Integration by substitution

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Complete the following integrals by using a suitable sunstitution.
Use the Geogebra App to check your answers.

1. $\int 2x\sqrt{(1-x^2)}dx$

2. $\int x^3e^{3x^4}dx$ Use $\exp(3x^4)$ for e^{3x^4}

3. $\int \frac{1}{x\ln x}dx$

4. $\int \sin^2 x\cos x dx$

5. $\int 2x\sqrt{x^2-5}dx$

6. $\int 6xe^{x^2}dx$

7. $\int \frac{\sin 2x}{(3+\cos 2x)^3}dx$

8. $\int \sec^2 x\tan^2 x dx$

9. $\int \sec^2 x(1+\tan^2 x)dx$

10. $\int \frac{\sin x\cos x}{\sqrt{(\cos 2x+3)}}dx$

11. $\int \frac{x}{x^2-1}dx$

12. $\int \sec^2 x e^{4\tan x}dx$

13. $\int \tan x dx$

14. $\int 3x^2 e^{x^3}dx$

15. $\int \frac{e^{-\sqrt{x}}}{\sqrt{x}}dx$

Evaluate the following definite integrals. Use the Geogebra App to check your answers.

16. $\int_0^1 2x\sqrt{(1-x^2)}dx$ Answer $[\frac{2}{3}]$

17. $\int_0^1 x^3 e^{3x^4}dx$ Answer $[\frac{e^3-1}{12}]$

18. $\int_e^{e^2} \frac{1}{x\ln x}dx$ Answer $[\ln 2]$

19. $\int_{\pi/6}^{\pi/2} \sin^2 x\cos x dx$ Answer $[\frac{7}{24}]$

20. $\int_3^4 2x\sqrt{x^2-5}dx$ Answer $[\frac{22\sqrt{11}-16}{3}]$

21. $\int_1^2 6xe^{x^2}dx$ Answer $[3e(e^3-1]$

22. $\int_0^{\pi/4} \frac{\sin 2x}{(3+\cos 2x)^3}dx$ Answer $[\frac{7}{576}]$

23. $\int_0^{\pi/3} \sec^2 x\tan^2 x dx$ Answer $[\sqrt{3}]$

24. $\int_0^{\pi/3} \sec^2 x(1+\tan^2 x)dx$ Answer $[2\sqrt{3}]$

25. $\int_{\pi/4}^{\pi/2} \frac{\sin x \cos x}{\sqrt{(\cos 2x+3)}}dx$ Answer $[\frac{\sqrt{3}-\sqrt{2}}{2}]$

26. $\int_3^5 \frac{x}{x^2-1}dx$ Answer $[\frac{\ln 3}{2}]$

27. $\int_{\pi/4}^{\pi/3} \sec^2 x e^{4\tan x}dx$ Answer $[\frac{e^{4\sqrt{3}}-e^4}{4}]$

28. $\int_0^{\pi/3} \tan x dx$ Answer $[\ln 2]$

29. $\int_{-1}^{1} 3x^2 e^{x^3}dx$ Answer $[\frac{e^2-1}{e}]$

30. $\int_0^{100} \frac{e^{-\sqrt{x}}}{\sqrt{x}}dx$ Hint : let $u = -\sqrt{x}$ Answer $[2]$