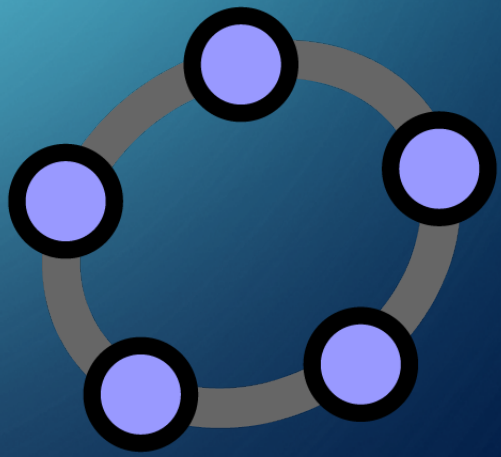
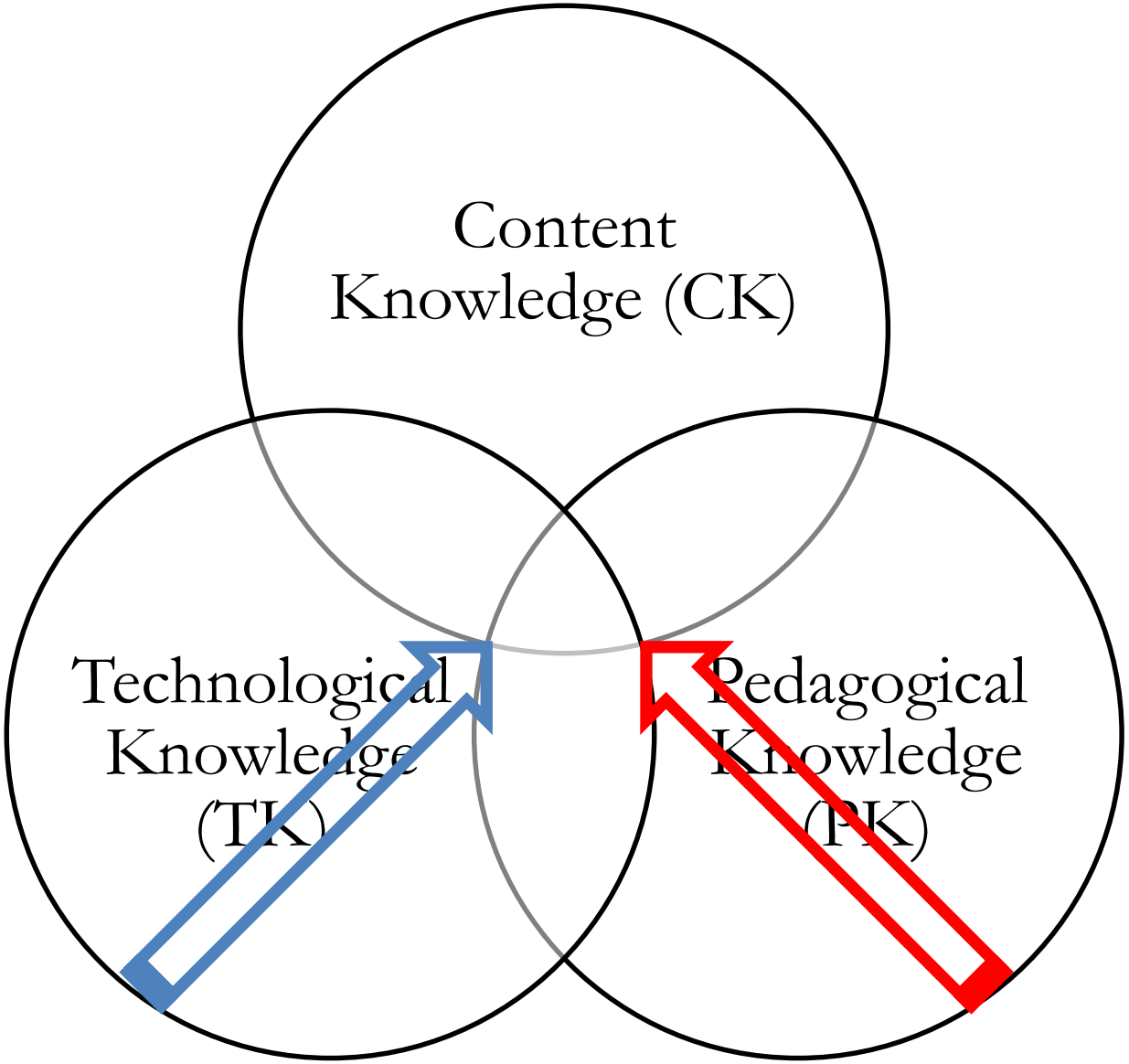
中學數學電子學習系列：(2) 在數學課堂有效運用資訊科技（進階程度）

教育局數學教育組

2021 年

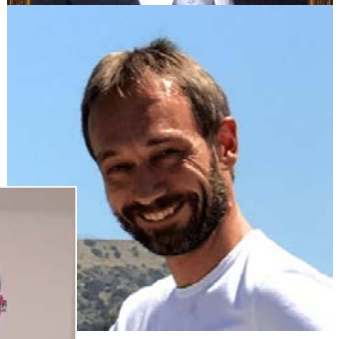


術 Technology & 道 Pedagogy



GeoGebra 簡介

- GeoGebra 是為了小學到大學的教學而設計的開源 (open source) 動態數學軟體。
- GeoGebra 是一套結合幾何（平面+立體）、代數、統計及微積分等的免費動態幾何軟件，它是在 2001 年由 Markus Hohenwarter 在奧地利的 Salzburg 薩爾茨堡大學所設計。
  - GeoGebra 其實就是他的碩士論文。
  - 目前在奧地利 Linz 的 Johannes Kepler 大學（克卜勒大學）擔任數學教學研究所所長。
- GeoGebra 是由 Java 寫成的，因此可以跨平台使用。
- GeoGebra 的一些學與教用途：
  - 教師用於課堂演示互動幾何圖像；
  - 學生用於探索與發現幾何概念，猜想幾何定理。
- 2011: 38 developers & 200 translators Celebrating 10 years of GeoGebra



If you want to go fast, go alone.
If you want to go far, go together.

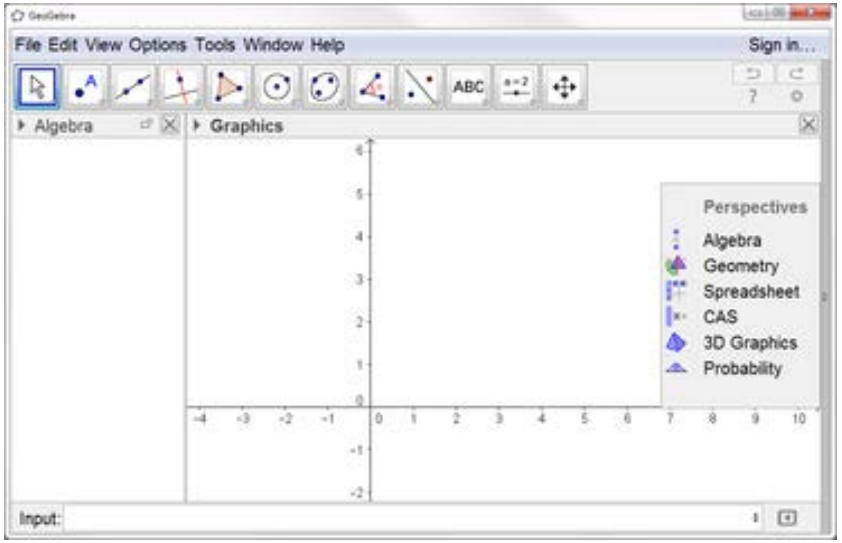
Stable release 6.0.452.0 (8 April 2018)
Stable release: 6.0.620 (1 December 2020)
https://en.wikipedia.org/wiki/GeoGebra

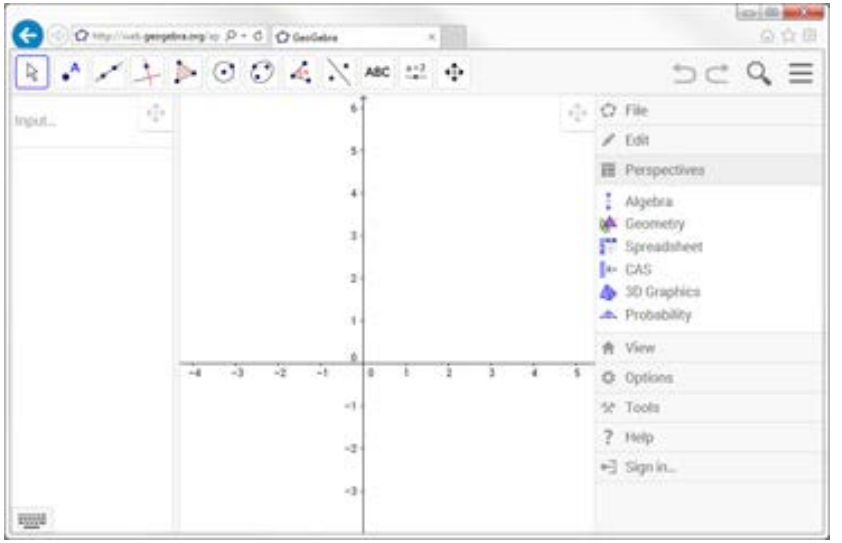
下載與安裝

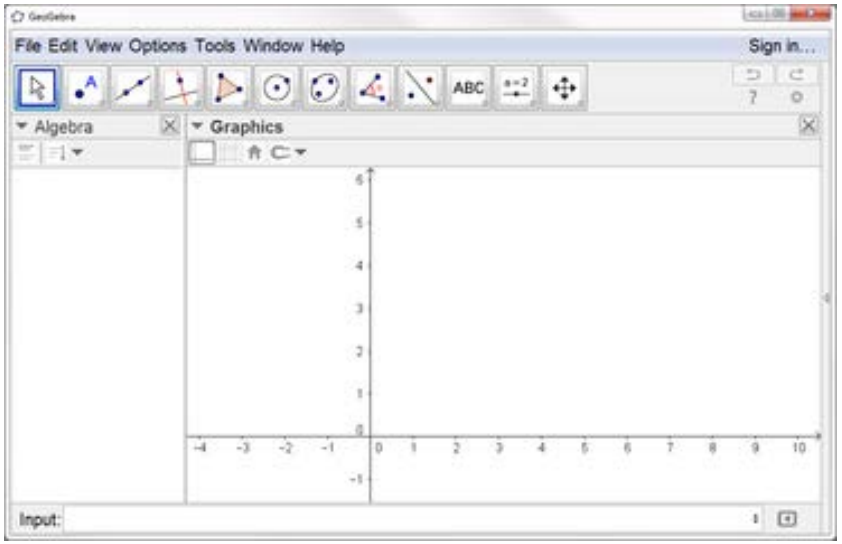
GeoGebra 经典 6

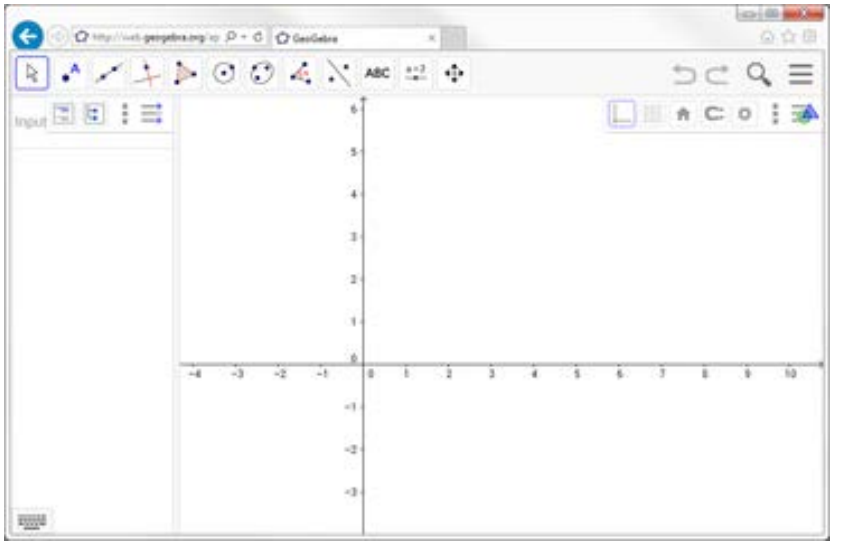
GeoGebra 经典 5

- GeoGebra Desktop 桌機版 vs. Web and Tablet App 線上版或平板版



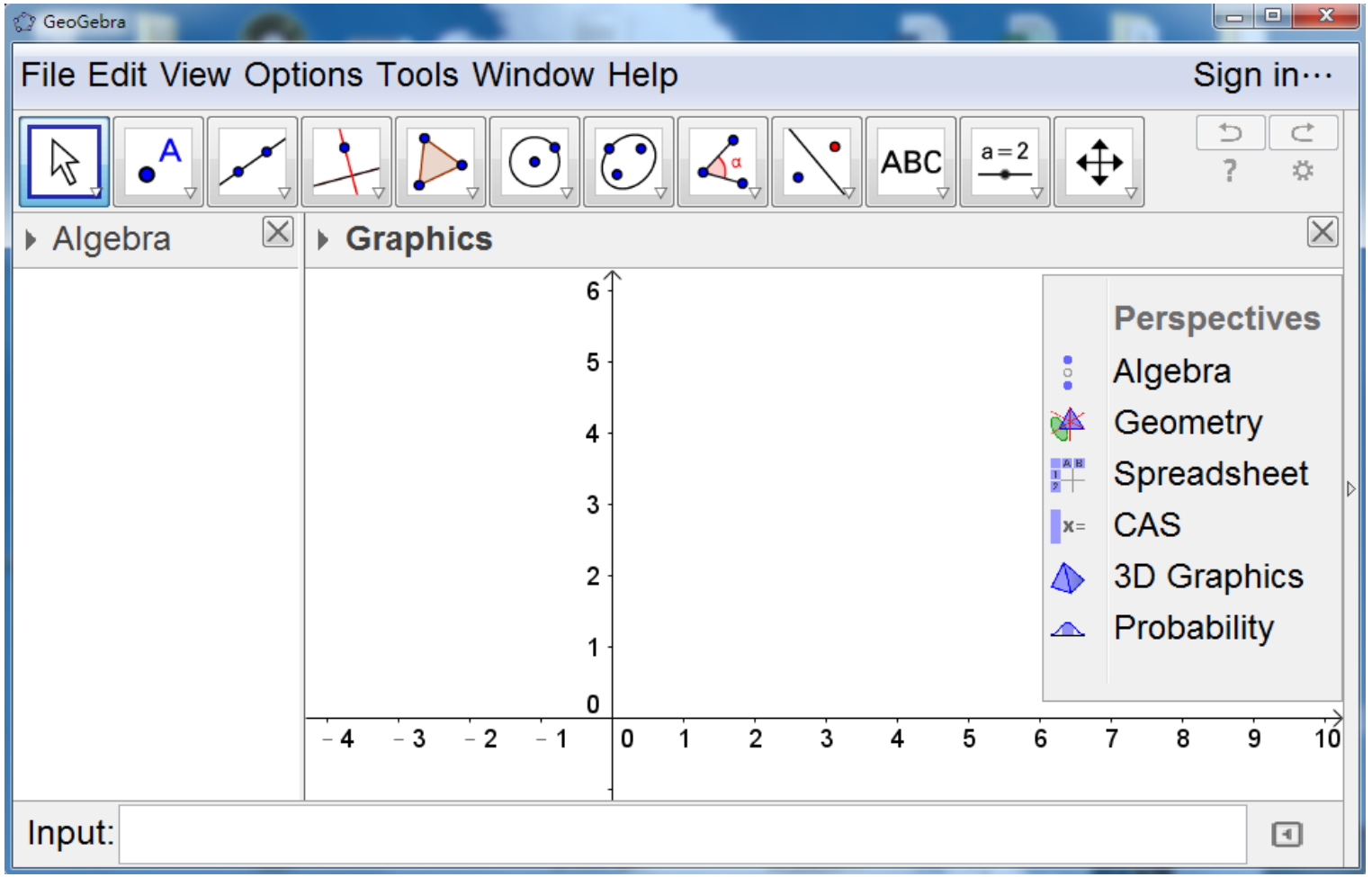






選擇 GeoGebra Classic 5 或 6？

- 您可以根據硬件設備與個人偏好，來選擇使用桌機版、線上版或平板版，兩者在介面的設計上，只有少許的差異。



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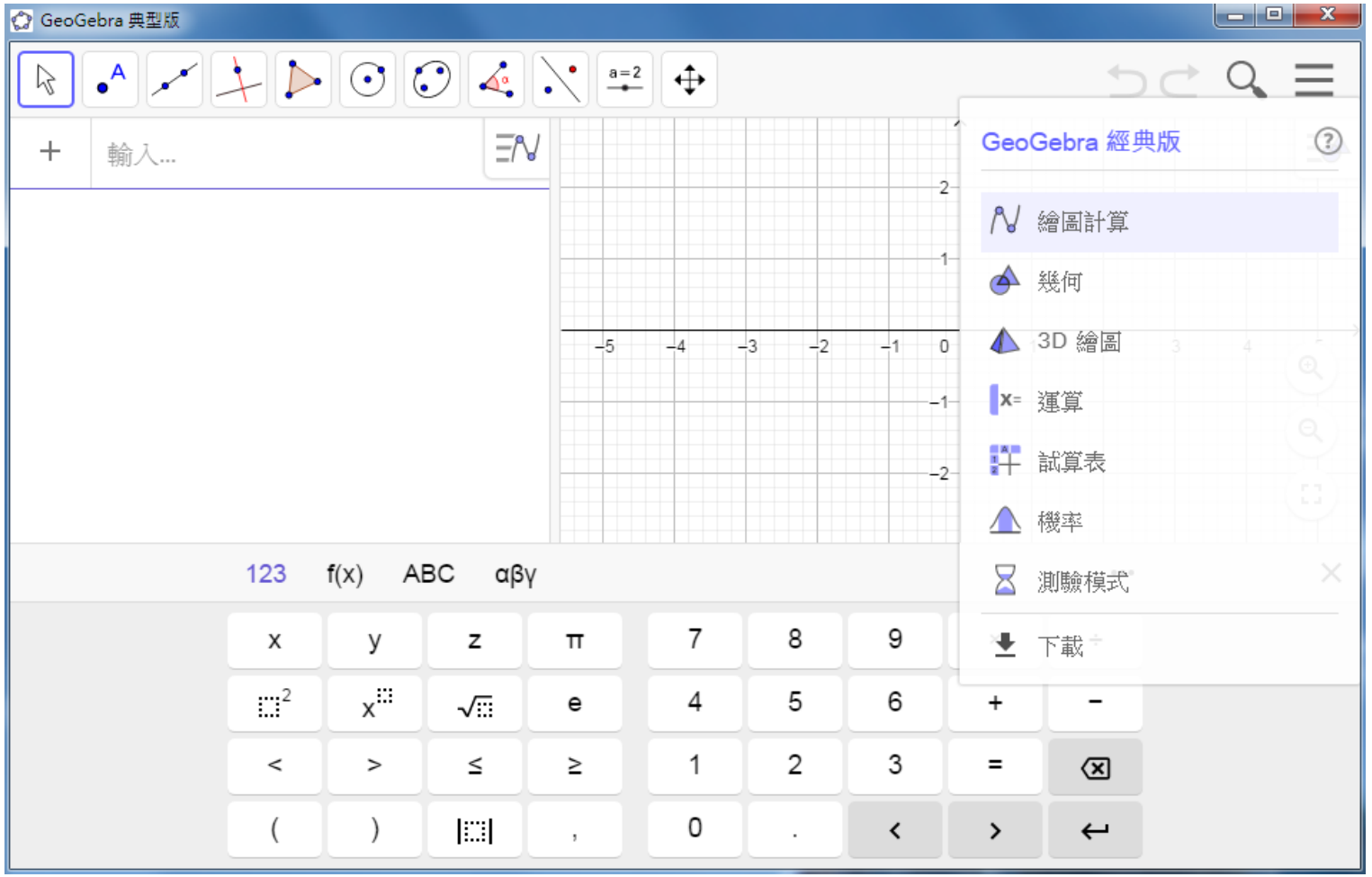


Comparison of GeoGebra Math Apps

apps / features	 Scientific	 Graphing	 Geometry	 Suite	 3D	 CAS	 Classic
Numeric calculations	✔	✔	✔	✔	✔	✔	✔
Function operations	✔	✔	✔	✔	✔	✔	✔
Fraction operations	✔	✔	✔	✔	✔	✔	✔
Graphing		✔	✔	✔	✔	✔	✔
Sliders		✔	✔	✔	✔	✔	✔
Vectors and matrices		✔	✔	✔	✔	✔	✔
Table of values		✔		✔		✔	✔
Geometric constructions			✔	✔	✔		✔
3D graphing				✔*	✔		✔
Symbolic calculations				✔*	✔	✔	✔
Derivatives & integrals				✔	✔	✔	✔
Equation solving				✔	✔	✔	✔

*coming soon

GeoGebra: Books

- Mathematical Modeling: Applications with GeoGebra
  - Hall, J., Lingefjärd T. (2016). *Mathematical Modeling: Applications with GeoGebra*. New York: Wiley. [568 pages]
- GeoGebra - 幾何與代數的美麗邂逅
  - 羅驥韡（2013）。《GeoGebra 幾何與代數的美麗邂逅》。臺北市：五南。

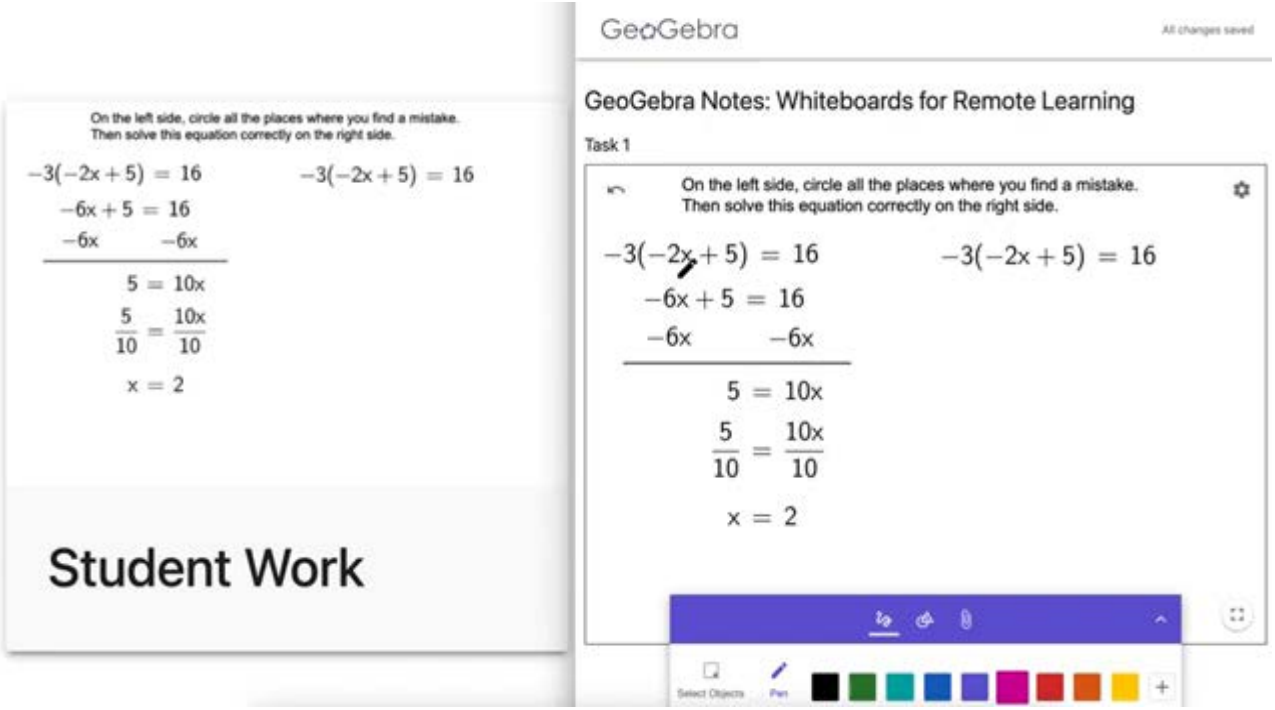
GeoGebra Classroom

GeoGebra Classroom is a virtual platform through which teachers can

- assign interactive and engaging tasks for students
- view live updated progress of students working on a specific task
- view which tasks students have (or have not) started
- ask the entire class questions and see all student answers instantly
- hide student names when displaying student responses to questions
- facilitate rich, interactive discussions among all students, groups of students, and individual students

Virtual Whiteboards for GeoGebra Classroom

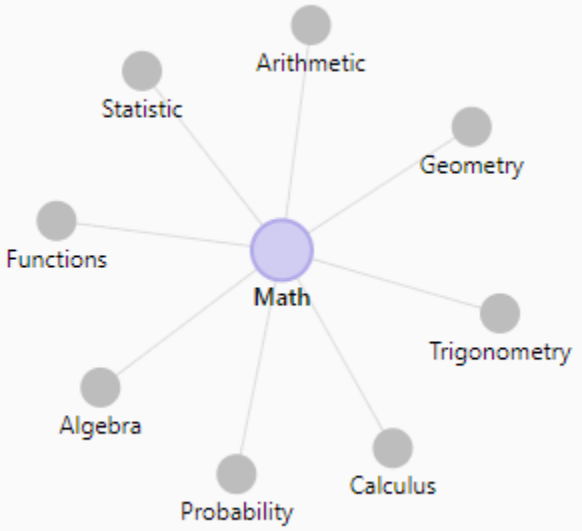
- https://youtu.be/jGFyXc1qRGw

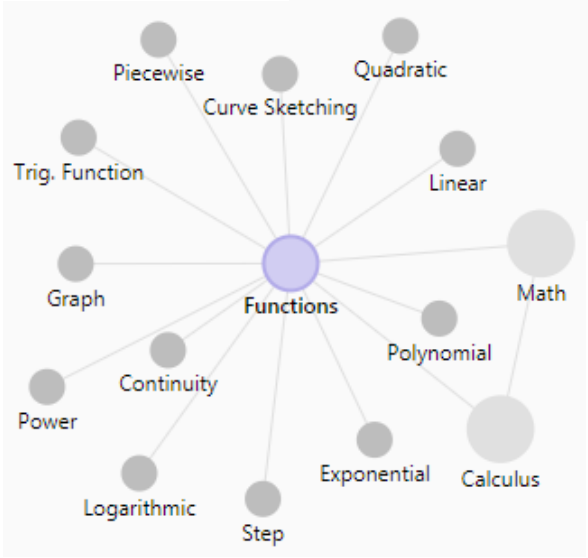


- Learn Notes <https://www.geogebra.org/m/fp7bctpr>

GeoGebra: Resources

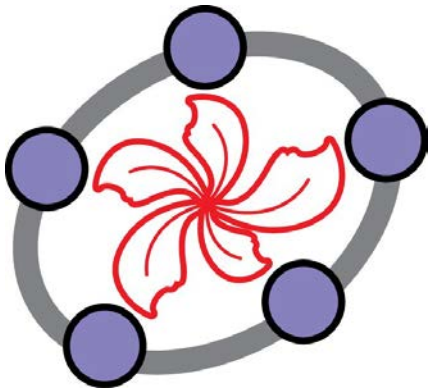
- Find over 1 million free activities, simulations, exercises, lessons, and games for math & science!
  - https://www.geogebra.org/materials
  - https://www.geogebra.org/t/math
  - https://www.geogebra.org/m/bgmn44x5
- Tutorials:
  - https://wiki.geogebra.org/en/Tutorials
  - https://www.geogebra.org/a/14





GeoGebra: Resources

- GeoGebra Institutes
  - https://www.geogebra.org/institutes



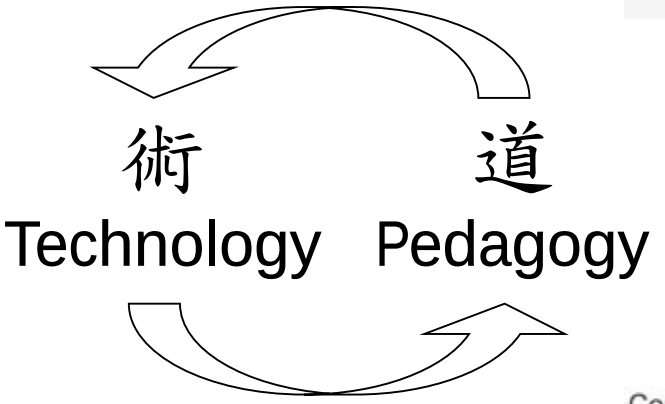
- GeoGebra Institute of Hong Kong (GIHK)
  - http://www.geogebra.org.hk

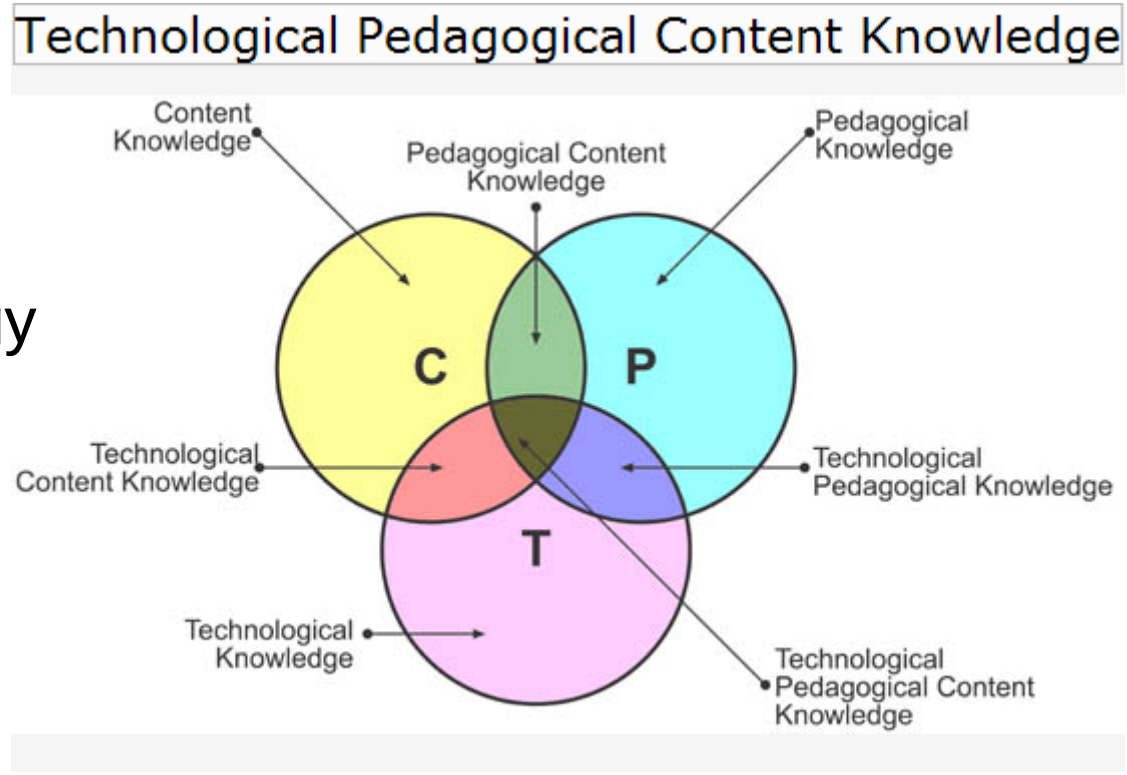
- Applets in Tablets: GeoGebra 數學電子教室
  - https://www.gmath.hk/
  - http://www.geogebra.hk/
  - http://www.geogebra.hk/reference

  - (Learning GeoGebra from Examples) https://www.geogebra.org/m/FZFjhBaa
  - (Graph Plotter 2018) https://www.geogebra.org/m/wQtHUaaa

TPACK

- Technological pedagogical and content knowledge (TPACK) refers to teachers integrating technology with pedagogy (teaching methods) and content.
  - https://julianaliebke.wordpress.com/literature-review/





How GeoGebra (and other IT tools) may enrich classroom L&T?

Mathematics Education Section

Advantages of GeoGebra

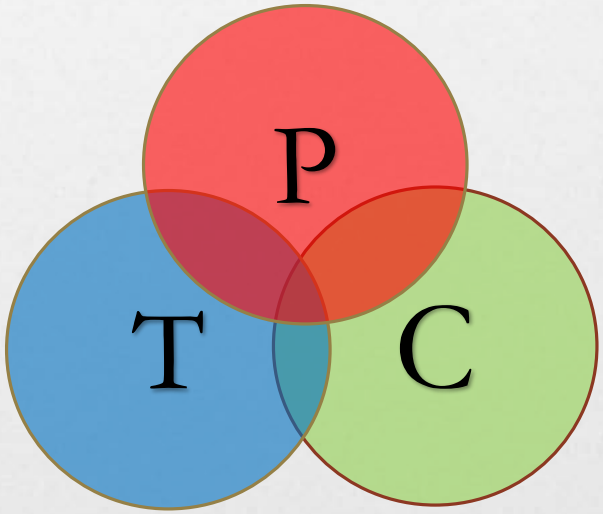
- Dynamic
- Extremely rich in functions
- Fast to observe and conclude

Disadvantages of GeoGebra

- Dynamic without knowing the mechanisms
- Too rich in functions without knowing the focus
- Too fast to observe and conclude only by superficial observations

Key question

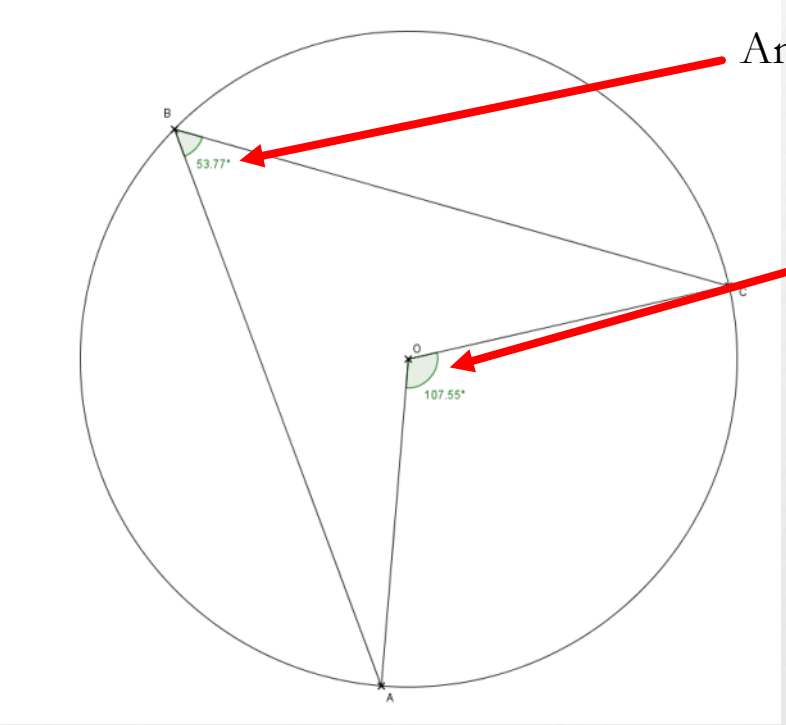
- What to teach?
- How to use?
- How to **refine**?



L&T in Properties of Circles

- A refinement process on L&T package using dynamic geometry software
- A theorem on the properties of circles

The theorem

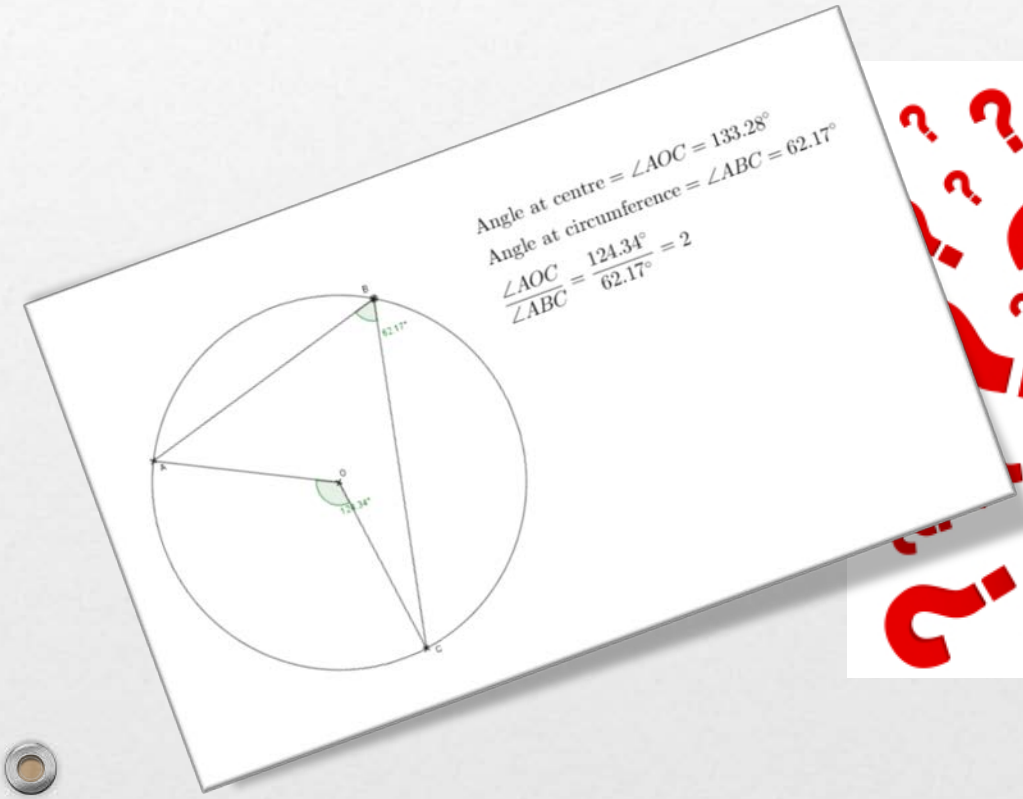


Angle at circumference

Angle at centre

"Angles at centre twice angles at circumference"

The proof


Angle at centre = $\angle AOC = 133.28°$
Angle at circumference = $\angle ABC = 62.17°$
$\frac{\angle AOC}{\angle ABC} = \frac{124.34°}{62.17°} = 2$
B
62.17°
A
O
124.34°
C

Does it work?

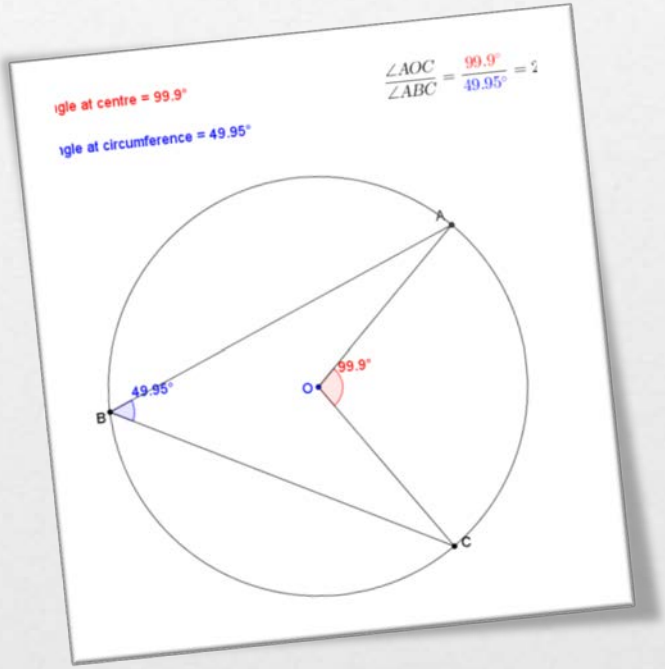
Pedagogical considerations?

How can a task aim at the difficulties?

What are students' difficulties?

Refinement:
From visualisation to abstraction

Understand the limitations

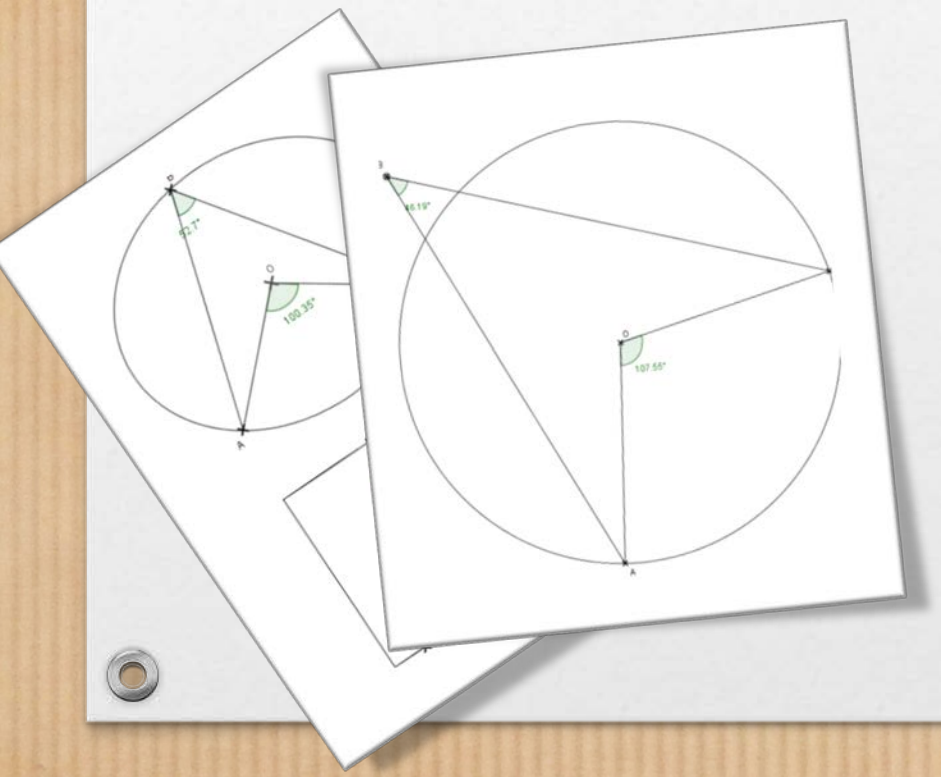


Dynamic geometry vs Euclidean Geometry?

Computation work vs Mind work

Refinement:
From visualisation to abstraction

Stretch the potential



Re-reading a geometric theorem:
In a circle, an angle at any points of the circumference is half of the angle subtended by the same arc of the circle at the centre.

Conditions and consequences:
Pedagogy of variations

A complete cycle of teaching

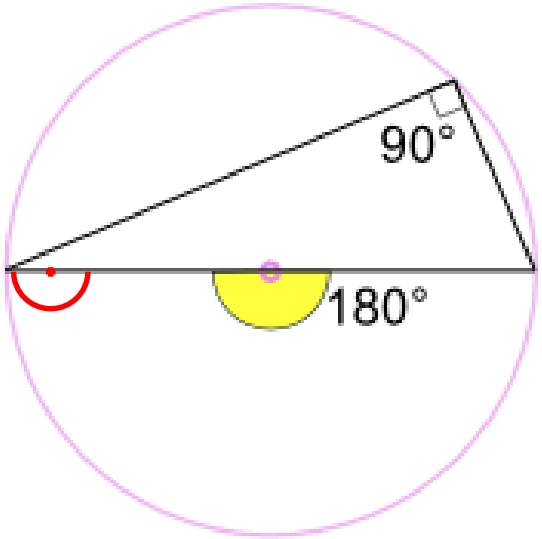
- Examples versus verifications
- Counter-examples versus constraints

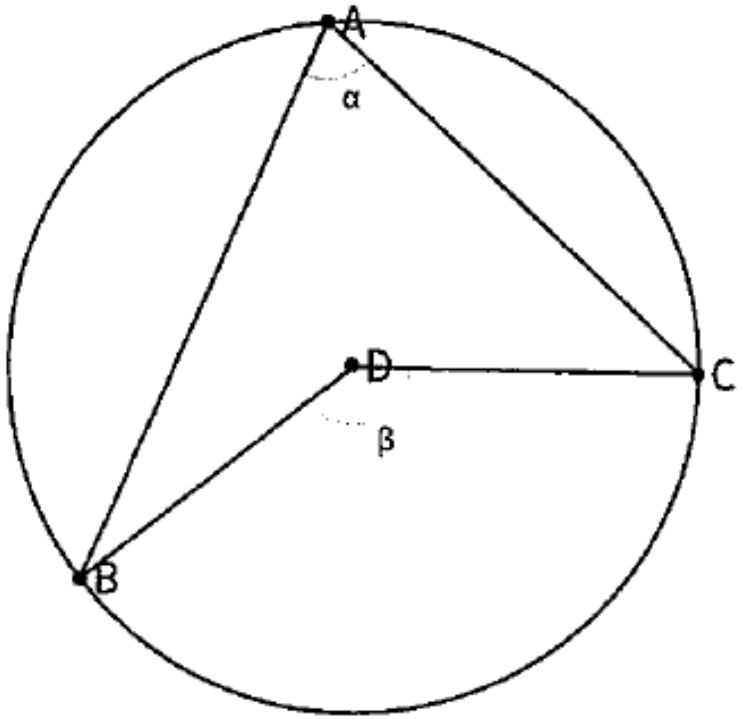
Re-visit the theorem: Think about it

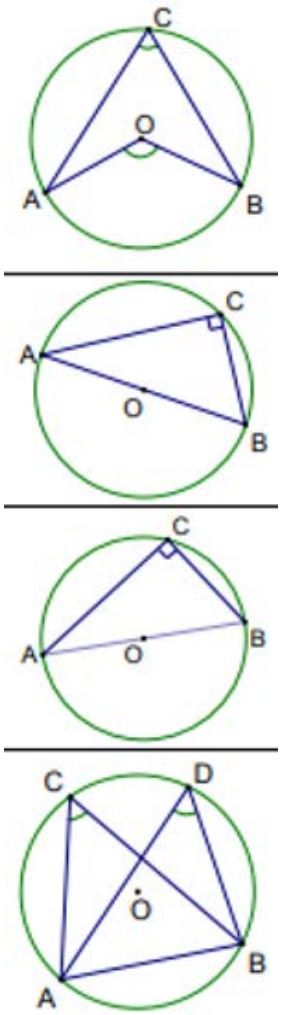
Section D: Think about it

Question 2

Cody claims that if $\beta = 2\alpha$, D must be the centre of the circle. Do you agree? Explain your answer.

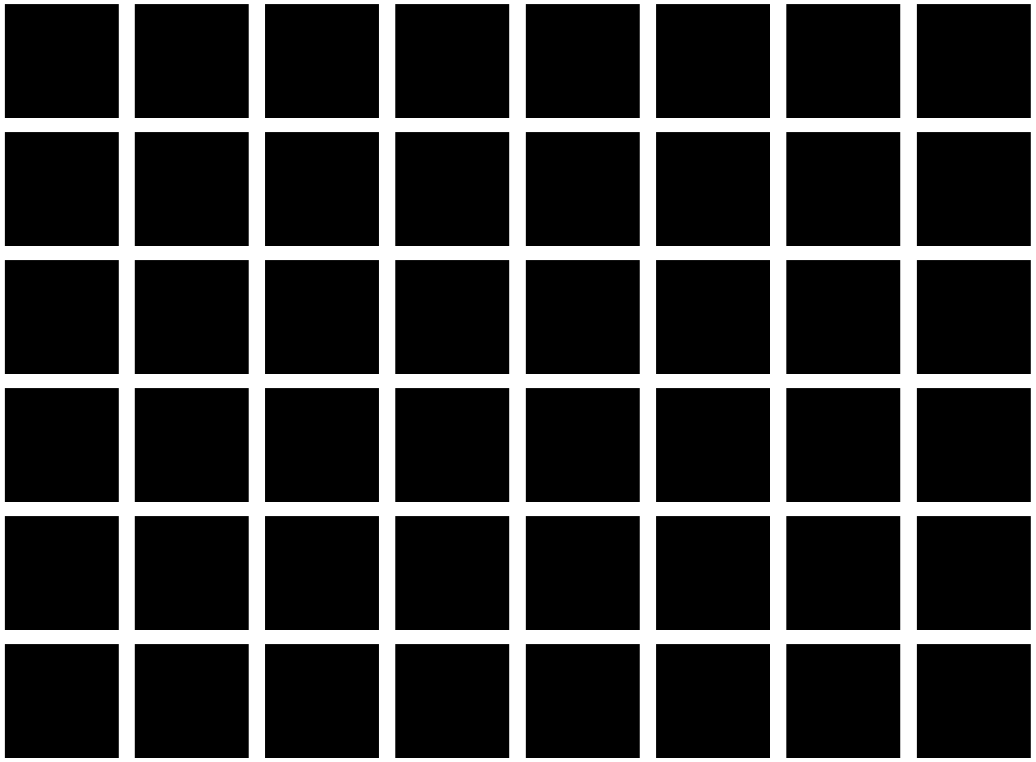




學習重點	時間	注釋
11.2 理解圓上角的性質 		圓上角的性質包括： • 一弧所對的圓心角為該弧所對的圓周角的兩倍 • 同弓形內的圓周角皆相等 • 弧與所對的圓周角成正比例 • 半圓內的圓周角為直角 • 若圓周角是一直角，則其所對的弦是一直徑

Thanks!
https://www.geogebra.org/u/cdoma7

- 多多指教！
- https://www.geogebra.org/u/cdoma7

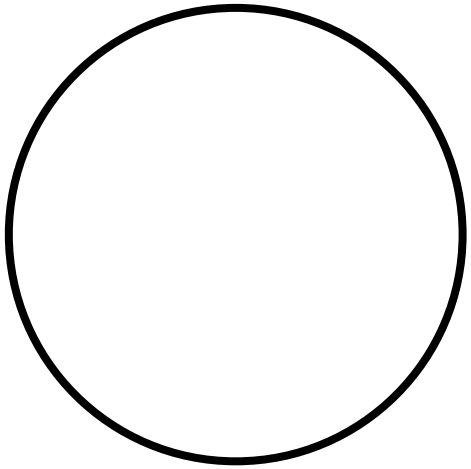


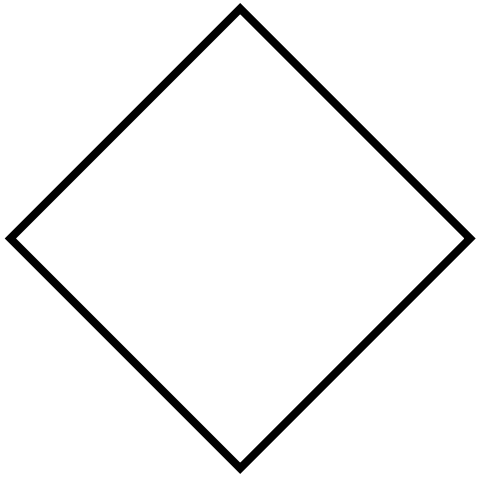
Hermann Grid 赫曼方格

Mr. Hui SL

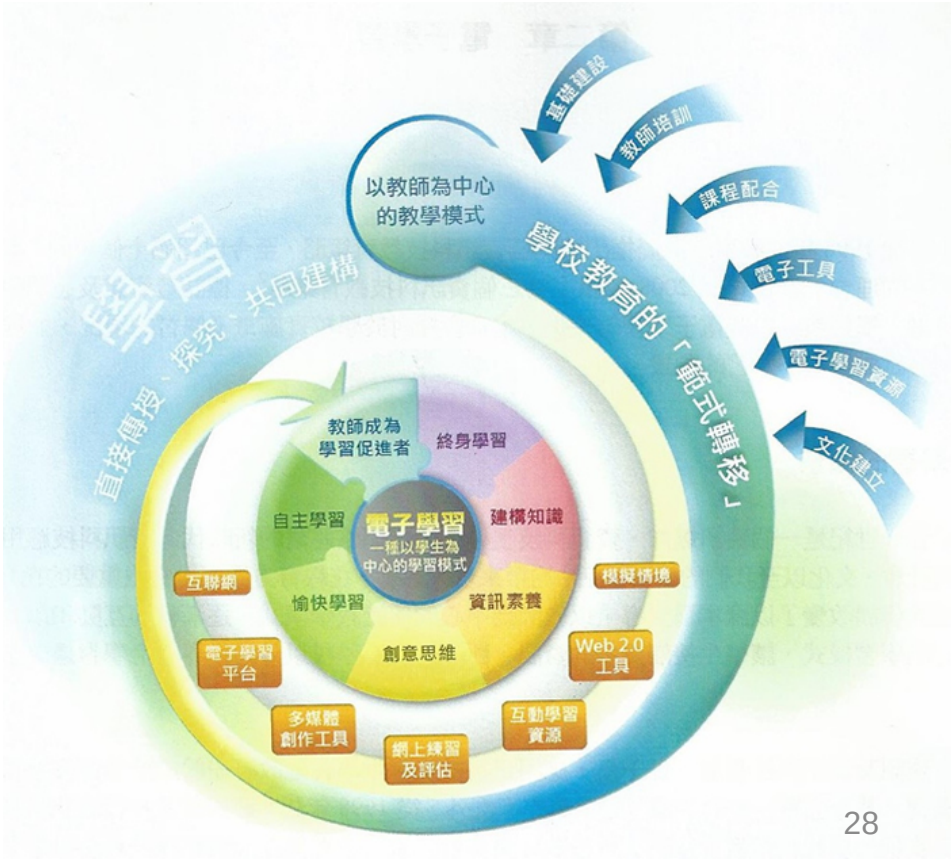
圓 = 圓，方 = 方，圓 ≠ 方！

- 化圓為方？（三大幾何問題）



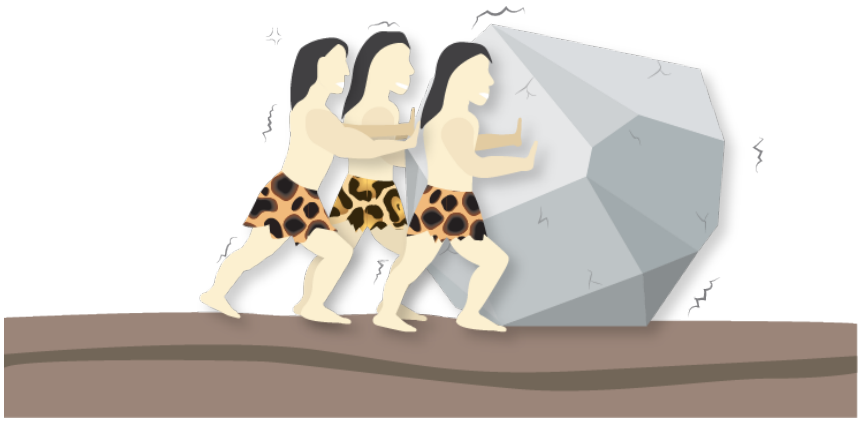


To conclude .....


基礎建設
教師培訓
課程配合
電子工具
電子學習資源
文化建立
學校教育的「範式轉移」
以教師為中心的教學模式
學習
直接傳授、探究、共同建構
教師成為學習促進者
終身學習
自主學習
電子學習
一種以學生為中心的學習模式
建構知識
互聯網
模擬情境
愉快學習
資訊素養
電子學習平台
創意思維
Web 2.0 工具
多媒體創作工具
互動學習資源
網上練習及評估

Technology = TOOL

Right technology at the Right time for the Right task



Thanks!
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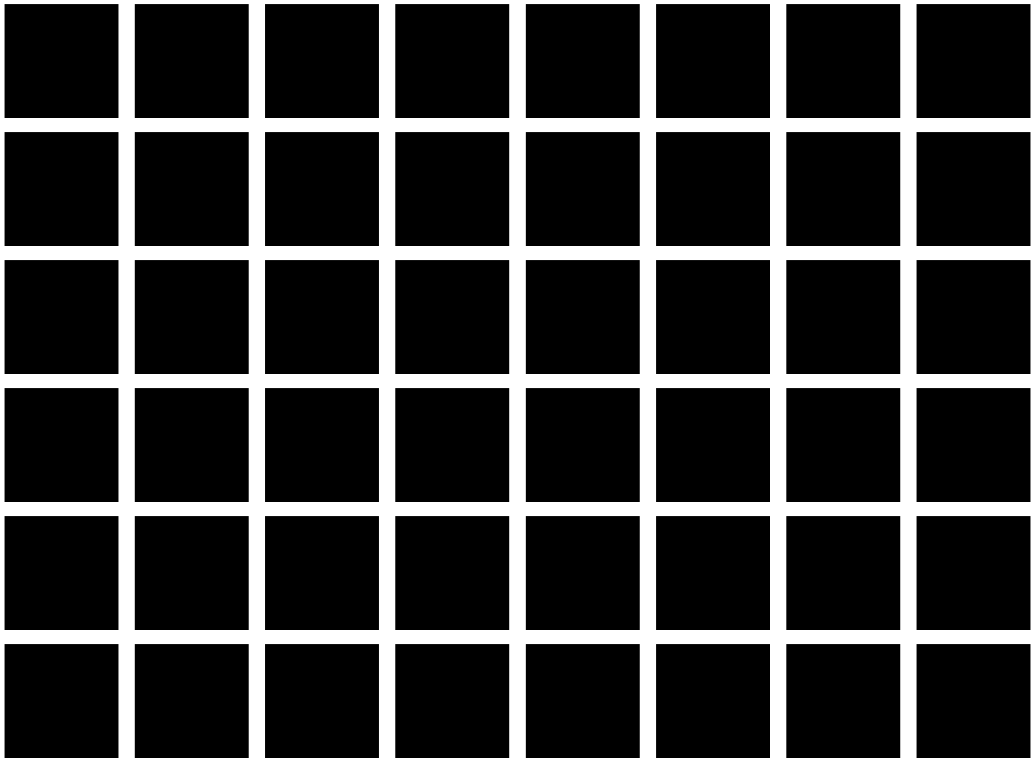

Hermann Grid