Time: 3:00 hrs   Optional Mathematics   **F.M. : 100**

Attempt all questions. Working must be shown.
Group 'A' $[5 \times (1+1) = 10]$

1. (a) If $f(x) = 2x - 3$, find $f^{-1}(x)$

   (b) If n is an even number, show that $(x - a)$ is a factor of $x^n - a^n$.

2. (a) Write the geometrical form of the numbers lying between $-\infty$ to ∞ .

   (b) What is the value of $(A^{-1})^{-1}$?

3. (a) If the lines $ax + by + c = 0$ and $px + qy + r = 0$ are parallel, prove that $aq = bp$.

   (b) If the intersecting plane is parallel to the base of cone, what conic section does it form ?

4. (a) Write the formula of $\cos 2A$ in form of $\tan^2 A$.

   (b) If $\sin \dfrac{\theta}{3} = \dfrac{1}{3}$ then find the value of $\sin \theta$.

5. (a) If $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OB} = \vec{b}$, what is the vector $\overrightarrow{AB}$?

   (b) If the radius of inversion circle is 8 cm, find the value of $OP \times OP'$.

Group 'B' $[13 \times 2 = 26]$

6. (a) What will be the points of intersection of the curve $f(x) = x^2 - 5$ and $f(x) = 4$?

   (b) If $f(4x - 15) = 8x - 27$, find $fof(2)$.

   (c) Find the 8th terms of the series $2 + 1 + \dfrac{1}{2} + \dfrac{1}{4} + ...$

7. (a) Find the matrix $\begin{bmatrix} a \\ b \end{bmatrix}$, if $\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$.

   (b) Find the values of D_1 and D_2 in the equations $3x + 5y = 11$ and $2x - 3y = 1$.

8. (a) Find the obtuse angle between two lines having slopes 2 and -3.

   (b) Find the center and radius of a circle represented by $x^2 + y^2 + 4x - 6y - 12 = 0$

9. (a) Prove that: $\dfrac{1 - \cos A}{1 + \cos A} = \tan^2 \dfrac{A}{2}$

   (b) Prove that: $\sin 75° + \sin 15° = \sqrt{\dfrac{3}{2}}$

   (c) Solve: $\sin^2 \theta - \sin \theta + \dfrac{1}{4} = 0 \ (0° \le \theta \le 90°)$

10. (a) If $\vec{a} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, find the vector perpendicular to $\vec{a}$ whose length along the X-axis is 6 unit.

    (b) In $\Delta ABC, \overrightarrow{OA} = 4\vec{i} - 5\vec{j}, \overrightarrow{OB} = 6\vec{i} - 4\vec{j}$ and the position vector of centroid G is $\overrightarrow{OG} = 2\vec{i} - \vec{j}$. Find $\overrightarrow{OC}$.

    (c) In a grouped data, value of first quartile is 2 m and quartile deviation is m. Find the third quartile and coefficient of quartile deviation.

Group 'C' $[11 \times 4 = 44]$

11. Solve: $x^3 - 9x^2 + 24x - 20 = 0$

12. Maximize the objective function $P = x + 2y$ subject to the conditions

$$2x + y \le 8, 2x + 3y \le 12, x \ge 0 \text{ and } y \ge 0$$

13. Examine the continuity or discontinuity of the function

$$f(x) = \begin{cases} 4x - 1 & \text{for } x < 1 \\ 7x & \text{for } x \ge 1 \end{cases} \text{ at } x = 1$$

by calculating left hand limit, right hand limit and functional value.

14. Solve by matrix method:

$$4x - \frac{9}{y} + 11 = 0 \text{ and } \frac{6}{y} - 3x = 8$$

15. Find the single equation of a pair of lines passing through the origin and perpendicular to the pair of lines represented by $2x^2 - xy - 3y^2 = 0$.

16. If $A + B + C = 180°$, Prove that:

$$\sin(B+C-A)+\sin(C+A-B)+\sin(A+B-C) = 4\sin A.\sin B.\sin C$$

17. Solve: $[0° \leq \theta \leq 360°]$

$$\cos 3\theta + \cos \theta = 2\cos \theta$$

18. From a point at a ground in front of a tower the angle of elevations of the top and bottom of the flag staff 6 m high situated at the top of the tower are robserved to be $60°$ and $45°$ respectively. Find the height of the tower and the distance between the base of the tower and the point of observation.

19. Find the image quadrilateral when the unit square is transformed by the matrix $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$.

20. Find the mean deviation from the median of the following data.

Marks obtained	0 - 10	10-20	20-30	30-40	40-50
No. of students	20	40	60	50	30

21. Find the standard deviation from given data:

Marks	20 - 40	40 - 60	60 - 80	60 - 100
Frequency	5	10	15	25

Group 'D' $[4 \times 5 = 20]$

22. If a, b and c are in AP and x, y z are in GP, prove that:

$$x^{b-c} \times y^{c-a} \times z^{a-b} = 1$$

23. Find the equations of the circle passing through the point (3,4) whose equations of two diameters are $x+y = 14$ and $2x-y = 4$

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24. Prove by vector method that the diagonals of a rhombus bisect each other at right angles.

25. Find the images of ΔMNP with vertices $M(3,4), N(1,1)$ and $P(4,1)$ under enlargement with centre at $(1,-1)$ and scale factor -2 followed by the rotation about origin through negative quarter turn. Also show the object and the image on the same graph paper.

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