of hypotenuse PQ of a right-angled triangle PQR is equidistant from its vertices.

25. E denotes the enlargement about the centre (3, 1) with a scale factor 2 and R denotes the reflection on the line y = x. Find the image of $\triangle$ ABC having the vertices A (2, 3), B (4, 5) and C (1, –2) under the combined transformation E_OR. Draw object and images triangles on the same graph paper.

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